Nodes Selection for Analog Circuits Fault Diagnosis Based on Condition Number

Yanjun Li, Yibing Shi, Wei Zhang
College of Automation Engineering
University of Electronic Science and Technology of China
Chengdu, China P.R.
email: yjli@uestc.edu.cn, ybshi@uestc.edu.cn, weizhang@uestc.edu.cn

***Abstract*—It was proposed that the method of nodes selection for analog circuits soft fault diagnosis, which is using sensitivity vector as the fault signature. The problem of fault diagnosis was formulated into the one of finding the solution for the smallest 2-norm of the diagnosis equations. Considering that the calculation accuracy of the solution was affected by the condition number of the diagnosis equations, a method of nodes selection toward minimal condition number was proposed. The method could be applied with low computation cost and the validity was proved by the simulation results.**

Keywords-sensitivity vector;condition number; analog circuit fault diagnosis;diagnosis equations

I. INTRODUCTION

Analog circuit fault diagnosis is a hot but very difficult topic because of the continuity of the parameters' variations. In recent years, the node voltage increment has been used as fault signature in analog circuit fault diagnosis, and it has made some promising progress [1-3]. It was further proved that the direction of the sensitivity vector is determined by the location of the corresponding element [4]. The sensitivity vector was used as fault signature to locate the faulty element and estimate the parameter variation, and the accuracy of fault diagnosis with tolerance was promoted. However, method for nodes selection was not discussed in the above research works.

In [3], [4], sensitivity or sensitivity vector was used to build the *diagnosis equations* between element parameter variations and node voltage increments. Then, the problem of fault diagnosis was formulated into solving the diagnosis equations. However, the measurement error for node voltage and the variation of node voltage increment caused by element parameter variation within tolerance could be enlarged by the condition number of the diagnosis equations. False fault diagnosis was always lead by big condition number. Nodes selection toward minimizing condition number could improve the accuracy of the minimal norm solution for the diagnosis equations.

The problem of nodes selection for analog circuit fault diagnosis could be formulated into picking row vectors from the *fault pattern matrix* to build the diagnosis equations with small condition number. Tabu search was used in [5] to solve a similar problem. However, the computation cost of tabu search is too big when the number of the row vectors is large. In this paper, it is proved that the condition number of a matrix is bigger than the *maximum row norm ratio*, which is the ratio between the largest row vector norm and the smallest row vector norm. A method toward minimizing maximum row norm ratio was proposed for nodes selection, and the computation cost is small.

II. FAULT DIAGNOSIS BASED ON SENSITIVITY VECTOR

A. Fault Signature

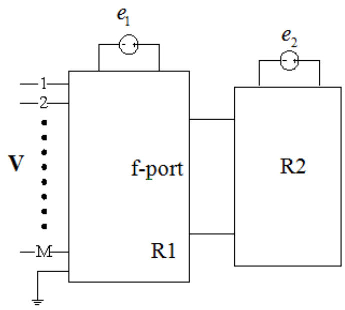

Figure 1. A time-invariant analog circuit with M accessible nodes

Consider the circuit in Fig. 1, in which both R1 and R2 are linear and time-invariant, connected by the f-port, and are connected to the independent source e_1 and e_2 respectively. Both R1 and R2 may include dependent sources; the ones in R2 may depend on the circuit variables in R1, but not vice versa. Let there be *M* nodes accessible for measurement in R1. Denote them $\mathbf{V} = (v_1, v_2, ..., v_M)$ the *measured node-voltage vector.*

It is proved in [4] that when R2 is at fault, there will be

$$\Delta\mathbf{V} = \mathbf{r}\Delta j \quad (1)$$

Where, $\Delta\mathbf{V}$ is called the *fault vector*, which is the node-voltage increment vector. $\mathbf{r}$ is called the *port signature*, which is the transfer resistance vector from the f-port to the *M* accessible nodes. And Δj is the current increment at f-port caused by the fault in R2. Thus, the direction of the vector $\Delta\mathbf{V}$ is the same as the one of the vector $\mathbf{r}$, which can be predetermined and depends only on the location of the f-port.

Consider a circuit with K elements, the current flowing through the *k-th* element can be expressed as

$$j_k = f_k(x_1, x_2, \ldots, x_K) = f_k(\mathbf{x}) \quad (2)$$

Where, x_k is the parameter of the *k-th* element and f_k is determined by the nominal circuit. For small parameter deviation of x_k, the current deviation at the *k-th* element, say the *k-th* f-port, can be expressed as

$$\Delta j_k \cong \frac{\partial f_k(\mathbf{x})}{\partial x_k} \Delta x_k \quad (3)$$

Where, Δx_k is the parameter deviation of x_k.

