Step 2 Calculate the zenith satellites number m with Formula (14) according to the total selected satellites number n. And Select the first m visible satellites from all-in-view satellites according to their elevation angles which are listed in descending order.

Step 3 Find out the visible satellite from all-in-view satellites which has the minimum elevation angle and denote it as A.

Step 4 Divide all-in-view satellites into *n-m* groups except the ones selected in Step 2 and Step 3.

The azimuths of satellites in the Group j should satisfy the inequality

$$azi_A + \frac{360(j-0.5)}{n-m} < azi \le azi_A + \frac{360(j+0.5)}{n-m} \quad (15)$$

Where azi_A is the azimuth of satellite A and azi denotes the azimuth of a visible satellite in the Group j $(j = 1, 2, \cdots, n-m)$.

The grouping can be shown in Fig. 2 which describes the orientation relationship between the receiver and all-in-view satellites in topocentric coordinate system. ***O*** is the position of the receiver. The farther the dot from ***O*** is, the lower the elevation angle of the satellite denoted by the dot will be. Circle ***C*** means horizon orientation. The yellow hexagons denote the zenith satellites selected in Step2.The green five-pointed star denotes satellite A selected in Step3. The red triangles are drawn according to the orientation of satellite A. They represent the virtual satellites which can constitute the optimal SRG with the visible satellite selected in Step 2 and Step 3. The purple dots denote the grouped satellites.

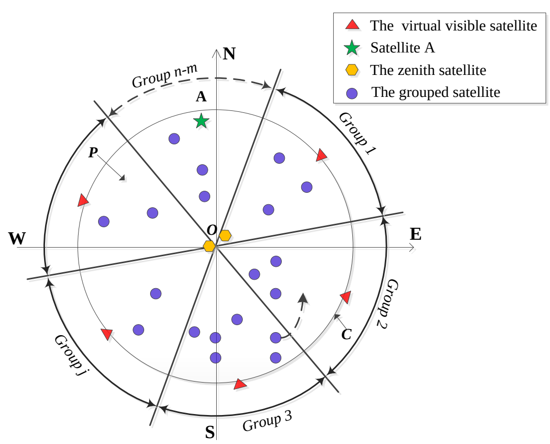

Fig. 2 Satellite selection process

Step 5 Adjust the grouping.

Sometimes, all the elevation angles of visible satellites in Group j are large enough to exceed a threshold (maybe 30˚), just like Group *2* in Fig. 3. If one of them is selected, the polygon composed by the selected satellites except the ones at the zenith may be a concave polygon in topocentric coordinate system. Thus the satellite selection will be disappointing. Therefore, the grouping should be adjusted.

The group adjustment process can be seen in Fig. 3.

