Dynamic Modeling and Vibration Control for Spacecraft's Solar Array

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Abstract. Dynamic modeling and active vibration control for a typical flexible solar array of spacecraft by using piezoelectric actuators is discussed. It is different from the simplified beam or plate model for the solar array in the literate, a coupled dynamic model of the solar array with piezoelectric actuators is developed firstly. Especially, a four-node rectangular piezoelectric plate element is presented to discretize the smart solar array. Secondly, a LQR controller is designed to control the vibration of the solar array. Numerical simulation shows that the vibration of solar array could be suppressed efficiently by using piezoelectric actuators.

Introduction

Solar array is the very important part of a spacecraft, which will supply the electric power. However, the spacecraft's solar array could be very large and flexible, whose modal frequencies and damping ratios are relatively low. In order to meet the higher precision requirement of spacecraft, the vibration control of solar array becomes more and more important [1].

In recent years, smart materials, such as piezoelectric materials, have been used extensively as distributed sensors and actuators for vibration control of flexible structures. The active suppression of vibration for spacecraft flexible booms using piezoelectric actuators was discussed in Ref.[2]. Hu and Ma[3] presented an approach to vibration reduction of flexible spacecraft actuated by on-off thrusters, by applying the variable structure control technique. The spacecraft to be investigated is a hub with a cantilever flexible beam appendage. Garcia [4] developed an active control method to suppress vibration of flexible ribbed antenna structures using piezoceramic components as both sensor and actuator simultaneously.

However, most published papers in the area of active vibration control for the flexible appendages of spacecraft are based on dynamic models of cantilever beam or plate structures[2]~[9]. Actually, most of the solar arrays of spacecraft are multi-panel deployable structures. This deployable solar array often employed rigid honeycomb panels interconnected with spring-driven hinges, binding mechanism and electrical harnessing. Thus, a simplified beam or plate model does not predict the dynamic characteristic accurately of the solar array system.

This paper is aim to study the vibration control of a typical deployable solar array using surface bonded piezoelectric actuators. Firstly, a coupled dynamic model of the solar array with piezoelectric actuators is developed based on the Hamilton's principle. After that, a vibration controller for the smart solar array is presented. Numerical results demonstrate the validity and efficiency of proposed design scheme. Finally, some concluding remarks are drawn.

Dynamic Models of the Smart Solar Array

Let us consider a typical multi-panel deployable solar array illustrated in Fig.1, which consists of solar panels, tripod, hinges and piezoelectric actuators. Each actuator consists of two piezoelectric patches bonded to opposite faces of the panel. The outer electrodes of the patches are electrically connected together and the plate, which is grounded, is used as the other electrode for both patches of the actuator pair. In this configuration, when an external voltage is applied to the patches, one patch will expand and the other contract, thus producing a curvature in the panel. The vibration of the solar panel will be suppressed in this way.

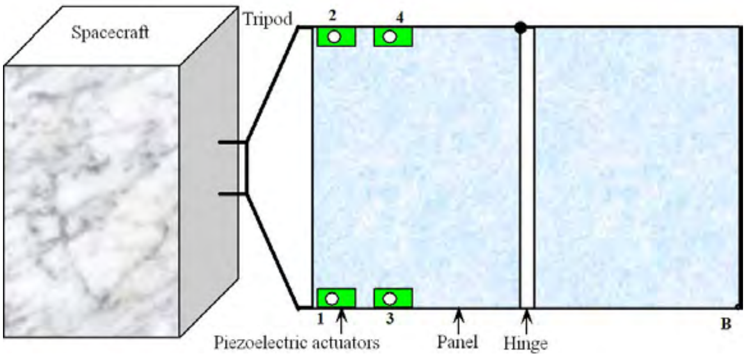

Fig. 1 A Typical Spacecraft'S Smart Solar Array

In order to develop the dynamic model of the multi-body deployable structure, the finite element method is employed. Thus, the tripod and hinges of the solar array are modeled by using a two-node 3D beam element. This element has six degrees of freedom (DOFs) at each node, which stiffness and mass matrices can be found in many textbook.