Denote $S_{x_k}^{V_l}$ the voltage sensitivity from parameter x_k to measurement node l, then in linear circuit $S_{x_k}^{V_l}$ can be formulated as

$$S_{x_k}^{V_l} = \frac{\partial V_l}{\partial x_k} = \frac{\partial V_l}{\partial j_k}\frac{\partial j_k}{\partial x_k} = r_{lk}\frac{\partial f_k(\mathbf{x})}{\partial x_k} \quad (4)$$

Where, r_{lk} is the transfer resistance from the *k-th* element to the *l-th* accessible node. Also, the node-voltage sensitivity vector for x_k can be shown as

$$\mathbf{S}_{x_k}^{\mathbf{V}} = \begin{bmatrix} S_{x_k}^{V_1} & S_{x_k}^{V_2} & \cdots & S_{x_k}^{V_M} \end{bmatrix}^T = \left[\frac{\partial f_k(\mathbf{x})}{\partial x_k}\right]\mathbf{r}_k \quad (5)$$

Where, $\mathbf{r}_k$ is the port signature of the *k-th* element. Obviously, the direction of $\mathbf{S}_{x_k}^{\mathbf{V}}$ is the same as the one of $\mathbf{r}_k$. And when the *k-th* element is at fault, the fault vector can be expressed as

$$\Delta\mathbf{V} \cong \mathbf{S}_{x_k}^{\mathbf{V}} \Delta x_k \quad (6)$$

It is stated in [4] that the direction of $\Delta\mathbf{V}$ is also the same as the one of $\mathbf{S}_{x_k}^{\mathbf{V}}$ when Δx_k is large. So, $\mathbf{S}_{x_k}^{\mathbf{V}}$ can be used as the *fault signature* of the *k-th* element.

B. Fault Diagnosis with Tolerance

Consider that all the elements parameters may deviate from their nominal values, then the fault vector can be expressed as

$$\Delta\mathbf{V} = \sum_{i=1}^{K} \mathbf{S}_{x_i}^{\mathbf{V}} \alpha_i(\Delta\mathbf{x}) \quad (7)$$

Where $\mathbf{S}_{x_i}^{\mathbf{V}}$ is the fault signature for x_i, and $\alpha_i(\Delta\mathbf{x})$ is a scalar nonlinear function of $\Delta\mathbf{x}$. In (7), the small parameter sensitivity theory is extended to large deviations. In analog circuit the circuit parameters always deviate from their nominal values but within their tolerance ranges. Therefore, the allowable percentage of the parameter deviation is usually used to determine whether the element is at fault. Usually, we cannot obtain the function $\alpha_i(\Delta\mathbf{x})$ but the bound of $\alpha_i(\Delta\mathbf{x})$, i.e.,

$$\alpha_i(\Delta\mathbf{x}) \le B_i, \text{ when } \|\Delta\mathbf{x}\| \text{ is within tolerance}$$

Such bound B_i can be obtained by simulation or estimation. Using the bounds, a strategy for fault diagnosis can be formulated as follows:

Equation (7) can be rewritten as

$$\Delta\mathbf{V} = \sum_{i=1}^{K} \mathbf{S}_{x_i}^{\mathbf{V}} B_i \frac{\alpha_i(\Delta\mathbf{x})}{B_i} \quad (8)$$

Or, in matrix form

$$\Delta\mathbf{V} = \mathbf{A}\boldsymbol{\eta} \quad (9)$$

Where,

$$\Delta\mathbf{V} = \begin{bmatrix} \Delta V_1 & \Delta V_2 & \cdots & \Delta V_M \end{bmatrix}^T \quad (10)$$

is the fault vector,

$$\mathbf{A} = \begin{bmatrix} S_{x_1}^{V_1} B_1 & S_{x_2}^{V_1} B_2 & \cdots & S_{x_K}^{V_1} B_K \\ S_{x_1}^{V_2} B_1 & S_{x_2}^{V_2} B_2 & \cdots & S_{x_K}^{V_2} B_K \\ \vdots & \vdots & \ddots & \vdots \\ S_{x_1}^{V_M} B_1 & S_{x_2}^{V_M} B_2 & \cdots & S_{x_K}^{V_M} B_K \end{bmatrix} \quad (11)$$

is called fault pattern matrix, and its column-vectors are called the fault pattern vectors,

$$\boldsymbol{\eta} = \begin{bmatrix} \frac{\alpha_1(\Delta\mathbf{x})}{B_1} & \frac{\alpha_2(\Delta\mathbf{x})}{B_2} & \cdots & \frac{\alpha_K(\Delta\mathbf{x})}{B_K} \end{bmatrix}^T \quad (12)$$

is called the percentage deviation vector.

