Image Matching Algorithm Based on an Improved Hausdorff Distance

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***Abstract*—As for the rotation, scaling, cropping in image matching, corresponding algorithm based on Hausdorff distance is deduced. The proposed algorithm extracts feature points by SIFT operator in original image and target image, and determines whether there is a kind of affine transformation between two images by an improved Hausdorff distance and box distance transformation. If there is an affine transformation which satisfies the requirement of the Hausdorff distance, the original image and target image can be matched. If there isn't an affine transformation which satisfies the requirement of the Hausdorff distance, the original image and target image couldn't be matched. The experimental results show that the algorithm can reduce affine space, improve the efficiency of image matching and eventually find the affine transformation between two images.**

Keywords-image matching; Husdorff distance; box distance transformation; affine transformation

I. INTRODUCTION

The research of image matching technology is an important issue in the area of digital image processing. And it is widely used in the domain of image retrieval[1]~[4], image authentication[5], multimedia annotation[6][7], multi-target tracking[8], and medical image analysis. Image matching is the process of spatial alternation of two or more images generated from different time, viewing angle, and mode, so as to make consistency on each image in geometrical space. There are two types of frequently used image matching algorithm, namely matching algorithm based on geometrical characteristic of image and matching algorithm based on gray information of pixel. The later is more suitable for the matching of images generated from the same kind of sensor, and usually fails to match those from different kind of sensors. The former makes use of the correspondence of features between images, so it is suitable for the matching of images generated from different kind of sensors.

In recent years, the application of local invariant feature has greatly improved the accuracy of image matching, making it a hot study object for many scholars. The SIFT algorithm[9] proposed by Lowe et al, satisfying the requirement of displacement, rotation and scale invariant for feature extraction, lays the foundation for image feature matching. Image matching algorithm based on geometric features includes two aspects: similarity measure and search strategy. The traditional geometric feature matching algorithm is often looking for one to one correspondence between the feature points and calculates the similarity between the corresponding feature points to obtain the matching relationship between the images. In this kind of algorithm, the complexity of process is the establishment of image feature pairs. If the number of feature pairs exceeds a certain amount, the calculation time will be multiplied, and when the feature extraction process produces false features or loses features, the matching algorithm based on the corresponding relation of feature points is hard to come to the right result. The traditional search strategy is exhaustive search in the feature space. However, this kind of search strategy requires large amount of calculation. In order to overcome the disadvantages of the above method, we put forward a kind of image matching algorithm based on an improved Hausdorff distance. Instead of establishing one to one correspondence between feature points, it just calculates the degree of similarity between two feature point sets, avoids the enormous calculation work in choosing feature points, solves the problem of time consuming in multiple feature matching and ensures the matching accuracy.

In the beginning, we extract the feature points in the original image and target image respectively. Then we calculate the minimum Hausdorff distance by constructing non-overlapping affine space and use iterative algorithm to search the best affine transformation in affine space. Finally we determine the matching relationship by the minimum Hausdorff distance. If the minimum Hausdorff distance is less than a given threshold, the match is successful and the transform relationship is determined. If not, the match fails.

II. HAUSDORFF DISTANCE

A. The traditional Hausdorff distance

Hausdorff distance is a kind of measurement tool describing the degree of similarity between two point sets. Supposedly there are two sets $A=\{a_1,a_2...a_p\}$ and $B=\{b_1,b_2...b_q\}$, the Hausdorff distance of the two point sets is defined as:

$$H(A,B)=\max\left(h(A,B),h(B,A)\right) \tag{1}$$

Where

$$h(A,B)=\max_{a_i\in A}\min_{b_j\in B}\left\|a_i-b_j\right\| \tag{2}$$

$$h(B,A)=\max_{b_j\in B}\min_{a_i\in A}\left\|b_j-a_i\right\| \tag{3}$$

Symbol ‖·‖ in (2), (3) is the Euclidean distance between point a_i and point b_j. Equation (1) is known as undirected Hausdorff distance, the basic form of Hausdorff distance; Equation (2) and (3) are defined as directed Hausdorff distance from point set *A* to point set *B* and from point set *B* to point set *A* respectively. Equation (2) sorts the distance $||a_i-b_j||$, the distance of each point a_i in point set *A* to the nearest point b_j in point set *B*, and takes the maximum distance. Equation (3) is in the same way. Equation (1) is the bigger one between directed distance (2) and (3). Equation (1) measures the largest degree of non-similarity between two point sets. The greater the Hausdorff distance is, the less similar the two point sets are.

