Common Credit Risk Factors in the Derivatives Option Pricing

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Abstract—**Credit risk is the most important function of derivatives market, and is also the basic reason of the developing of stock market. As one of the most important species of financial derivatives, derivatives option pricing is important to avoid the systemic risk of stock markets. As the main method of risk aversion, Option Pricing is used to manage risk by hedgers, in order to lock profits. The use of Black----Scholes with the risk-neutral option pricing for reference, application of martingale pricing and probability methods. The research work of paper will be helpful to enrich the study derivatives pricing with credit risk.**

Keywords-derivatives; hedge; the optimal hedging ratio; credit risk; martingale

I. Introduction

European options on the spot have identical prices [1] when the option contracts expire simultaneously. The pricing model developed in this study is closely related to Black [2]'s option pricing model except that the variance component in the Black formulation, $\sigma^2\tau$, is replaced by an integral of the form $\int_0^\tau \sigma^2(s)ds$, where τ is the time to expiration of the option and $\sigma^2 s$ depends on the form of the bond pricing model assumed.

In an efficient market with no riskless arbitrage opportunity, and portfolio [3] with a zero market risk must have an expected rate of return equal to the riskless interest rate. The Black-Scholes formulation establishes the equilibrium condition between the expected return on the option, the expected return on the stock and the riskless interest rate.

The pricing problem of option derivatives has been proposed for several years. Now, the pricing method has: partial different equation; martingale theory and numerical schemes. The pricing model of option bond under credit risk is studied. The method of solve equation and pricing formula is gained, pricing method is also given for other derivatives , In addition, the pricing of European option is also studied, in various assumptions and conditions, the pricing model and concrete pricing formula is gained.

Considering a writer of a European call option a stock, it is exposed to the risk of unlimited liability if the stock price rises acutely above the strike price. In an efficient market with no riskless arbitrage opportunity, portfolio [4] with a zero market risk must have an expected rate of return equal to the riskless interest rate. The Black-Scholes formulation establishes the equilibrium condition between the expected return on the option, the expected return on the stock and the riskless interest rate. The paper considers the financial derivatives model with the credit risk, though the value of corporation value model, the credit risk will be introduced to options pricing.

II. Black-Scholes Option Pricing

A. Terminal payoffs

Consider a European call option with strike price X and let S_T denote the price of the underlying asset on the date of expiration T. If $S_T > X$, then the holder of the can option will choose to exercise since he can buy the asset, which is worth S_T dollars, at the cost of X dollars. The gain to the holder from the call option is then $S_T - X$ .However, if $S_T \le X$, then the holder will forfeit the right to exercise the option since he can buy the asset in the market at a cost less than or equal to the predetermined strike Price X .The terminal payoff from the long position in a European call is then

$$\max(S_T - X, 0) \tag{1}$$

The above parabolic partial differential equation is called the Black-scholes equation. Note that the parameter, which is the expected rate of return of the asset, does not appear in the equation .To complete the formulation of the option pricing model, we need to prescribe the auxiliary condition for the European call option.

Similarly, the terminal payoff from the long position in a European put can be shown to be

$$\max(X - S_T, 0) \tag{2}$$

These properties reflect the very nature of options whereby they are exercised only if this results in positive payoffs.

B. **Riskless Hedging Principle**

How to use the riskless hedging principle [5] to derive the governing partial differential equation for the price of a European call [6]. The derivation follows the approach used by Black and scholes in their seminal paper (1973).They made the following assumptions on the financial market:

(i) trading takes place continuously in time;
(ii) the riskless interest rate r is known and constant over time;
(iii) the asset pays no dividend;
(iv) there are no transaction cists in buying or selling the asset or the option, and no taxes;
(v) the assets are perfectly divisible;
(vi) there are no penalties to short selling and the full use of proceeds is permitted ;
(vii) there are no riskless arbitrage opportunities.

The evolution of the asset price S at time t is assumed to follow the Geometric Brownian motion

$$dS = \mu S dt + \sigma S dZ \tag{3}$$

where μ is the expected rate of return and σ is the volatility, and dZ is the standard Wiener process.Both σ and μ are assumed to be constant.Consider a portfolio which involves short selling of one unit of a European call option and long holding of $\triangle$ units of the underlying asset.The value of the portfolio $\prod$ is given by

$$\prod = -c + \Delta S \tag{4}$$

where c=c(S,t)denotes the call price.The call price is a function of the asset price and time.Note that $\triangle$S here refers to $\triangle$ times S,not infinitesimal change in S.The use of this confusing symbol $\triangle$will be justified later .Since both c and $\prod$ are random variables,we apply the Ito Lemma to compute their stochastic differentials as follows:

$$dc = \frac{\partial c}{\partial t} dt + \frac{\partial c}{\partial S} dS + \frac{1}{2} \frac{\partial^2 c}{\partial S^2} \sigma^2 S^2 dt \tag{5}$$

So $\frac{\partial C}{\partial t} + \frac{\partial C}{\partial S} rS + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 = rC$