In (9), the fault pattern matrix can be pre-calculated, and the fault vector is measured. Thus, the percentage deviation vector can be estimated by solving (9) as

$$\boldsymbol{\eta} = \mathbf{A}^{\dagger}\Delta\mathbf{V} \quad (13)$$

Where, $\mathbf{A}^{\dagger}$ is the pseudo-inverse of $\mathbf{A}$.

The faulty element can be located by considering each element of $\boldsymbol{\eta}$. The element of $\boldsymbol{\eta}$ bigger than 1 designates the faulty circuit element.

C. Error of Fault Diagnosis

Fault diagnosis using (13) is not always correct because of the calculation error, especially when the matrix **A** is ill conditioned. In this section, the reason will be discussed in detail. For simplicity, Equation (9) is rewritten as

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{14}$$

Where, $\mathbf{x} = \boldsymbol{\eta}$, and $\mathbf{b} = \Delta\mathbf{V}$.

Consider the elements tolerance and the measurement errors at the accessible nodes, Eq. (14) should be

$$\mathbf{A}(\mathbf{x} + \delta\mathbf{x}) = \mathbf{b} + \delta\mathbf{b} \tag{15}$$

Where, $\delta\mathbf{b}$ is caused by the elements tolerance, the measurement errors or the other factors. And $\delta\mathbf{x}$ is caused by $\delta\mathbf{b}$. Comparing Eq. (14) and Eq. (15), there will be

$$\mathbf{A}\delta\mathbf{x} = \delta\mathbf{b} \tag{16}$$

And

$$\delta\mathbf{x} = \mathbf{A}^{\dagger}\delta\mathbf{b} \tag{17}$$

It can be easily got by using Cauchy-Schwarz inequality that

$$\|\delta\mathbf{x}\|_2 \le \|\mathbf{A}^{\dagger}\|_2 \|\delta\mathbf{b}\|_2 \tag{18}$$

Where, $\|\bullet\|_2$ is spectral norm. Also, from Eq. (16) it can be got that

$$\|\mathbf{b}\|_2 \le \|\mathbf{A}\|_2 \|\mathbf{x}\|_2 \tag{19}$$

Thus, from Eq. (18) and Eq. (19) there will be

$$\frac{\|\delta\mathbf{x}\|_2}{\|\mathbf{x}\|_2} \le \|\mathbf{A}^{\dagger}\|_2 \|\mathbf{A}\|_2 \frac{\|\delta\mathbf{b}\|_2}{\|\mathbf{b}\|_2} \tag{20}$$

Where, $K(\mathbf{A}) = \|\mathbf{A}^{\dagger}\|_2 \|\mathbf{A}\|_2$ is the condition number of matrix **A** in spectral norm. As shown in (20), the fractional error $\|\delta\mathbf{b}\| / \|\mathbf{b}\|$ is magnified $K(\mathbf{A})$ times. When matrix **A** is ill conditioned, say $K(\mathbf{A})$ is big, the result of fault diagnosis may be wrong.

III. NODES SELECTION BASED ON CONDITION NUMBER

A. Proposition of Nodes Selection Problem

Suppose that the number of elements is n, the number of accessible nodes is M. And suppose that no more than m nodes can be selected due to some reasons, such as the cost of test and cost of computation. The problem of nodes selection for analog circuit fault diagnosis can be formulated into the problem as follows:

Let the matrix **A** in (14) be expressed as

$$\mathbf{A} = \begin{bmatrix} S_{x_1}^{V_1} B_1 & S_{x_2}^{V_1} B_2 & \cdots & S_{x_K}^{V_1} B_K \\ S_{x_1}^{V_2} B_1 & S_{x_2}^{V_2} B_2 & \cdots & S_{x_K}^{V_2} B_K \\ \vdots & \vdots & \ddots & \vdots \\ S_{x_1}^{V_M} B_1 & S_{x_2}^{V_M} B_2 & \cdots & S_{x_K}^{V_M} B_K \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_M \end{bmatrix} \tag{21}$$

