Partial Transmit Sequence (PTS)-PAPR Reduction Technique in OFDM Systems with Reduced Complexity

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Abstract
OFDM systems have the inherent problem of a high peak to average power ratio (PAPR). OFDM Suffers as the no of Subcarriers operating in the large dynamic range operates in the non-linear region of amplifier due to OFDM suffer the PAPR problem. Application of high power amplifiers results in increased component cost. In general, there has been a trade-off between PAPR reduction and computational complexity in partial transmits sequence (PTS) OFDM. The complexity reduction of PTS PAPR reduction scheme in OFDM systems by reducing the complexity of the IFFT architecture is investigated in this paper. In the IFFT architecture of PTS OFDM scheme, there are a lot of additions and multiplications with zero, which are obviously unnecessary. We can efficiently reduce the computational complexity without changing the resulting signal or degrading the performance of PAPR reduction by eliminating the additions and multiplications with zero from the architecture. In this paper PTS SUB-BLOCKS PAPR reduction techniques have been proposed and analyzed.

Keywords:-OFDM, PAPR, PTS, Interleaved PTS, Adjacent PTS, Sub-optimal PTS.

I. Introduction
In broadband wireless communications high bit rate transmission is required for high quality communications. OFDM is a very attractive technique for high speed data transmission over multipath fading channels. The PAPR problem is one of the most important issues for developing multicarrier transmission systems [1]. Recently many works have been done in developing a method to reduce the PAPR. The simple and widely used method is clipping the signal to limit the PAPR below a threshold level. Selected mapping (SLM) [3] and partial transmit sequence (PTS) [4] were proposed to lower the PAPR with a relatively small increase in redundancy but without any signal distortion. In this paper, we focus on PTS OFDM scheme to reduce the computational complexity.

The block diagram of OFDM system is shown in figure 1. The transmit signal can be generated by a simple IDFT operation, which can replace the bank of modulators [2] and at the receiver; a DFT can be performed to recover the transmitted signal. OFDM signal consists of n data symbols transmitted over N0 subcarriers. Let P = { Pk, k=0, 1,2,- ,N0-1} be a block of n data symbols and each symbol modulating a set of subcarriers {fk, k=0,1,- ,N0-1}

Fig1:-Block diagram of OFDM system
PAPR of the OFDM signal $x(t)$ is defined as the ratio between peak power and its average power during the OFDM signal.

The transmitted signal having a complex envelope is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X(n) e^{j2\pi nt}, \quad 0 \leq t \leq T \quad (1)$$

The PAPR of the transmitted signal in (1) is defined by

$$\text{PAPR} = \frac{\text{max} \left| x(t) \right|^2}{E \left| x(t) \right|^2} \quad (2)$$

Where $E \left[ . \right]$ denotes expectation and complementary cumulative distribution function (CCDF) for OFDM signal can be written as $\text{CCDF} = \text{probability (PAPR} > P_0)$, where $P_0$ is the Threshold. PAPR of OFDM signal is mathematically defined as given in

$$\text{PAPR} = 10 \log_{10} \left( \frac{\text{max} \left| x(t) \right|^2}{E \left| x(t) \right|^2} \right) \quad (3)$$

It is easy to manipulate the above equation by decreasing the numerator $\text{max} \left| x(t) \right|^2$ or increasing the denominator $E \left| x(t) \right|^2$ or both [2].

In principle, PAPR reduction techniques are concerned with reducing $\text{max} \left| x(t) \right|$. However, since most systems employ discrete-time signals, the amplitude of samples of $\left| x(t) \right|$ is dealt with in many of the PAPR reduction techniques. Since symbol-spaced sampling of (3) sometimes misses some of the Signal peaks and results in optimistic results for the PAPR, Oversampling by the factor $L=4$ to avoid the ISI. Oversampled time domain samples are obtained by $LN$-point IDFT of the data block with $(L - 1) N$ zero-padding. It was shown in that $L = 4$ are sufficient to capture the peaks [7].

