Comparison between Bit Error Rate And Signal To Noise Ratio in OFDM Using LSE Algorithm

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Abstract

In this paper we are going to investigate the performance of Least Squares Error (LSE) Algorithm in OFDM channel estimation. It is based on the energy of pilot tones and data symbols. We first calculate the mean square error (MSE) of the LS channel estimate. LSE algorithm is used to enhance the channel estimation. Through simulation we learn that there is a considerable gain in signal to noise ratio (SNR) and minimization of bit error rate (BER).

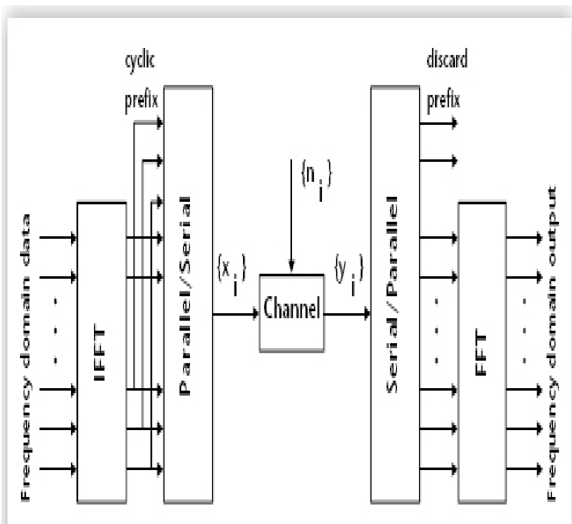
1. Introduction

In recent years high data rate techniques have gained considerable interests in communication systems. In a basic communication system, the data are modulated onto a single frequency. The available bandwidth is then totally occupied by each symbol. These kinds of systems can lead to inter –symbol- interference. The basic idea of orthogonal frequency division multiplexing is to divide the available spectrum into several orthogonal sub channels as a result each narrowband sub channel experiences almost flat fading. In OFDM the overlapping sub channels are in frequency domain hence increasing the data transmission rate.

Signal –to-noise ratio (SNR) is defined as the ratio of the desired signal power to noise power.SNR indicates the reliability of link between the transmitter and receiver. In OFDM based multi-carrier modulation system, the channel bandwidth is wide and interference is not constant over the whole band in use. It is very likely that there is variation of spectral content over the OFDM sub carriers i.e. some part of spectrum is more affected by interference than the other parts of spectrum

The most meaningful criterion for evaluation of performance of communication systems is the bit error rate(BER).In case of orthogonal communication systems, the main sources affecting its BER performances are additive white Gaussian noise(AWGN) and Intercarrier Interference(ICI). In most of the cases, the IC1 is assumed to have a normal distribution; however such assumption is not accurate theoretically. With the help of well developed conventional single carrier

communication systems, the BER expression of OFDM systems can be obtained without knowing the IC1 distribution function.
Fig. 1 shows the baseband equivalent model of an OFDM system that is considered in this paper. The analysis is in-dependent of mapping of the transmitted data as complex values a0,i………an-1,i and is therefore applicable to all forms of modulation which is utilized in OFDM systems. Before transmission, these values are processed by an IFFT, and a cyclic prefix of is added.



1.1. System model description

Total number of sub channels, N, are 256 having 32 pilots, P. Total number of data sub channels S, is the difference in the number of sub channels and pilots i.e. 224. Guard interval is taken to be 64 and pilot position interval is 8. Channel length L is 16.There are 500 iterations in each evaluation. SNR is taken from 0 to 27 in dBs.

Location of pilot and data is specified. After that fast Fourier transform matrix is generated. Then random channel coefficients are generated using random function. Data is transmitted with the help of quadrature amplitude modulation technique. While data transmission, AWGN noise is added and channel estimation of transmitted and received pilot is done for the computation of bit error rate with respect to signal to noise ratio.