The composite panel of solar array with surface bonded piezoelectric material becomes typical laminated plate. In order to formulate the governing equations for this laminated plate, the following assumptions are adopted: (a) the plate is thin and piezoelectric actuators are integrated into the laminated composite substrate as plies; (b) the laminate is perfectly bonded, elastic and orthotropic in behavior with small strains and displacements; (c) piezoelectric actuators are made of homogenous and isotropic dielectric materials and high electric fields and cyclic fields are not involved.

Based on the classical laminated plate theory (CLPT), the displacement field of the laminated plate will be written as

$$u(x,y,z)=u_0(x,y)-z\,\partial w/\partial y, \qquad v(x,y,z)=v_0(x,y)+z\,\partial w/\partial x, \qquad w(x,y,z)=w_0(x,y) \tag{1}$$

where (u_0, v_0, w_0) are the displacement of the point on the reference plane of the laminate plate. An assumption of this theory is that each layer undergoes the same displacement as the mid-plane of the laminated plate, which also means the displacement filed vector would be the same for the each layer. According to this, the strain equations would be written as

$$\varepsilon_{xx}=\frac{\partial u_0}{\partial x}-z\frac{\partial^2 w}{\partial x^2}, \qquad \varepsilon_{yy}=\frac{\partial v_0}{\partial y}-z\frac{\partial^2 w}{\partial y^2}, \qquad \gamma_{xy}=\left(\frac{\partial u_0}{\partial y}+\frac{\partial v_0}{\partial x}\right)-2z\frac{\partial^2 w}{\partial x\partial y} \tag{2}$$

Based on these assumptions a linear constitutive relationship is adopted for the piezoelectric composite panel, which can be expressed as

$$\boldsymbol{\zeta}=\bar{\boldsymbol{Q}}\bar{\boldsymbol{\varepsilon}}-\bar{\boldsymbol{e}}\bar{\boldsymbol{E}} \tag{3}$$

$$\bar{\boldsymbol{D}}=\bar{\boldsymbol{e}}^T\bar{\boldsymbol{\varepsilon}}+\bar{\boldsymbol{p}}\bar{\boldsymbol{E}} \tag{4}$$

where $\boldsymbol{\zeta}=\left[\sigma_{xx}\ \sigma_{yy}\ \sigma_{xy}\right]^T$ is the elastic stress vector and $\bar{\boldsymbol{\varepsilon}}=\left[\varepsilon_{xx}\ \varepsilon_{yy}\ \gamma_{xy}\right]^T$ the elastic strain vector, $\bar{\boldsymbol{Q}}$ the symmetric elastic constitutive matrix, $\bar{\boldsymbol{e}}$ the piezoelectric stress coefficients matrix, $\bar{\boldsymbol{E}}=\left[\bar{E}_x\ \bar{E}_y\ \bar{E}_z\right]^T$ the electric field vector, $\bar{\boldsymbol{D}}=\left[\bar{D}_x\ \bar{D}_y\ \bar{D}_z\right]^T$ the electric displacement vector and $\bar{\boldsymbol{p}}$ the symmetric dielectric matrix, in the element local system (x, y, z) of the laminate.

A four-node rectangular plate element with five degrees of freedom at each node is used. We denote the nodal displacement vector of each node i by $\boldsymbol{\delta}_i^T = \left[u_i, v_i, w_i, \theta_{xi}, \theta_{yi}\right]$ and denote the displacement vector of all nodes in the element by $\boldsymbol{q}_p^e = \left[\boldsymbol{\delta}_1^T \quad \boldsymbol{\delta}_2^T \quad \boldsymbol{\delta}_3^T \quad \boldsymbol{\delta}_4^T\right]^T$. Consequently, the displacement vector of any point within an element can be expressed in terms of all nodal displacement by means of interpolation functions $\boldsymbol{N}_i$ such as

$$\boldsymbol{u} = \left[u_0 \; v_0 \; w_0 \; \theta_x \; \theta_y\right]^T = \sum_{i=1}^{4} \boldsymbol{N}_i \boldsymbol{\delta}_i = \boldsymbol{N}\boldsymbol{q}_p^e \tag{5}$$