Let us investigate how the PAPR grows with no of sub-carriers. Consider $N$ Gaussian i.i.d random variables $x(n), 0 \leq n \leq N-1$, with mean zero and unit power. The average signal power is $E[a \left| x(n) \right|^2]$ is then

$$E \left[ \frac{1}{N} \left| x(0) + x(1) + \ldots + x(N-1) \right|^2 \right]$$

$$= \frac{1}{N} E \left[ \left| x(0) + x(1) + \ldots + x(N-1) \right|^2 \right]$$

$$= 1$$

The maximum value occurs when $x_i$ s add coherently i.e.

$$\text{max} \left[ \frac{1}{N} \left| x(0) + x(1) + \ldots + x(N-1) \right|^2 \right]$$

$$= \left( \frac{1}{\sqrt{N}} \right)^2 = N$$

Hence the maximum PAPR for $N$ sub-carriers is $N$ [8].

### III. PAPR reduction using PTS

In the PTS scheme as shown in figure 2, the input symbol sequence is partitioned into a number of disjoint symbol subsequences. IFFT is then applied to each symbol subsequence and the resulting signal subsequences are summed after being multiplied by a set of distinct rotating vectors. Next the PAPR is computed for each resulting sequence and then the signal sequence with the minimum PAPR is transmitted. As the number of subcarriers and the order of modulation are increased, reducing the computational complexity becomes more important than decreasing redundancy [4].

![Fig 2: PTS-PAPR reduction scheme](image)
IV. New PTS scheme with reduced complexity

a) Interleaved sub-block partition

Let $X=[X(0) X(1) \ldots \ldots X(N_0-1)]$ is OFDM input symbol sequence with length $N_0$. And $X_0, \ldots \ldots X_{M-1}$ with length $N$ are OFDM symbols after portioning and the $X_0, \ldots \ldots X_{M-1}$ with length $N_0$ after zero padding. Where $M$ is the no. of sub-blocks. Consider that the interleaving partition method is used in PTS OFDM scheme, $X_m$ would be expressed as $X_m=[X(m)X(M+m)]_{1,N}$ and $X_{m-1}$ would be expressed as $X_{m-1}=[0\ldots 0X(m)0\ldots 0X(NM-M+m)]_{1,N}$ where $0\leq m \leq M-1$ and $N=N_0/M$. It is clear that the most elements of $X_m$ are zeros, therefore there are many unnecessary multiplications and additions to zeros while applying $N_0*N_0$ IFFT would be replaced by $N*N$ IFFT [5].

b) Adjacent sub-block partition

In this scheme, the complexity of PTS scheme would be reduced by eliminating these multiplications and additions in figure 2. Consider that the adjacent partition method is used in PTS OFDM scheme, $X_m=[X(m)X(m+1)\ldots X(Nm+N-1)]_{1,N}$ and $X_{m-1}=[0\ldots 0X(m)0\ldots 0X(Nm+N-1)0\ldots 0X(Nm+N-1)]_{1,N}$ where $0\leq m \leq M-1$ and $N=N_0/M$.

In fig 2, the complexity reduction of partial transmit sequence (PTS) PAPR reduction scheme in OFDM systems by reducing the complexity of the IFFT architecture are investigated in this scheme. In the IFFT architecture of PTS OFDM scheme, there a lot of additions and multiplications with zero, which are obviously unnecessary. We can efficiently reduce the computational complexity without changing the resulting signal or degrading the performance of PAPR reduction by eliminating the addition and multiplications with zero from the architecture [5].

c) Sub-optimal sub-block partition

In the fig 2, there are steps to be taken in sub-optimal method, these are as follows:

1. Set $b_{index} = 1$, $m = 1, 2 \ldots M$ using (2) and (3), we can calculate PAPR of OFDM signals with the value of $PAPR_1$, and set $index = 1$;
2. Set $b_{index} = 1$, PAPR at this time is calculated by the same method with the value of $PAPR_2$.
3. If $PAPR_1 > PAPR_2$, $b_{index} = 1$; otherwise $PAPR_1 = PAPR_2$, $index = index + 1$;
4. $index = index + 1$;
5. Repeat from step 2-4 if $index < M + 1$.

