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Taming Spatiotemporal Chaos by Impurities in the Parametrically Driven Damped Nonlinear Schrödinger Equation

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Abstract

Solitons of the parametrically driven, damped nonlinear Schrödinger equation become unstable and seed spatiotemporal chaos for sufficiently large driving amplitudes. We show that the chaos can be suppressed by introducing localized inhomogeneities in the parameters of the equation. The pinning of the soliton on an "attractive" inhomogeneity expands its stability region whereas "repulsive" impurities produce an effective partitioning of the interval. We also show that attractive impurities may spontaneously nucleate solitons which subsequently remain pinned on these defects. A brief account of these results has appeared in patt-sol/9906001, where the interested reader can also find multicolor versions of the figures.

Motivation. The ability to synchronize populations of coupled nonlinear oscillators would afford enormous technological benefits. A textbook example is provided by chains of Josephson junctions. A single junction can serve as an unparalleled source of ultrahigh-frequency voltage oscillations; however, its industrial utilization was hindered by anomalously low power outputs. A natural way out would be to assemble a large array of coupled identical junctions, in anticipation that the coupling would force them to pulsate in unison. However — even if individual oscillators are nonchaotic — the synchronized regime may be unstable and evolve into a highly incoherent state, usually referred to as the spatio-temporal chaos.

In an exciting twist of events, recent numerical simulations revealed that the introduction of slight uncorrelated differences between the oscillators may result in a significant improvement of the synchronization of the array [1, 2, 3]. The disorder was seen to suppress the chaos! In an attempt to gain a deeper insight into the nature of this counter-intuitive phenomenon, a numerical study of the effect of a single impurity on an otherwise homogeneous array was carried out [4]. Surprisingly, a single impurity was found to be sufficient to "tame" the chaotic behaviour and produce simple spatiotemporal patterns in very long chains.

As a prototype nonlinear array, the authors of [4] (see also [2]) chose a chain of pendula coupled to their nearest neighbours and driven by a periodic external torque. All individual pendula in the chain were in their chaotic parameter regime whereas the natural frequency of the central pendulum (the impurity) was out of the chaotic range. Consequently, the proposed mechanism of stabilization was through the formation of a nonchaotic cluster around the defect which subsequently pulls the whole array out of chaos [2, 4].

In the present note we study the effect of an impurity on a damped driven system by considering it from the viewpoint of nonlinear waves. In other words we explore *collective* stabilization mechanisms. As in Ref. [4] we study a chain of coupled pendula but unlike [4], we focus on the regime where all individual pendula are nonchaotic. (This does not mean of course that the array as a whole may not fall into the state of the spatiotemporal chaos.) Another distinction from Ref. [4] is that we are considering the *parametrically*, not externally, driven chain. (The main reason for this is the availability of explicit solutions.)

The collective stabilization mechanisms are activated when pendula are strongly coupled. In this case they tend to form soliton-like clusters of coherent behaviour. Similarly to externally driven systems, stationary solitons in parametrically driven chains are known to be stable for small driving strengths but lose their stability to oscillating solitons as the driver's amplitude is increased [5, 6, 7]. Increasing the driving amplitude still further, a spatiotemporal chaotic state sets in — with or without a series of intermediate bifurcations [6].

We will demonstrate that "attractive" impurities may act as centres of *spontaneous* nucleation of solitons and hence in extended systems with impurities, solitons are even more generic and natural occurrences than in their homogeneous counterparts. We will prove that pinning of a stationary soliton on an "attractive" (or "long") impurity expands its region of stability. In particular, by choosing a sufficiently long impurity pendulum, the soliton can be stabilized in the parameter region where in the absence of the inhomogeneity it would ignite the spatiotemporal chaos. On the other hand, although solitons pinned on "short" pendula will turn out to be more prone to oscillatory instabilities, we will show that such a pinning is an unlikely occurrence due to the repulsive nature of the short defects.