Therefore, the strains within an element can be expressed in terms of all nodal displacements in the element as

$$\bar{\boldsymbol{\varepsilon}} = \left[\varepsilon_{xx} \; \varepsilon_{yy} \; \gamma_{xy}\right]^T = \left(\boldsymbol{B}_0 + z\boldsymbol{B}_\kappa\right)\boldsymbol{q}_e = \boldsymbol{B}\boldsymbol{q}_p^e \tag{6}$$

The electrical potentials ϕ are assumed constant for each piezoelectric layer within each element. Hence there is one additional potential degree of freedom for each piezoelectric layer to represent the piezoelectric behavior and the vector of electrical degrees of freedom is

$$\boldsymbol{\varphi}^e = \left\{\cdots \phi_i \cdots\right\}^T \quad i = 1, \cdots, n \tag{7}$$

The electric field vector can be represented for an arbitrary element

$$\bar{\boldsymbol{E}} = -\boldsymbol{B}_\phi \boldsymbol{\varphi}^e \tag{8}$$

where $\boldsymbol{B}_\phi$ is the electric-potential matrix relating the electric field vector with the potentials.

By using the generalized Hamilton's principle, the finite element equations of motion for the laminated plate with coupled electromechanical properties can be obtained.

$$\begin{aligned} &\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} + \boldsymbol{K}_{u\varphi}\boldsymbol{\varphi} = \boldsymbol{F} \\ &\boldsymbol{K}_{\varphi u}\boldsymbol{q} - \boldsymbol{K}_{\varphi\varphi}\boldsymbol{\varphi} = \boldsymbol{Q} \end{aligned} \tag{9}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{K}$ is the elastic stiffness matrix, $\boldsymbol{K}_{\varphi\varphi}$ is the dielectric 'stiffness' matrix and $\boldsymbol{K}_{u\varphi} = \boldsymbol{K}_{\varphi u}^T$ are the coupling matrices between elastic mechanical and electrical effects. $\boldsymbol{q}, \boldsymbol{\varphi}$ are the system generalized displacements vector and voltages at actuator, respectively. $\boldsymbol{F}, \boldsymbol{Q}$ are the mechanical loads vector and the applied electrical charges, respectively.

Vibration Control of Smart Solar Array

An optimal controller is designed to minimize a cost function, or performance index, which is proportional to the required measure of the system's response and to the control inputs required to attenuate the response, such as the linear quadratic regulator (LQR). The feedback gain $\boldsymbol{G}$ is chosen to minimize a quadratic performance index of the form

$$J = \frac{1}{2}\int_0^\infty \left(\boldsymbol{x}^T \boldsymbol{Q}\boldsymbol{x} + \boldsymbol{\varphi}^T \boldsymbol{R}\boldsymbol{\varphi}\right) dt \tag{10}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are symmetric positive semi definite weighting matrices, and their elements are selected to provide suitable performance.

Assuming the control voltage $\boldsymbol{\varphi} = -\boldsymbol{G}\boldsymbol{x}$, then the determination of control gain $\boldsymbol{G}$ can be reduced to solving a matrix Riccati equation

$$A^T P + PA - PBR^{-1}B_\varphi^T P + Q = 0 \tag{11}$$

Thus the control gain for state feedback is then obtained by $G = R^{-1}B^T P$.

The weighting matrix Q for the modal space quadratic performance index is usually assumed to be diagonal and takes the form $Q = \mathrm{diag}\left(\omega_1^2, \cdots \omega_r^2, 1, \cdots, 1\right)$. However, there no general guidelines for the choice of R, the weighting matrix for control forces. A diagonal R, as assumed in most applications, may be written as $R = \mathrm{diag}\left(r_1, r_2 \cdots .r_m\right)$.