B. The improved Hausdorff distance

Hausdorff distance describes the degree of non-similarity between two point sets, but it is very sensitive to sudden noises. As shown in figure (1), point set *M* and point set *N* are very similar except for a single point in *N* far from *M*, making the value of *h(N,M)* bigger, so being the *H(N,M)*. This sensitivity to outliers is not acceptable in practical recognition tasks.

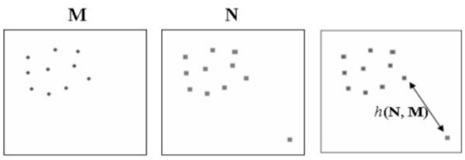


Fig. 1. The sensitivity of Hausdorff distance to isolated point

In order to avoid this problem, Huttenlocher put forward the concept of partial Hausdorff distance[10]:

$$H_{LK}\left(A,B\right)=\max(\ h_L(A,B),h_K(B,A)) \quad (4)$$

Where

$$h_L\left(A,B\right)=L^{th}_{a_i\in A}\min_{b_j\in B}\left\|a_i-b_j\right\| \quad (5)$$

$$h_K\left(B,A\right)=K^{th}_{b_i\in B}\min_{a_i\in A}\left\|b_j-a_i\right\| \quad (6)$$

Equation (5) represents the L^{th} value in the sorted distance value set from point set *A* to point set *B* and (6) represents the K^{th} value in the sorted distance value set from point set *B* to point set *A*. Equation (4) is the undirected Hausdorff distance when *K=L=1*. The partial Hausdorff distance ignores the influence of the disturbance of isolated point and reduces the sensitivity of the Hausdorff distance. However there are some occasions that the degree of similarity between two point sets cannot be accurately measured by the partial Hausdorff distance. To this end, we use average Hausdorff distance:

$$H_{mean}(A,B)=\max\left(h_{mean}\left(A,B\right),h_{mean}\left(B,A\right)\right) \quad (7)$$

Where

$$h_{mean}\left(A,B\right)=\frac{1}{N_A}\sum_{a_i\in A}\left(\min_{b_j\in B}\left\|a_i-b_j\right\|\right) \quad (8)$$

$$h_{mean}\left(B,A\right)=\frac{1}{N_B}\sum_{b_j\in B}\left(\min_{a_i\in A}\left\|b_j-a_i\right\|\right) \quad (9)$$

Symbol N_A, N_B in (8), (9) represent the number of feature points contained in point set *A* and point set *B*. The average Hausdorff distance contains the contribution of all points in point set and it is more real and accurate in describing the degree of similarity between two point sets.

III. MATCHING ALGORITHM

After we obtained feature point set *A* and *B* respectively correspond to the original image and target image by SIFT operator, we apply affine transform with point set *B* ,then calculate the improved Hausdorff distance with point set *A* and *B*:

$$H\left(t_{affine}\left[A\right],B\right)=\max\left(h\left(t_{affine}\left[A\right],B\right),h\left(B,t_{affine}\left[A\right]\right)\right) \quad (10)$$

Symbol t_{affine} in (10) is the transform vector of point set *A*. The task of image matching becomes the task to find out the best affine transform which makes the minimize Hausdorff distance in affine space. The image matching algorithm based on improved Hausdorff distance is as follows[11]~[13]:

(1) Constructing affine space *R* by using 6 parameters of affine transformation, the affine transformation equation is as follows:

$$\begin{pmatrix}X_2\\Y_2\end{pmatrix}=\begin{pmatrix}a_{00}&a_{01}\\a_{10}&a_{11}\end{pmatrix}\begin{pmatrix}X_1\\Y_1\end{pmatrix}+\begin{pmatrix}t_x\\t_y\end{pmatrix} \quad (11)$$