The model. The angle the n-th pendulum in our chain makes to the vertical, satisfies

$$ml_n^2\ddot{\theta}_n+\alpha l_n\dot{\theta}_n-k(\theta_{n+1}-2\theta_n+\theta_{n-1}) = -ml_n\left(g+4\omega^2\rho\cos 2\omega t\right)\sin\theta_n, \tag{1}$$

where k is the torsion-spring constant, α the friction in the pivots, ρ and ω are the amplitude and frequency of the driver, and the length $l_n = 1$ for all $n \neq 0$. In what follows we set $g = m = 1$. Assume that the spring is very hard, or, equivalently, the distance between the neighbouring pendula (which we define as $a \equiv 1/\sqrt{k}$) is very small: $a \to 0$. Then we can neglect the $O\left(a^4\right)$ terms in the Taylor expansion $\theta_{n\pm1} = \theta_n \pm \theta_n' a + \frac{1}{2}\theta_n'' a^2 + \cdots$, so that $\theta_{n+1}-2\theta_n+\theta_{n-1} \approx a^2\theta''(z_n)$. Here the function $\theta(z)$ is assumed to be differentiable at all sites $z_n = na$, $-N \leq n \leq N$, except the site with $n = 0$. At the site z_0 it will only have the left and right derivatives and hence the above expression should be replaced by

$$\theta_1-2\theta_0+\theta_{-1} \approx a\theta'\Big|_{z=-0}^{z=+0}+a^2\frac{\theta''(+0)+\theta''(-0)}{2}. \tag{2}$$

It is convenient to choose the square of the inter-pendulum distance as a small parameter: $\varepsilon = a^2$. For simplicity, we confine ourselves to the case when the driving frequency is just below the edge of the continuous spectrum of linear waves, $\omega^2 = 1 - \varepsilon^2$, and α and

ρ are small: $\alpha = \varepsilon^2\gamma$, $4\omega^2\rho = 2h\varepsilon^2$. In this case we can assume that the pendula execute nearly-synchronized small-amplitude librations of the form

$$\theta = 2\varepsilon\psi(T,X)e^{-i\omega t} + \text{c.c.} + \mathrm{O}\left(\varepsilon^3\right), \tag{3}$$

where the envelope ψ is only slowly varying in space and time: $X = \varepsilon z$, $T = \varepsilon^2 t/2$. It is this variable amplitude that contains the information on how disorganized the array is. If ψ were constant, the pendula would be perfectly synchronized; however, ψ will not generally be constant. Substituting (3) into eq. (1) with $n \neq 0$ and sending $a \to 0$ we obtain the amplitude equation

$$i\psi_T + \psi_{XX} + 2|\psi|^2\psi - \psi = h\psi^* - i\gamma\psi, \quad X \neq 0. \tag{4}$$

Next, let the central pendulum be slightly longer or slightly shorter than the rest of the chain, $l_0 = 1 + 2q\sqrt{\varepsilon}$, with q being positive or negative, respectively. Using (2) and sending $a \to 0$, the equation (1) with $n = 0$ gives rise to the boundary condition

$$2q\psi(0) + \psi_X\Big|_{-0}^{+0} = 0. \tag{5}$$

Eqs. (4) and (5) can be combined into the parametrically driven damped nonlinear Schrödinger equation with a δ-function inhomogeneity:

$$i\psi_T + \psi_{XX} + 2|\psi|^2\psi - \psi + 2q\delta(X)\psi = h\psi^* - i\gamma\psi, \quad -\infty < X < \infty. \tag{6}$$

Equation (6) with $q = 0$ was previously used to model the nonlinear Faraday resonance in a long narrow water trough [8]. The inhomogeneous term $2q\delta(X)\psi$ represents a local widening (for $q > 0$) or narrowing ($q < 0$) of the trough. The same equation describes an easy-plane ferromagnet with a combination of a static and hf field in the easy plane [9, 5] and the planar weakly anisotropic XY model [10]; in both cases the inhomogeneous term accounts for an impurity spin. Equation (6) was also used in studies of the effect of phase-sensitive parametric amplifiers on solitons in optical fibers [11].

As in the spatially homogeneous case [5], the zero solution of eq. (6) is unstable against continuous spectrum excitations for $h > \sqrt{1+\gamma^2}$. Next, the $q > 0$-impurities host a discrete mode:

$$\delta\psi = \epsilon\left(\cos\Omega T - i\frac{1+h-q^2}{\Omega}\sin\Omega T\right)e^{-\gamma T - q|X|},$$

where $\Omega^2 = \left(1-q^2\right)^2 - h^2$ and $\epsilon \ll 1$ is the linearization parameter. When h exceeds the value $\hbar_{q,\gamma}$,

$$\hbar_{q,\gamma} \equiv \sqrt{(1-q^2)^2 + \gamma^2}, \tag{7}$$

this localized mode also produces instability. For positive $q < \sqrt{2}$ (in which case the curve $\hbar_{q,\gamma}$ lies below $\sqrt{1+\gamma^2}$), the nonlinear development of this instability leads to the formation of solitons (Fig. 1), stable or unstable.