Numerical Applications

In this section, the accuracy of the coupled dynamic model is demonstrated firstly by comparing with the results from the Nastran software. Then, the efficiently of suppressing the vibration induced by an initial displacement is evaluated. The piezoelectric actuators are made of PZT-5H piezoelectric ceramics, which properties can be given in [10]. The geometry size of each actuator is 300×100×1 mm^3. The finite element model includes 40 3D beam elements, 300 plate elements and 388 nodes. Uniform structural damping with a typical value of 0.5% is assumed. Vertical displacement at point B (as shown in Figure 1) is selected as the performance output.

Table 1 shows the first six natural frequencies of the solar array from the proposed model and the Nastran software. Furthermore, the frequencies of the smart solar array with piezoelectric actuators are also listed in the Table 1. It tells us that the present model provides quite good results, and the piezoelectric actuators would change the natural frequency. Especially, the maximize difference of the first frequency is 13.78% because of considering the four piezoelectric actuators. Thus, the accurate dynamic model should include the influence of the piezoelectric patches.

Tab. 1 The First Six Natural Frequencies from Nastran Software and the Proposed Model

Number	Nastran	Present model without actuators	Difference	Present model with actuators	Difference
1	0.2344	0.2401	2.43%	0.2667	13.78%
2	0.7276	0.7274	-0.027%	0.7247	-0.39%
3	1.3366	1.364	0.02%	1.4034	4.99%
4	1.3499	1.3781	0.02%	1.4064	4.19%
5	3.777	3.8069	0.79%	3.7410	-0.95%
6	4.1565	4.2007	0.01%	4.1312	-0.61%

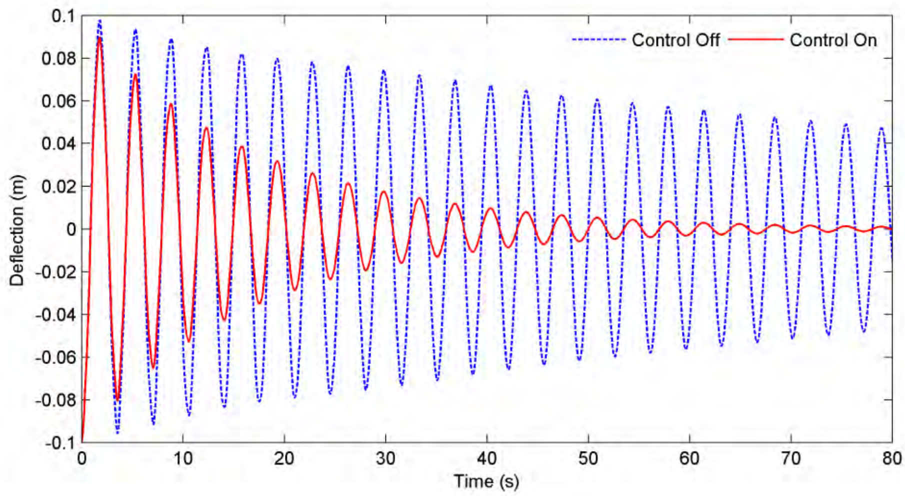

Fig. 2 Tip Displacement of the Open and Closed Loop System

Consider an initial displacement field applied to the solar array with a tip displacement equal to 0.1m. Transient responses of the solar array tip displacement (at point B shown in Fig.1) are shown in Fig.2. Results when the control is off are also shown for comparison. Corresponding applied actuation voltages are plotted in Figure 3.

As can be seen in Fig. 2, the LQR controller manages to significantly attenuate the free tip displacement with an admissible control voltage. The closed-loop 5% settling time is equal to 52s, which reveals a great improvement on the response attenuation when compared with the open-loop one. Note that structure natural settling time corresponds to the state when active control is turned off, and the structure vibration is solely suppressed by structural damping (0.5% ratio in the present case). Higher damping ratio will result in shorter settling time. However, the damping ratio of large flexible space structures is usually low.