Our investigation is based on following analysis:-

1.2. Channel estimation analysis

In this section, the MSE of the LS channel estimate is computed. Optimal pilot sequences and optimal placement of the pilot tones w.r.t. this MSE are then derived. From (11), the MSE of the LS channel estimate is given by

$$\begin{aligned}\mathrm{MSE} &= \frac{1}{LN_t}\mathcal{E}\left\{\left\|\hat{\mathbf{h}}^q - \mathbf{h}^q\right\|^2\right\}\\ &= \frac{1}{LN_t}\mathcal{E}\left\{\left\|\tilde{\mathbf{A}}^{\dagger}\tilde{\Xi}^q\right\|^2\right\}\\ &= \frac{1}{LN_t}\mathrm{tr}\left\{\tilde{\mathbf{A}}^{\dagger}\mathcal{E}\left\{\tilde{\Xi}^q\tilde{\Xi}^{q^H}\right\}\tilde{\mathbf{A}}^{\dagger^H}\right\}.\end{aligned} \quad (1)$$

For zero-mean white noise, we have $\mathcal{E}\{\tilde{\Xi}^q\tilde{\Xi}^{q^H}\} = \sigma_n^2\mathbf{I}_P.$ (2)

Then MSE can be written as

$$\mathrm{MSE} = \frac{\sigma_n^2}{LN_t}\mathrm{tr}\{(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}})^{-1}\}. \quad (3)$$

Using a similar argument as in [1], we can show that in order to obtain the minimum MSE of the LS channel estimate subject to a fixed power dedicated for training, we require

$$\tilde{\mathbf{A}}^H\hat{\mathbf{A}} = \mathcal{P}\mathbf{I}_{LN_t}. \quad (4)$$

The minimum MSE is given by

$$\mathrm{MSE}_{\min} = \frac{\sigma_n^2}{\mathcal{P}}. \quad (5)$$

2. Least Squares Channel Estimation

In this section, a least squares (LS) channel estimation scheme is derived. Let $\mathbf{X}^r(n) = \mathbf{S}^r(n) + \mathbf{B}^r(n)$,where $\mathbf{S}^r(n)$ is some arbitrary K×1 data vector, and $\mathbf{B}^r(n)$ is some arbitrary K×1 pilot sequence vector. Then we can say that-

$$\mathbf{Y}^q(n) = \sum_{r=1}^{N_t} \mathrm{diag}\{\mathbf{X}^r(n)\}\mathbf{F}\mathbf{h}^{q,r} + \Xi^q(n)$$
$$= \sum_{r=1}^{N_t} (\mathrm{diag}\{\mathbf{S}^r(n)\} + \mathrm{diag}\{\mathbf{B}^r(n)\})$$
$$\times \mathbf{F}\mathbf{h}^{q,r} + \Xi^q(n)$$

Defining

$\mathbf{S}^r_{\mathrm{diag}}(n) = \mathrm{diag}\{\mathbf{S}^r(n)\}$ and $\mathbf{B}^r_{\mathrm{diag}}(n) = \mathrm{diag}\{\mathbf{B}^r(n)\}$

We can write,

$$\mathbf{Y}^q(n) = \sum_{r=1}^{N_t} \mathbf{S}^r_{\mathrm{diag}}(n)\mathbf{F}\mathbf{h}^{q,r} + \sum_{r=1}^{N_t} \mathbf{B}^r_{\mathrm{diag}}(n)\mathbf{F}\mathbf{h}^{q,r} + \Xi^q(n). \quad (6)$$