Solitons. Thus we are naturally led to the consideration of localized solutions. For all q, h and γ the soliton solutions of eq. (6) are available in closed form. In fact, there are two stationary soliton solutions, ψ_+ and ψ_-, each having a cusp at the origin:

$$\psi_\pm(X) = A_\pm \operatorname{sech}\left(A_\pm|X| + \tilde{x}\right)e^{-i\theta_\pm}. \tag{8}$$

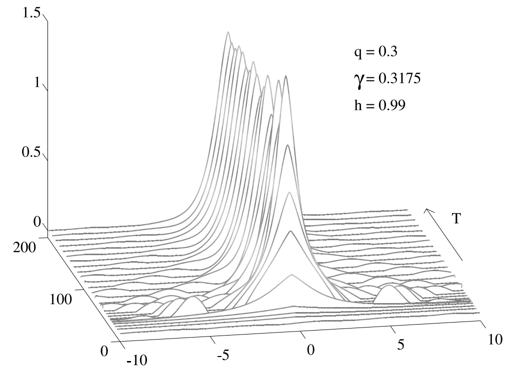

Figure 1. A stable pinned soliton emerging from a small-amplitude random spatial distribution.

Here $\cos 2\theta_\pm = \pm\sqrt{1-\gamma^2/h^2}$, $A_\pm^2 = 1 \pm \sqrt{h^2-\gamma^2}$, and $\tilde{x} = \operatorname{arctanh}(q/A_\pm)$. For weak impurities, $|q| < 1$, the ψ_+ soliton exists for any h and γ satisfying $h > \gamma$, whereas the ψ_- requires, in addition, that $h < \hbar_{q,\gamma}$ with $\hbar_{q,\gamma}$ as in (7). Strong impurities ($|q| > 1$) do not support the ψ_- soliton at all whereas the ψ_+ exists only if $h > \hbar_{q,\gamma}$.

First we demonstrate that the soliton ψ_- is always unstable so we can safely forget about it for the remainder of this study. Taking the linear perturbation in the form $\delta\psi_\pm = \epsilon(f+ig)e^{-i\theta_\pm-\gamma T}$ gives

$$-g_\tau - \Gamma g = L_1 f, \qquad f_\tau - \Gamma f = L_0 g, \tag{9}$$

where $\Gamma = \gamma/A_\pm^2$, $\tau = A_\pm^2 T$, and the Schrödinger operators L_0 and L_1 are given by

$$L_1 = -\partial_x^2 + 1 - 6\operatorname{sech}^2(|x|+\tilde{x}) - 2Q\delta(x), \tag{10}$$

$$L_0 = -\partial_x^2 + 1 \mp 2H - 2\operatorname{sech}^2(|x|+\tilde{x}) - 2Q\delta(x), \tag{11}$$

with $Q = q/A_\pm = q\sqrt{1\mp H}$, $H = \sqrt{h^2-\gamma^2}/A_\pm^2$, and $x = A_\pm X$. The minimum eigenvalue of the operator L_0 associated with a nodeless eigenfunction $\operatorname{sech}(|x|+\tilde{x})$, is $\nu_0 = \mp 2H$. Consequently, in the case of the ψ_- soliton the operator L_0 is positive definite and Eqs. (9) can be rewritten as

$$L_0^{-1} f_{\tau\tau} = \Gamma^2 L_0^{-1} f - L_1 f. \tag{12}$$

The maximum exponential growth rate λ of solutions to Eq. (12) is given by [12]

$$\lambda^2 = \Gamma^2 + \sup_f \frac{\langle f(x)| - L_1 | f(x)\rangle}{\langle f(x)| L_0^{-1} | f(x)\rangle}. \tag{13}$$

For any Q the operator L_1 has a negative eigenvalue $\mu_0 = 1-\kappa^2$ associated with an even eigenfunction

$$y_0(x) = e^{-\kappa\xi}\left(3\tanh^2\xi + 3\kappa\tanh\xi + \kappa^2 - 1\right), \tag{14}$$

where $\xi = |x| + \tilde{x}$ and κ is the root of $\kappa^3 + 2\kappa^2 Q + \kappa\left(3Q^2 - 5Q - 4\right) + 3Q^3 = 0$ with $\kappa > 1$. Hence the supremum in (13) is positive, λ is $> \Gamma$ and the soliton ψ_- is unstable against a symmetric nonoscillatory mode for all q, h and γ.