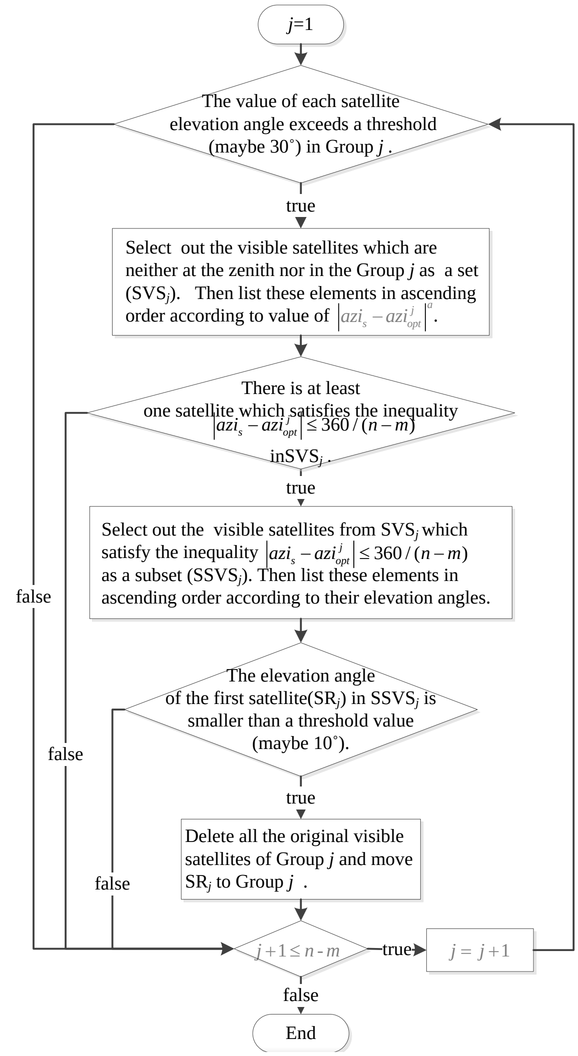

Fig. 3 The group adjustment process

[a] azi_s denotes the azimuth of the visible satellite. $s = 1, 2, \cdots, n-m-1-l_j$, l_j is the total number of visible satellites in Group j. $azi_{opt}^j = 360j/(n-m) + azi_A$.

Step 6 Select out one satellite in each group except the Group *n-m* and combine them with the satellites selected in Step 3 and Step 4 as a subset. Thus there will be $l_1 \times l_2 \times \cdots \times l_{n-m-1}$ different subsets, where l_j is the total satellites number of Group j ($j = 1, 2, \cdots, n-m-1$).Traverse all the subsets and find out the one minimize the GDOP value. This subset is the satellite selection result.

To illustrate the validity of ASMS, two simulation experiments are designed. In both experiments, the GDOP value of the point S at North latitude 39˚, East longitude 116˚ is calculated from 2011-1-1 5:00 to 2011-1-1 7:00 with the sampling interval of 2 min. The satellites are selected from GPS, GLONASS and Galileo. The number of all-in-view satellites is from 31 to 35 under this default scene.

In the first experiment, each GDOP value of selected 8 satellites using ASMS is compared with the minimum one among all the GDOP values of optionally random selecting 8 satellites for 1×10^6 times. The simulation result can be seen in Fig. 4 and TABLE Ⅱ.

Analysing the statistical data in TABLE Ⅱ, it's undeniable that the satellite selection result of ASMS is not the optimal. However there will be about ten million random combinations of different 8 satellites (C_{35}^{8}). It is impossible to traverse all the combinations and find the one which minimize the GDOP value for its huge amount of calculation and time-consuming (tens of minutes for once). ASMS can finish the calculation process within less than 100 milliseconds. Under the premise of sacrificing the GDOP value a little, it reduces the computation and improves the satellite selection speed.

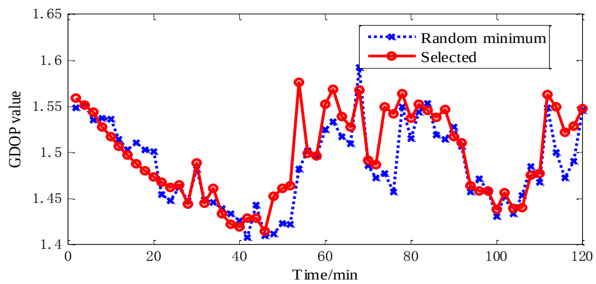

Fig. 4 GDOP value for selected 8 satellites and random 8 satellites in the first simulation experiment

TABLE Ⅱ The statistical data of the first experiment

The GDOP value	Mean	Variance	Standard deviation
Selected 8satellites	1.4973	0.0022	0.0467
Random 8 satellites	1.4780	0.0020	0.0443

In the second experiment, ASMS is used to respectively select 10, 14, 18 and 22 satellites from GPS and Galileo. The simulation result is shown in Fig. 5.

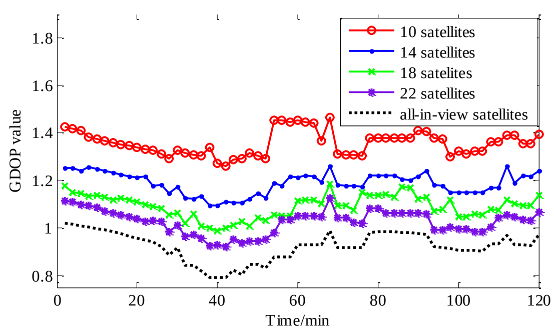

Fig. 5 GDOP value for different number of selected satellites

As seen in Fig. 5, with the increasing of the number of satellites selected by ASMS, the GDOP value decreases. The GDOP value is close to that of all-in-view satellites with the increasing of selected satellites number.