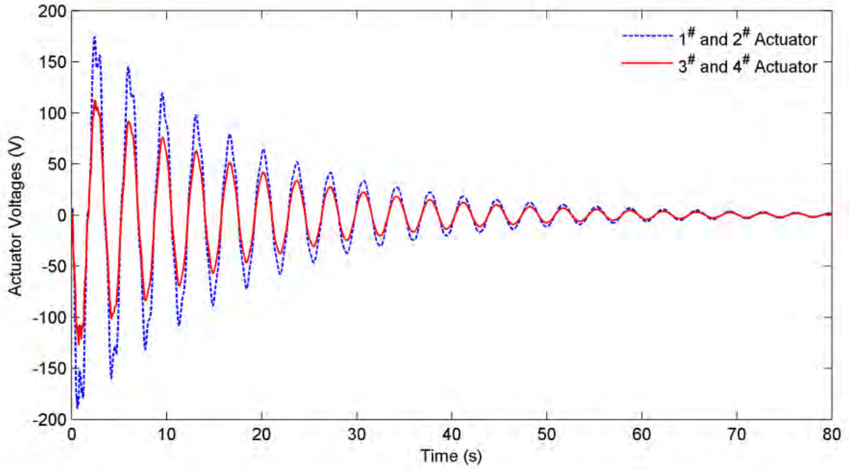

Fig.3 The Control Voltages of the Piezoelectric Actuatros

The maximum actuator voltages are 189V for actuator 1, 2 and 127V for actuator 3, 4, respectively. Because of the symmetry about x-axis, actuators 1 and 2, 3 and 4, have identical control voltages response. In practical design practice, it is important to ensure that the maximum voltages on actuators are within certain ranges so that piezoelectric patches will not experience electric breakdown. Note that these ranges are predefined based on material properties and device characteristics. If the linear piezoelectric constitutive equations are used to design the structures as in present study, it is also imperative to restrict actuators within the linear range. The maximum actuation voltage in above two cases is less than 200V, which does not exceed the linear range of most piezoelectric ceramics.

Fig 4 presents the frequency response (bode plot) at the solar array tip (point B in Fig.1). A 20 dB and 10 dB attenuation is achieved with the LQR controller for the first and second resonant frequencies, respectively. It is also demonstrated the efficiently of the proposed active vibration scheme.

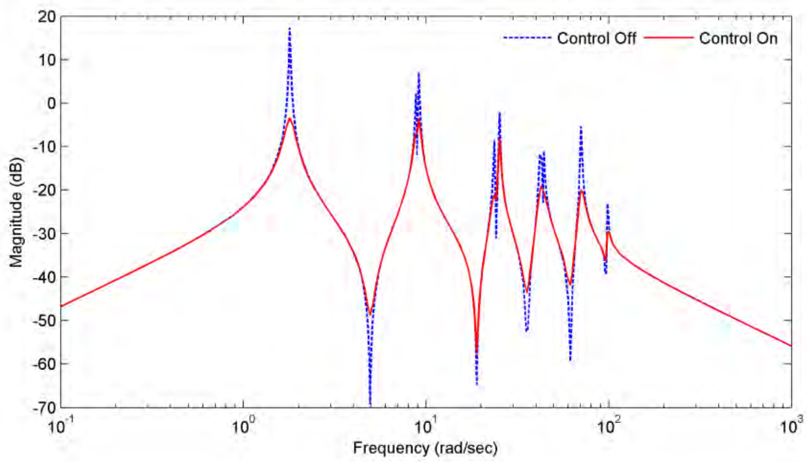

Fig. 4 The Frequency Response of the Open and Closed Loop System

Conclusions

This paper discusses the coupled dynamic model and vibration control of the solar array with piezoelectric actuators. Towards this end, the solar array bonded piezoelectric actuators is modeled using finite element methods. The controller design is carried out by using LQR. Numerical results demonstrate the accurately and validity of the presented dynamic model and the control scheme. Especially, it is different from the simplified beam or plate model for the solar array in the literate, the coupled dynamic model is based on the multi-body deployable structures, and the influence of the piezoelectric actuators is considered also. The vibration control experiments of the smart solar array will be carried out to demonstrate the efficiently of the proposed model and control scheme.

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