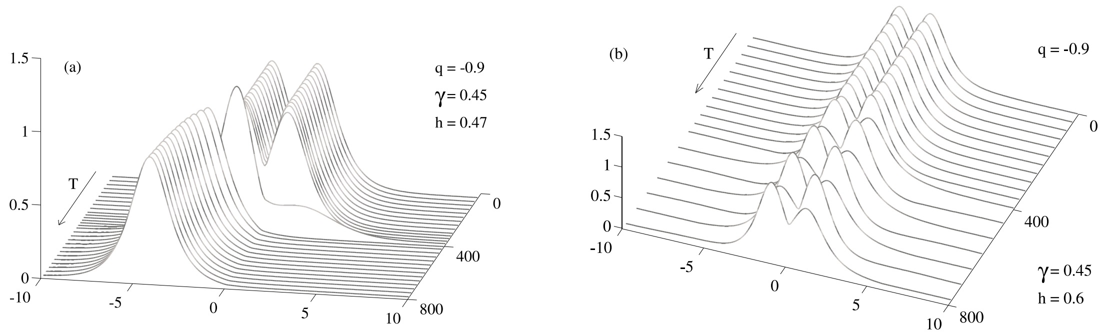

Figure 2. The effect of the $q < 0$ impurity. For $h < \hbar_{q,\gamma}$, the growth of the asymmetric instability leads to the soliton's unpinning and repulsion from the impurity (a), while for $h > \hbar_{q,\gamma}$, the asymmetric Hopf instability evolves into a two-humped soliton staggering in a seesaw fashion (b).

***q* < 0: Unpinning instability**. A similar argument can be used to detect the disengagement instability of the other soliton, ψ_+, arising for impurities with $q < 0$. We simply notice that the second lowest eigenvalue of the operator L_0 which is associated with an odd eigenfunction

$$w_1(x) = \mathrm{sgn}(x) e^{Q(|x|+\tilde{x})} \left\{ \tanh(|x| + \tilde{x}) - Q \right\},$$

is equal to $\nu_1 = 1 - 2H - Q^2$. For $2H < 1 - Q^2$ or equivalently for $h < \hbar_{q,\gamma}$, L_0 is positive definite on the subspace of odd functions. On the other hand, for $Q < 0$ the operator L_1 has a negative eigenvalue $\mu_1 = 1 - \kappa^2$ associated with an odd eigenfunction $y_1(x) = \mathrm{sgn}\, x\; y_0(x)$, with y_0 as in (14) and κ the root of $\kappa^2 + 3\kappa Q + 3Q^2 - 1 = 0$ satisfying $\kappa > 1$. Hence if we restrict ourselves to the space of odd functions, the variational principle (13) remains applicable, λ is $> \Gamma$ and the soliton ψ_+ is unstable. The interpretation of this instability is straightforward if one invokes the energy considerations. In the undamped case ($\gamma = 0$) the equation (6) conserves the energy integral,

$$E = \int \left\{ |\psi_X|^2 + |\psi|^2 - |\psi|^4 - 2q\delta(X)|\psi|^2 + h \,\mathrm{Re}\, \psi^2 \right\} dX. \tag{15}$$

The inhomogeneous term produces a local decrease respectively increase of the energy density for $q > 0$ respectively $q < 0$. Consequently, in the conservative and weakly dissipative cases, the $q > 0$-impurity will attract and the one with $q < 0$ repel small-amplitude tails of *distant* solitons. On the other hand, the energy of the *pinned* soliton is $E_Q = \frac{4}{3} A^3 \left(1 - 3Q + 2Q^3\right)$. For $Q > 0$ ($Q < 0$) this is smaller (greater) than the energy E_0 of the infinitely remote soliton. These two facts indicate that $Q > 0$-impurities should attract and trap solitons (cf. [13]). In the $Q < 0$ case, conversely, distant solitons should be repelled while an initially pinned soliton is expected to unpin and move away from the impurity regaining its cusp-free shape. This was indeed confirmed by simulations of Eq. (6) (Fig. 2(a)).