The current affine space can be divided into 26 non-overlapping subspaces by dividing every dimension of the affine space into high-dimensional spaces with low-dimensional spaces:

$$R_i=(R^1_{i+1},R^2_{i+1},R^3_{i+1},...,R^{62}_{i+1},R^{63}_{i+1},R^{64}_{i+1}) \quad (12)$$

Finally, the affine space can be decomposed into unit vectors which comprised of high-dimensional components and low-dimensional components by the iteration algorithm of subspace decomposition. $t_i^{low}=\left[a_{00}^{low},a_{01}^{low},a_{10}^{low},a_{11}^{low},t_x^{low},t_y^{low}\right]$ represents the unit vector composed of the minimum value of all parameters in R_i and $t_i^{high}=\left[a_{00}^{high},a_{01}^{high},a_{10}^{high},a_{11}^{high},t_x^{high},t_y^{high}\right]$ represents the unit vector composed of the maximum value of all parameters in R_i (R_i represents the i^{th} level subspace).

(2) To exam every current subspace R_i, we transform every feature point a_j in the point set *A* by t_i^{low} and calculate the box distance $\Delta_{w,h}'\left(\left[t_i^{low}\left(a_j\right)\right]\right)$. If the number of the points which satisfied $\Delta_{w,h}'\left(\left[t_i^{low}\left(a_j\right)\right]\right)\le\tau$ is greater than a certain threshold, R_i is remarked as interested affine space; if not, exclude this subspace. We are difficult to determine whether the selection of and the threshold is appropriate due to different experimental materials. There is a solution for improvement. After calculated all the values of the box distance in the same level of affine space and sorted the values of the box distance, we select the affine spaces which makes the minimum box distance in the first *n* as interested subspaces (*n=4* under the experiment condition). The purpose is to reduce the amount of calculation and guarantee the accuracy in the process of iteration.

(3) After examining all the same level of subspaces, a list of interested subspaces will be generated. Each interested subspace can be decomposed into lower level subspaces in

accordance with the principle of iteration algorithm. Repeat steps (2), (3) until the current subspace contains only one unit transform.

The key of the algorithm is step (2). The box distance transformation of point set B is described as:

$$\Delta_{w,h}'[x,y] = \min_{\substack{0 \le x' \le w \\ 0 \le y' \le h}} \Delta[x+x', y+y'] \tag{13}$$

Where $\Delta[x,y] = \min_{b_j \in B} \| (x,y) - b_j \|$ represents the shortest distance from point a_j (x,y) to all points in point set B, and $\Delta_{w,h}'([t_i^{low}(a_j)])$ represents the shortest distance between from point a_j to point set B by affine transform t_i^{low}. $t_i^{low}(a_j)$ changes with the change in R_i, but the variation range is controlled by w and h. No matter the change of $t_i^{low}(a_j)$, $\Delta_{w,h}'([t_i^{low}(a_j)])$ always describes the shortest distance between from point a_j to point set B by affine transform t_i^{low}. Generally speaking, w and h are described as:

$$w = \left(a_{00}^h - a_{00}^l\right) + \left(a_{01}^h - a_{01}^l\right) + \left(t_x^h - t_x^l\right) \tag{14}$$

$$h = \left(a_{10}^h - a_{10}^l\right) + \left(a_{11}^h - a_{11}^l\right) + \left(t_y^h - t_y^l\right) \tag{15}$$

The description means that point a_j will reflect on a fixed rectangle measuring $(w+1)\times(h+1)$, with $t_i^{low}(a_j)$ as upper-left angle, by any affine transform in affine space R_i. This way of defining strictly limits the variation range of the box distance, but lacks accuracy. In order to calculate the box distance more accurate, we improve the description of boundary in box distance transform: $w_i = t_i^h(x) - t_i^l(x)$, $h_i = t_i^h(y) - t_i^l(y)$. This description means that point a_j will reflect on an unfixed rectangle measuring $(w_i+1)\times(h_i+1)$, with $t_i^{low}(a)$ as upper-left angle, $t_i^{high}(a)$ as lower-right angle, by different affine transformation $t_i \in R_i$.

(4) The feature point set of the original image is reflected to a new feature point set by the above series of unit transformations, and the Hausdorff distance between new feature point set and target feature point set can be calculated by the average Hausdorff distance. When the minimum value of the Hausdorff distance is less than the threshold , the corresponding affine transformation describes the relationship between the original image and target image, and the match is successful. If not, two images have no affine relation, and the match fails.