Where $\mathbf{a}_i$ is the *i-th* row vector. We want to select m rows of A of which the condition number is minimum, i.e., select

$$\mathbf{A}_j = \begin{bmatrix} \mathbf{b}_1^H & \mathbf{b}_2^H & \cdots & \mathbf{b}_m^H \end{bmatrix}^H \tag{22}$$

that minimize its condition number

$$K(\mathbf{A}_j)$$

subject to

$$\mathbf{b}_k = \mathbf{a}_i \text{ for some } i, \text{ and } \mathbf{b}_i \ne \mathbf{b}_j (i \ne j)$$
$$\text{for } (k = 1, 2, \ldots, m).$$

B. Condition Number and Maximum Row Norm Ratio

In this section, the relation between the condition number and the maximum row norm ratio will be derived. Let

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_M \end{bmatrix}$$

Where $\mathbf{c}_i$ is the *i-th* column vector. If matrix **C** is non-singular, let the eigenvalues of $\mathbf{C}^H\mathbf{C}$ be $\lambda_1 \ge \lambda_1 \ge \cdots \ge \lambda_M > 0$. The condition number of **C** is given by

$$K(\mathbf{C}) = \sqrt{\lambda_1 / \lambda_M}$$

Consider the equation $\mathbf{C}\mathbf{x} = \mathbf{b}$, where $\mathbf{x} \ne \mathbf{0}$. We have

$$\|\mathbf{C}\mathbf{x}\|_2 = \|\mathbf{b}\|_2 \le \|\mathbf{C}\|_2 \|\mathbf{x}\|_2$$

Because of $\|\mathbf{C}\|_2 = \sqrt{\lambda_1}$, we have

$$\frac{\|\mathbf{C}\mathbf{x}\|_2}{\|\mathbf{x}\|_2} = \frac{\|\mathbf{b}\|_2}{\|\mathbf{x}\|_2} \le \|\mathbf{C}\|_2 = \sqrt{\lambda_1} \tag{23}$$

Let $\mathbf{C}^{\dagger}$ be the pseudo-inverse of $\mathbf{C}$. We have

$$\mathbf{x} = \mathbf{C}^{\dagger}\mathbf{b}$$

Similar to (23), it can be shown that

$$\frac{\|\mathbf{Cx}\|_2}{\|\mathbf{x}\|_2} = \frac{\|\mathbf{b}\|_2}{\|\mathbf{x}\|_2} \geq \frac{1}{\|\mathbf{C}^{\dagger}\|_2} = \sqrt{\lambda_M} \quad (24)$$

Then, (23) and (24) can be merged into

$$\sqrt{\lambda_M} \leq \frac{\|\mathbf{b}\|_2}{\|\mathbf{x}\|_2} = \frac{\|\mathbf{Cx}\|_2}{\|\mathbf{x}\|_2} \leq \sqrt{\lambda_1} \quad (25)$$

Let

$$L = \max\left\{\|\mathbf{c}_i\|_2 \mid i = 1, 2, \ldots, M\right\} = \|\mathbf{c}_r\|_2$$
$$l = \min\left\{\|\mathbf{c}_i\|_2 \mid i = 1, 2, \ldots, M\right\} = \|\mathbf{c}_s\|_2$$

Theorem 1

$$K(\mathbf{C}) \geq L / l$$

Proof

Let $\mathbf{Ce}_r = \mathbf{c}_r$, where $\mathbf{e}_r$ is a standard basis. From (23), we have

$$\frac{\|\mathbf{Ce}_r\|_2}{\|\mathbf{e}_r\|_2} = \frac{\|\mathbf{c}_r\|_2}{\|\mathbf{e}_r\|_2} = L \leq \sqrt{\lambda_1} \quad (26)$$

Similar to the derivation above, let $\mathbf{Ce}_s = \mathbf{c}_s$ and $\mathbf{e}_s$ is also a standard basis. From (24) there will be

$$l \geq \sqrt{\lambda_M} \quad (27)$$

From (26) and (27), it can be easily got that

$$L / l \leq \sqrt{\lambda_1 / \lambda_M} = K(\mathbf{C})$$

The theorem is proved.