The amount of computation can be effectively reduced by sub-optimal PTS algorithm. Compared to $2^M-1$ IFFT operations of optimum PTS, the computational cost of PTS is only $M$ IFFT operations [6].

V. Simulation Results

The section deals with the simulation results of PAPR reduction of OFDM system through PTS SUB-BLOCK partition schemes.

Simulated work is concluded in the way, QAM modulation scheme i.e. 16-QAM has been used in OFDM simulation, as QAM is very effective modulation techniques in 4G technologies having a bandwidth conserving modulation technique. The no of sub-carriers is $N=64$ having a sampling frequency of $F_s = 1000Hz$, satisfies the condition of orthogonality. $PAPR_0$ of the original OFDM is computed by oversampling the no. of sub-carriers $N=64$ by the oversampling factor of $L=4$, by insertion of $(L-1)N$ zeros to reduce the ISI. Thus at, $CCDF=10^9$, $PAPR_0$ (dB) = 8.8 dB. After applying the PTS-SUBBLOCK partition technique i.e. Interleaved-PTS scheme, at $CCDF=10^9$, $PAPR_0$ (dB) = 5.98, but this technique having a high complexity as there are many unnecessary multiplications and additions to zeros while applying $N_0*N_0$ IFFT would be replaced by $N*N$ IFFT. The next scheme has a no of sub-blocks = 4, and no of phase rotating factor = 4. PTS-Sub-block partition technique is the Adjacent-PTS scheme having no sub-block = 4, no of phase rotating factor = 4, with reduce complexity than Interleaved. At $CCDF=10^9$, $PAPR_0$ (dB) = 6.56 as shown in Figure 4.A, $CCDF=10^9$, $PAPR_0$ (dB) = 8.2, another SUB-BLOCK partition known as Sub-optimal-PTS with a reduce complexity than the above two discuss techniques in which no. of sub-blocks=4, but the no of phase rotating factor=2, by reducing the no of phase rotating factor the search complexity is reduce having a reduce complexity in PTS with less
PAPR0 than the original OFDM PAPR0 as shown in figure 4.

![Fig 3: PAPR Reduction for different N](image1)

![Fig 4: Comparison of different PTS sub-block PAPR reduction scheme](image2)

**VI. Conclusion**

In figure 3, the simulation results shows that at CCDF=10^0, the original OFDM PAPR0 (dB) = 8.8 by taking 16-ary QAM modulation technique and using L=4 and N=64 and by increasing the N, PAPR0 increases shown by blue color. After applying, Interleaved-PTS scheme, PAPR0 reduce at CCDF=10^0, PAPR0 = 5.5 dB, shown by red color.

In figure 4, there is reduction of PAPR0, with the reduction of complexity in PTS, as in case of Interleaved-PTS which is having high complexity i.e. the no. of (L-1)*N ZEROS are there, this amount of complexity is reduce by Adjacent-PTS in which the extras zeros are eliminated which makes the complex no of additions and multiplications are reduce by 81.25% and 68.75%, in both the techniques no. of sub-blocks=4, no. of phase rotating factor =4,thus a future sub-block technique known as Sub-optimal-

PTS is introduced in which no. of phase factor=2, thus the search complexity is reduce having a maximum combination of 8,which is just half as taken in above two schemes with a maximum combination of 16.

In this paper, we introduced a new PTS OFDM scheme with low complexity IFFT implement architecture. It is shown that the new proposed scheme is equivalent to the traditional PTS OFDM scheme, therefore its performance of PAPR reduction are also the same as the traditional PTS OFDM scheme. The new scheme reduces the computational complexity significantly.

**VII. References**