It is important to emphasize that we are using the energy considerations only for the *interpretation* of the instability detected by some other, rigorous, methods. The negativity of q does not itself guarantee that the soliton pinned on the repulsive impurity will necessarily unpin and escape. Moreover, the variational approach of the present section works only for $h < \hbar_{q,\gamma}$ while for greater h we can't even claim that the soliton pinned

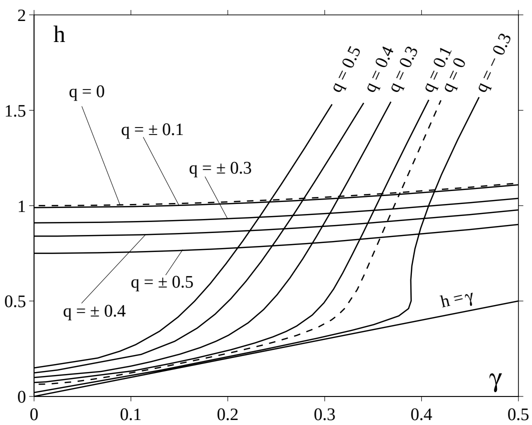

Figure 3. An atlas of stability charts on the (γ, h)-plane. Above the dashed "almost-horizontal" curve, $h = \sqrt{1+\gamma^2}$, all localized solutions are unstable w.r.t. continuous spectrum waves. Solid "horizontal" curves are given by $h = \hbar_{q,\gamma}$. Below their corresponding $\hbar_{q,\gamma}$-curves, impurities with $q < 0$ repel solitons. The family of "parabolas" depict the onset of (symmetric) instability of the pinned soliton; the greater is q the larger is the stability domain. Finally, in the band between the line $\sqrt{1+\gamma^2}$ and its corresponding $\hbar_{q,\gamma}$-curve, an attractive impurity will spontaneously nucleate solitons.

on the repulsive impurity is unstable! Another remark is that the unpinning instability is not connected with overdriving the chain; it occurs already in the undriven NLS [13].

***q* > 0: Numerical stability analysis**. In the region $h > \hbar_{q,\gamma}$ (as well as in the case of symmetric instabilities, and also for the attractive impurities) the variational principle (13) is not applicable and we have to resort to the help of computer. Here we let $f(x,t) = u(x)e^{i\Omega\tau}$ and $g(x,t) = -\omega(\Gamma + i\Omega)^{-1}v(x)e^{i\Omega\tau}$, where $\Omega^2 = \omega^2 - \Gamma^2$. Then eq. (9) reduces to an eigenvalue problem

$$L_1 u = \omega v, \qquad L_0 v = \omega u. \tag{16}$$

Notice that a three-parameter (q, h and γ) linear system has been reduced to a two-parameter (q and H) eigenproblem. Having found $\omega = \omega(q, H)$, one immediately recovers the instability growth rate $|\mathrm{Im}\,\Omega(q,H)| - \Gamma$ for all q, h, and γ.

For small $h < \hbar_{q,\gamma}$ the numerical analysis of the soliton pinned on the "repulsive" impurity ($q < 0$) shows that there is only one unstable pair of imaginary eigenvalues $\pm\omega_1$, with the associated u and v being odd. As h is increased beyond $\hbar_{q,\gamma}$, the imaginary eigenvalues move onto the real axis (i.e. the soliton restabilizes) while another real doublet $\pm\omega_2$ detaches from the continuous spectrum. Subsequently, the two collide and emerge as a complex quadruplet after which the real and imaginary parts grow until an asymmetric instability sets in. This is *not* a disengagement instability now; the stationary soliton is replaced by a two-humped structure (still pinned on the impurity) whose left and right wings stagger 180° out of phase (Fig. 2(b)).

For the soliton pinned on the "attractive" impurity, $q > 0$, the motion of eigenvalues ω on the complex plane is similar to the homogeneous case [5]. The stationary soliton ψ_+ is stable for h close to γ but loses its stability to a symmetric oscillating soliton as h is increased. Fig. 3 shows the Hopf bifurcation curves $h = h_q(\gamma)$ obtained from the relation $|\mathrm{Im}\,\Omega(q,H)| = \Gamma$ for $q = 0.1$, 0.3, 0.4, 0.5 and -0.3. For $q > 0$ the stability domain is wider than without an impurity. For example, taking $q = 0.5$ is sufficient to *double* the size of this domain. On the contrary, the $q < 0$-impurity narrows the stability region.