It can be confirmed from above two experiments that although the satellite selection may not be the optimal, compared with traversing all of satellites combinations, ASMS greatly reduces computation. Moreover, with the increasing of selected satellites number, the GDOP value decreases. Therefore it can be studied determining the number of selected satellites according to the receiver's needs of positioning accuracy.

V. Conclusions

Starting with derivation the relationship of the optimal SRG and the minimum GDOP value in 2-D, a rule to obtain the minimum GDOP value has been found, which is to make matrix G^TG a diagonal matrix. Through this rule, a formula (Formula (13)) that is used to calculate the minimum GDOP value of n satellites in 3-D was deduced and its reliability was verified by simulation. Meanwhile the optimal SRG to minimize the GDOP value in 3-D was summarized: for n satellites ($n\geq4$), there should be m at the zenith and m-n in the horizontal plane to constitute a regular polygon.

After analysing the minimum GDOP value and the optimal SRG of n satellites in 3-D, an algorithm of selecting more than 4 satellites from GNSS (ASMS) was proposed. ASMS can get the small GDOP value and improve satellite selection efficiency by selecting the satellites whose SRG is close to the optimal one. By two simulation experiments, the good performance of ASMS is illustrated.

Because the thresholds of elevation angle in Step 5 are to be determined, ASMS should be studied further.

References

[1] Nuria Blanco-Delgado, Fernando D. Nunes and Gonzalo Seco-Granados, "Relation between GDOP and the Geometry of the Satellite Constellation", *2011 International Conference on Localization and GNSS,USA*, pp.75-180,June 1987.

[2] Lu Yun and Yang Qiangwen, "Study on GNSS Interoperability". *SCIENTIA SINICA Phys, Mech & Astron*, vol.40, no. 9, pp. 534-540, 2010.

[3] Miaoyan Zhang and Jun Zhang, "A Fast Satellite Selection Algorithm: Beyond Four Satellites", *Journal of Selected Topics in Signal Processing*,vol.30, no. 5,pp. 740-747, 2009.

[4] Mok E. and Cross P.A., "A fast satellite selection algorithm for combined GPS and GLONASS receivers", *Journal of Navigation*, vol. 40, pp. 283-389,1994.

[5] CONG Li, Ahmed I Abidat and TAN Zhan-zhong, "Analysis and Simulation of the GDOP of Satellite Navigation", *ACTA ELECTRONICA SINICA*, vol. 34, no. 9,pp. 2204-2208, 2006.

[6] Hofman-Wellenhof, Lichtenegger and Wasle, GNSS-Global Navigation Satellite Systems GPS, GLONASS, Galileo & more, Spring-Verlag, 2008, pp. 197-199.

[7] Zhang Miaoyan, Zhang Jun and Qin Yong, Satellite Selection for Multi-Constellation, *2008 IEEE/ION Position, Location and Navigation Symposium, USA*, pp. 1053-1059, Monterey, May 2008.

[8] ZHOU ShanShi, et al, "Positioning accuracy assessment for the 4GEO/5IGSO/2MEO constellation of COMPASS", *SCIENCE CHINA Physics, Mechanics & Astronomy*, vol.55, No.12, pp. 2290-2299, 2012.

[9] Massatt P. and Rudnick K., "Geometric Formulas for Dilution of Precision Calculations", *Journal of the Institute of Navigation*, vol.37, no. 4, pp. 379–391,1991.

[10] ZHENG Zuoya, et al, "Selection of GPS Satellites for the Optimum Geometry", *Chinese Astronomy and Astrophysics*, vol.28, pp. 80-87, 2003.

[11] Nuria Blanco-Delgado and Fernando D. Nunes, "Satellite Selection Method for Multi-Constellation GNSS Using Convex Geometry", *IEEE Translated. Vehicular Technology, USA*, vol.59, pp. 4289-4297, 2010.