Assuming training over g consecutive OFDM symbols, e.g., over the time indices

$n \in \{0, \ldots, g-1\}$,

Now, we consider the data model

$$\mathbf{Y}^q = \mathbf{T}\mathbf{h}^q + \mathbf{A}\mathbf{h}^q + \Xi^q$$

$$\mathbf{T} = \begin{bmatrix} \mathbf{S}^1_{\mathrm{diag}}(0)\mathbf{F} & \cdots & \mathbf{S}^{N_t}_{\mathrm{diag}}(0)\mathbf{F} \\ \vdots & \cdots & \vdots \\ \mathbf{S}^1_{\mathrm{diag}}(g-1)\mathbf{F} & \cdots & \mathbf{S}^{N_t}_{\mathrm{diag}}(g-1)\mathbf{F} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{B}^1_{\mathrm{diag}}(0)\mathbf{F} & \cdots & \mathbf{B}^{N_t}_{\mathrm{diag}}(0)\mathbf{F} \\ \vdots & \cdots & \vdots \\ \mathbf{B}^1_{\mathrm{diag}}(g-1)\mathbf{F} & \cdots & \mathbf{B}^{N_t}_{\mathrm{diag}}(g-1)\mathbf{F} \end{bmatrix}$$

And $\mathbf{h}^q = [\mathbf{h}^{q,1^T}, \ldots, \mathbf{h}^{q,N_t^T}]^T$.

The LS estimate of $\mathbf{h}^q$ can then be obtained as

$$\hat{\mathbf{h}}^q = \mathbf{A}^{\dagger}\mathbf{Y}^q.$$

We assume that the pilot sequences are designed such that the

$gK \times LN_t$ Matrix is of full column rank, which requires

$gK \geq LN_t$. The pseudo-inverse of **A** can thus be written as

We then obtain

$$\hat{\mathbf{h}}^q = \mathbf{h}^q + \mathbf{A}^{\dagger}\mathbf{T}\mathbf{h}^q + \mathbf{A}^{\dagger}\Xi^q. \quad (7)$$

To eliminate the interference term due to the data, we impose the following condition:

As mentioned earlier, we will design the $gK \times LN_t$ matrix A to have full column rank LN_t. following the above design, this is equivalent to the $P \times LN_t$ matrix $\tilde{\mathbf{A}}$ having full column rank LN_t, which requires $P \geq LN_t$. It can easily be checked that the design we will propose later satisfies this full rank condition.

3. **SNR and BER analysis**

In this section, we derive the expression for the SNR and BER for downlink OFDM systems in noisy fading Doppler channels. The SNR of the mth subcarrier is given as:

$$SNR_m = \frac{P_{D_m}}{P_{I_m} + P_{N_m}},$$

Where PDm is the average power of the desired signal, PIm is the average interference power and PNm is the average noise power. The average power of the desired, interference and noise is defined a PDm = E |Dm |2, PIm = E |Im |2, and PNm = E |Nm |2 respectively. Hence, the average SNR on the mth subcarrier is formulated as:

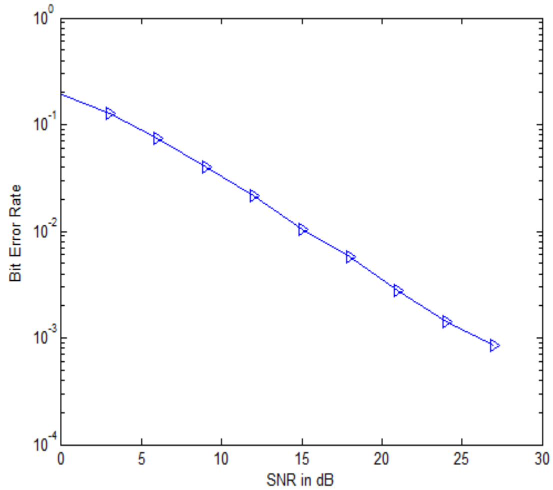
$$SNR_m = \frac{E\left[|D_m|^2\right]}{E\left[|I_m + N_m|^2\right]}, \quad (8)$$

Where the average power of the desired signal is calculated as:

$$E\left[|D_m|^2\right] = E\left[|c_o \alpha_{m,i} a_{m,i}|^2\right] \quad (9)$$

4. Result-

By investigation of LSE algorithm in OFDM channel estimation. We realize that SNR is increased to a large extent with simultaneous decrease in bit error rate. This is only possible with PAM and QAM as in PSK bit error rate increases to a large extent. After doing MATLAB coding and running the program, we obtained the following result:



As the SNR increase, BER constantly decreases proving the system to be efficient.

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