Let

$$\mathbf{A}^H = \mathbf{C} = \begin{bmatrix} \mathbf{a}_1^H & \mathbf{a}_2^H & \cdots & \mathbf{a}_M^H \end{bmatrix}$$

From theorem 1, there should be

$$K(\mathbf{A}) = K(\mathbf{A}^H) \geq S / s \quad (28)$$

Where,

$$S = \max\left\{\|\mathbf{a}_i^H\|_2 \mid i = 1, 2, \ldots, M\right\}$$
$$s = \min\left\{\|\mathbf{a}_i^H\|_2 \mid i = 1, 2, \ldots, M\right\}$$

The relation between the condition number and the row norms of $\mathbf{A}$ is described in (28), because $\|\mathbf{a}_i\|_2 = \|\mathbf{a}_i^H\|_2$.

Definition 1

Define $p = S / s$ be the maximum row norm ratio of matrix $\mathbf{A}$.

C. Nodes Selection toward Minimal p

According to (28), $\mathbf{A}_j$ should be constructed by the row vectors picked from $\mathbf{A}$ to make p minimal. However, the condition number of a matrix is not only related to p but also related to the linear dependence between the row vectors of $\mathbf{A}_j$. If there are several vectors in $\mathbf{A}_j$ that are linearly dependent, the matrix $\mathbf{A}_j$ is ill-conditioned. In this case, the method of nodes selection toward minimal p is designed as follows:

- Compute the norm of $\mathbf{a}_i (i = 1, 2, ..., M)$ in (21) and the mean of the norms of $\mathbf{a}_i (i = 1, 2, ..., M)$.
- If $\mathbf{A}$ is row full rank, skip to the next step. Or, if there are k row vectors in $\mathbf{A}$ that are linearly dependent, keep the vector with the norm nearest to the mean of all the vector norms and remove the rest k-1 vectors.
- Sort the row vectors in $\mathbf{A}$ by norm from the smallest to the largest as

$$\|\mathbf{b}_1\| \leq \|\mathbf{b}_2\| \leq \cdots \leq \|\mathbf{b}_{M-k+1}\|$$

  Where, $\mathbf{b}_k = a_j$ for $k, j = 1, 2, ..., M - k + 1$ and $\mathbf{b}_i \neq \mathbf{b}_j (i \neq j)$. Record the row index of $\mathbf{b}_k$ in $\mathbf{A}$. Consider m nodes will be selected. Let

$$p_i = \frac{\|\mathbf{b}_{i+m-1}\|_2}{\|\mathbf{b}_i\|_2}, i = 1, 2, \ldots, M - k - m + 2$$

- Find p_j subject to

$$p_j = \min\left\{p_i \mid i = 1, 2, \ldots, M - k - m + 2\right\}$$

  The matrix $\mathbf{A}_j$ toward minimal p is

$$\mathbf{A}_j = \begin{bmatrix} \mathbf{b}_j^H & \mathbf{b}_{j+1}^H & \cdots & \mathbf{b}_{j+m-1}^H \end{bmatrix}^H \quad (29)$$

  The row indexes of $\mathbf{b}_i, i = j, j+1, \ldots, j+m-1$ in $\mathbf{A}$ designate the nodes selected from the circuit.

IV. SIMULATION RESULTS

For the purpose of discussion, as shown in Table I a matrix $\mathbf{A}_{20\times 25}$ was used to simulate the fault pattern matrix of a circuit, which has 20 nodes and 25 elements. And each element of $\mathbf{A}$ is generated randomly. The condition number