IV. Experiment And Analysis

This section describes the experiment and result analysis of the algorithm. All of the images which undergone affine transformation in experiment are generated by Stirmark Benchmark software, and SIFT feature points are attached in the images. The experiment verifies two images related by affine transformation can be matched by the proposed algorithm, and the best affine transformation can be found out.

Fig. 2. original image

(a)

$R = \begin{pmatrix} 0.8984 & 0 \\ 0.0039 & 0.9023 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$H = 0.0539$

(b)

$R = \begin{pmatrix} 1.4961 & 0 \\ 0 & 1.5000 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

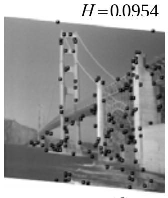
$H = 0.0954$

(c)

$R = \begin{pmatrix} 1.0039 & 0 \\ -0.0898 & 0.8984 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$H = 0.0457$

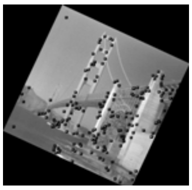


(d)

$R = \begin{pmatrix} 0.8984 & 0.0898 \\ -0.0039 & 1.0000 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$H = 0.0461$

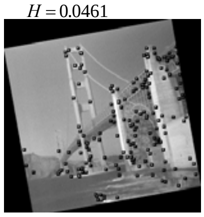


(e)

$R = \begin{pmatrix} 0.9063 & 0.4492 \\ -0.4219 & 0.9102 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$H = 0.0724$

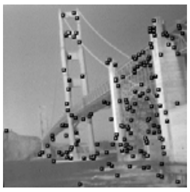


(f)

$R = \begin{pmatrix} 0.9805 & -0.1836 \\ 0.1758 & 0.9844 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$H = 0.0593$



(g)

$R = \begin{pmatrix} 0.9961 & -0.0938 \\ 0.1172 & 0.9922 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$H = 0.0794$

(h)

$R = \begin{pmatrix} 0.9961 & 0 \\ 0 & 0.9961 \end{pmatrix}$

$t = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$H = 0.1853$

Fig. 3 . Image under attack: 90% decrease (a). 150% amplification (b). 90% horizontal reduce and vertical cutting (c). 90% vertical reduce and horizontal cutting (d). 25° clockwise rotation (e). 10° counterclockwise rotation (f). 5° counterclockwise rotation and 15% cutting (g). tampering (h).

The selection of threshold for the minimum Hausdorff distance implies great importance on the result of the experiment. A large number of experiments show that: if $H < \tilde{\iota}$, the corresponding affine transformation of the minimum Hausdorff distance describes the affine transformation between the original image and target image; if $H > \tilde{\iota}$, we can't get the appropriate affine transform between original image and target image.

As we have known, the traditional SIFT matching algorithm uses Euclidean distance among feature vectors as judgment basis for similarity between two images. For example, we are going to extract a certain key point in original image and then find out the nearest two key points in target image by iteration algorithm. If the result of the nearest distance divided by the second nearest distance is smaller than a certain threshold, the result can be concluded that this is a pair of matching points. If not, it cannot match the feature points. In this kind of algorithm, the first step is to find out all the feature points in original and target image. The second step is to use RANSAC algorithm to eliminate false match. It is necessary to traverse all the feature points in target image to determine each pair of feature points in original image and target image. It leads to inefficiency.

This paper proposes that the relation between the minimum Hausdorff distance and the threshold . is enough for judging the match of images rather than finding out every feature point pairs. It requires less calculation than SIFT matching algorithm and we can determine the affine transformation between two images.

V. CONCLUSION

Based on image matching experiment, this paper puts forward a kind of image matching algorithm based on an improved Hausdorff distance to solve the problem of image matching. The first step is to use SIFT operator to extract the feature point set of the original image and target image. The second step is to construct affine space according to the scope of the affine parameters. The third step is to search the best affine transform in the affine space by iteration algorithm. Finally the matching relationship between the original image and target image is judged by the improved Hausdorff distance. The experimental results show that the algorithm can accurately determine the matching relationship between two images.

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