C. **The Option Pricing with Risk Neutrality Argument**

We would like to present an alternative approach of deriving the Black-Scholes equation for the option pricing model, by which the argument of risk neutrality can be explained in a more succinct manner (Cox and Ross [7], 1976). Suppose we write the stochastic process followed by the option price as

$$\frac{dc}{c} = \mu dt + \sigma dZ \tag{6}$$

where μ is the expected rate of return of c and σ^2 the corresponding variance of the rate of return .
From Eq. (3).we have

$$dc = (\frac{\partial c}{\partial t} + \mu S \frac{\partial c}{\partial S} + \frac{1}{2} \frac{\partial^2 c}{\partial S^2} \sigma^2 S^2) dt + \sigma S \frac{\partial c}{\partial S} dZ \tag{7}$$

The above statement of risk neutrality can be represented mathematically as

$$C = C(S,t) = e^{-r\tau} E[\max(S_T - K, 0)] \tag{8}$$

The option pricing model takes the following form

$$C = SN(d_1) - Ke^{-r\tau}N(d_2) \tag{9}$$

where

$$d_1 = \frac{\ln\left(S/K\right) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} ,$$

$$d_2 = d_1 - \sigma\sqrt{\tau} = \frac{\ln\left(S/K\right) + (r - \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} ,$$

$$\tau = T - t , \sigma = \sqrt{\text{var}(dS/S)} .$$

The above call price formula can be interpreted using the language of probability. First, $N(d_2)$ is seen as the probability of the call option being in-the-money at expiry and so $KN(d_2)$ can be interpreted as the risk neutral expectation of the payment made by the holder of the call option at expiry on exercising the option. Hence, the expectation of the call value at expiry is $Se^{r\tau}N(d_1) - KN(d_2)$, which is then discounted by the factor $e^{-r\tau}$ in the risk neutral world to give the present value of the call price.

III.Derivatives Option Pricing with Common Credit Risk Factors

Within the Black-Scholes model the current value of a call is dependent on five parameters: the current stock price S, the exercise price K, time to maturity $T - t$, the risk-free interest rate r, and volatility s. As the parameters K, r, and s are supposed to be constant, the current stock price and the time to maturity are changing from one to another trading point of time. It is for this reason that the stock price S. In order to improve the original Black-Scholes formulation by relaxing some of the assumption in the model with credit risk, we consider the valuation of the chooser option, which has the feature that the holder can choose whether the option is a call or a put after a specified period of time from the starting date of the option contract.

$$V(S_{T_c}, T_c) = \max(c(S_{T_c}, T - T_c; K), P(S_{T_c}, T - T_c; K))$$

where $T - T_c$ is the time to expiry in both call and put price formulas above, and S_{T_c} is the asset price at time T_c.We

assume the underlying asset pays a continuous dividend yield at the rate q. By the put-call parity relation, the above payoff function can be expressed as

$$V(S_{T_c},T_c) = \max(c, c + Ke^{-r(T-T_c)} - S_{T_c}e^{-q(T-T_c)})$$
$$= c + e^{-q(T-T_c)}\max(0, Ke^{-(r-q)(T-T_c)} - S_{T_c}) \quad (10)$$

Hence, the risk option is equivalent to the combination of one call with exercise price K. Option at the current time is found to be

$$V(S,0) = Se^{-qT}N(x) - Ke^{-rT}N(x - \sigma\sqrt{T})$$
$$+ Ke^{-rT}N(-y + \sigma\sqrt{T_c}) - Se^{-qT}N(-y) \quad (11)$$

where S is the current asset price and

$$x = \frac{\ln\frac{S}{K} + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, y = \frac{\ln\frac{S}{K} + (r-q)T + \frac{\sigma^2}{2}T_c}{\sigma\sqrt{T_c}}.$$

Risk aversion [8] is the most important function of future market, and is also the basic reason of the developing of future market. As one of the most important species of financial futures, stock index future is important to avoid the systemic risk of stock markets. As the main method of risk aversion, Hedging is used to manage risk by hedgers, in order to lock profits. Finally, the results of empirical study show that the capital is closely related to the hedging ratio, and the hedging is very important to risk aversion, the hedging based on this model can improve the return of spot, so the model is appropriate.

IV. Conclusion

Several kinds of option pricing formula of European foreign option with credit risk are obtained. Under different assumptions, the derivatives include two styles: one is the pricing model where the domestic and foreign bond rates are underlying asset and assumed stochastic, using the two approach of martingale theory and partial different equation, another the pricing formula of the derivatives is established which exchange rates, and bond rates follow lognormal diffusion process.

At first, we will derive the lower and upper bounds for the value of an option, which are independent of the modelling of the price development of the stock. If one of these bounds is violated, there is a possibility to pursue time arbitrage, given that both the option and the underlying stock are tradable simultaneously. This strategy implies setting up a portfolio, whose liquidating value is never negative, for any price development of the stock, and whose setup is associated with a positive payment surplus. This strategy is called profitable time arbitrage.

The paper derives a number of special cases of the pricing formulation with common credit risk, using the martingale and probability method to deduce the pricing formula of derivatives option with common credit risk.

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