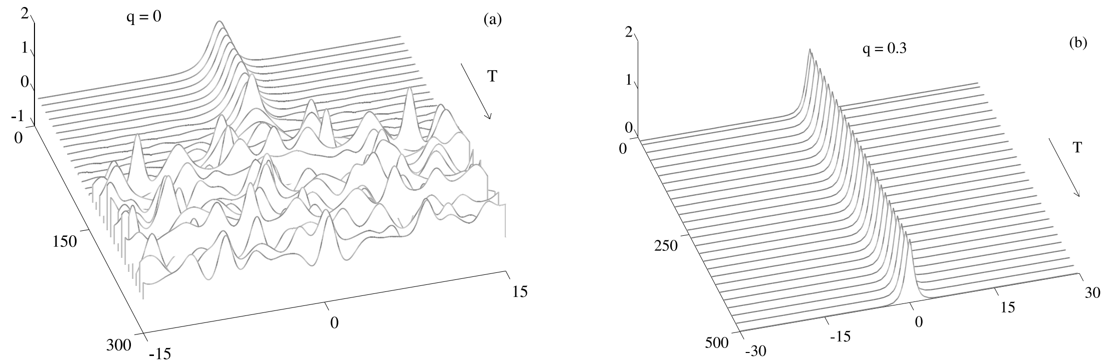

Figure 4. In the homogeneous equation with $\gamma = 0.315, h = 0.95$ the unstable soliton seeds spatiotemporal chaos (a). Introducing an impurity with $q = 0.3$ is sufficient to stabilize it (b).

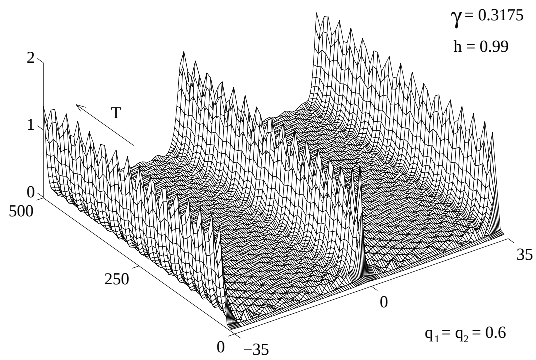

Figure 5. Two $q > 0$ impurities placed in the middle and at the end of the periodic interval, respectively.

Taming chaos with disorder? Finally let us summarize our conclusions and discuss possible applications for chains of pendula with random distributions of widely separated impurities. We have shown that for small h ($h < \hbar_{q,\gamma}$) "long" impurities ($q > 0$) attract and trap solitons while for $h > \hbar_{q,\gamma}$, pinned solitons are *spontaneously* formed around the long defects. (This can be looked upon the other way around: for given h and γ the solitons will nucleate around attractive impurities with q close enough to 1: $\left(q^2 - 1\right)^2 < h^2 - \gamma^2$.) On the other hand, the soliton with h and γ such that it would ignite spatiotemporal chaos in the homogeneous case [6], is stabilized when pinned on a sufficiently "long" impurity (Fig. 4). Therefore the $q > 0$ defects should have a stabilizing effect on the chain. One should keep in mind, however, that spatiotemporal chaotic states are not localized and a single stable soliton will clearly be insufficient to suppress chaos in a long chain. The chaos can always be triggered by choosing the initial condition far enough from the soliton. In order to suppress chaos in a larger phase volume multiple impurities should be introduced; one is therefore led to the necessity of examining stability of solitons and periodic waves on *finite* intervals.

Next, the fact that *short* impurities enhance the symmetric instability should not play a destabilizing role since solitons tend to avoid "short" defects. On the contrary, the repulsive inhomogeneities will essentially partition the chain into smaller subintervals and this will generally have a stabilizing effect since long-wavelength instabilities will not fit in [14].

The analysis of the effect of multiple impurities (as well as the related case of finite intervals) is beyond the scope of this work. Here we only mention that introducing more than one impurity can result in more complicated (though still regular) patterns. We illustrate this by simulating the case of two impurities. As one could expect, when two stable stationary solitons are pinned *very* far from each other, they do not interact and remain time-independent. However, if the separation is smaller than a certain critical distance, they start exchanging weak radiation waves and develop spontaneous oscillations ... which subsequently synchronize (Fig. 5)! It is important to emphasize that this latter synchronization occurs not among individual elements but among solitons, i.e. *clusters* of pendula. We have therefore a hierarchy of synchronizations: firstly, the pendula form clusters of synchronous oscillation; secondly, different clusters start oscillating in unison.

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