TABLE I FAULT PATTERN MATRIX WITH 20 ROWS AND 25 COLUMNS

	Elements Number																								
Nodes Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0.15	0.20	0.53	0.11	1.00	0.75	0.88	0.71	0.40	0.65	0.76	0.48	0.74	0.56	0.92	0.93	0.47	0.54	0.77	0.76	0.86	0.57	0.19	0.49	0.57
2	0.66	0.96	0.86	0.18	0.00	0.12	0.39	0.94	0.12	0.03	0.93	0.99	0.44	0.72	0.01	0.97	0.15	0.71	1.00	0.81	0.26	0.38	0.49	0.97	0.43
3	0.63	0.02	0.90	0.75	0.54	0.51	0.38	0.17	0.08	0.79	0.83	0.60	0.05	0.11	0.38	0.36	0.99	0.01	0.23	0.10	0.88	0.21	0.84	0.11	0.08
4	0.23	0.96	0.19	0.05	0.86	0.17	0.27	0.24	0.36	0.92	0.26	0.94	0.09	0.22	0.17	0.64	0.43	0.78	0.92	0.47	0.19	0.79	0.14	0.74	0.29
5	0.18	0.03	0.66	0.07	0.91	0.83	0.87	0.64	0.83	0.49	0.21	0.49	0.80	0.81	0.54	0.07	0.96	0.93	0.64	0.22	0.76	0.15	0.73	0.64	0.56
6	0.17	0.97	0.94	0.49	0.85	0.93	0.74	0.81	0.21	0.83	0.52	0.44	0.66	0.14	0.10	0.21	0.72	0.01	0.11	0.92	0.03	0.49	0.69	0.59	0.63
7	0.15	0.30	0.98	0.85	0.88	0.17	0.45	0.85	0.79	0.13	0.40	0.77	0.03	0.88	0.04	0.04	0.58	0.82	0.27	0.32	0.64	0.01	0.03	0.50	0.93
8	9.78	6.71	2.93	4.53	1.32	1.93	1.18	3.50	1.66	7.54	4.61	1.87	1.49	8.29	4.81	6.51	3.00	2.19	4.53	7.02	1.25	6.73	2.74	9.25	4.23
9	0.94	9.60	0.52	7.37	6.18	6.89	4.10	2.09	2.31	6.22	6.45	6.11	0.28	8.42	0.67	1.34	3.40	8.82	0.10	8.88	3.64	2.03	8.72	5.05	4.68
10	6.62	0.89	5.04	5.10	3.83	0.50	1.20	6.66	0.52	3.94	5.14	0.52	7.57	6.65	8.98	6.39	9.19	0.20	5.99	0.55	6.76	8.69	6.01	6.28	0.23
11	6.03	7.98	7.68	3.83	9.91	1.84	5.72	9.73	9.02	3.59	8.14	5.76	7.96	9.60	4.97	3.85	4.56	3.42	6.02	0.98	3.76	7.51	3.21	7.19	0.65
12	4.74	5.91	2.83	9.05	2.87	0.46	9.49	6.23	7.93	0.89	0.97	8.42	2.94	9.43	7.71	7.66	4.43	7.66	6.49	6.50	8.63	4.19	2.84	0.24	9.24
13	3.56	9.12	2.25	9.65	7.06	8.85	2.56	0.64	3.73	3.42	4.64	5.00	1.15	1.13	0.60	6.53	4.54	3.43	3.43	7.64	2.92	0.00	4.35	5.75	5.34
14	4.76	1.01	3.31	6.28	5.35	8.40	9.90	3.74	8.32	5.49	5.90	4.39	3.75	6.48	2.62	3.81	9.45	6.19	4.93	9.88	1.33	1.49	9.04	0.47	3.67
15	1.82	2.43	3.28	2.49	2.67	3.19	2.34	2.03	4.58	0.34	4.85	0.37	4.02	2.09	1.10	2.04	0.82	3.16	0.27	2.93	4.81	0.02	3.01	0.55	4.49
16	0.76	2.32	4.92	4.25	2.55	2.24	4.85	2.33	2.48	0.34	4.99	1.56	1.23	1.91	1.20	1.24	4.42	3.55	2.50	4.20	2.67	2.56	4.57	2.73	1.54
17	0.75	0.66	4.90	0.95	1.93	1.22	4.21	4.76	0.83	0.83	4.94	4.48	0.32	4.43	0.15	3.26	3.33	3.44	2.16	2.34	2.40	3.39	3.41	3.44	0.31
18	1.75	4.43	1.25	0.62	1.55	4.02	0.39	4.83	1.63	4.74	0.75	4.17	1.32	2.10	3.51	1.60	4.24	1.60	4.52	2.73	3.97	2.83	4.73	0.74	1.10
19	1.68	3.37	3.12	0.01	0.02	4.12	1.19	3.83	1.48	4.06	4.79	0.01	0.51	1.42	0.04	0.52	3.81	2.66	3.15	0.90	0.46	2.39	0.50	3.89	0.41
20	3.92	4.18	3.64	0.76	4.08	4.26	4.09	2.87	2.79	3.55	2.65	3.20	2.42	0.24	3.05	2.68	4.04	4.37	4.92	3.17	4.40	1.60	2.56	2.00	4.75

of **A** in Table I is 158.54 computed by the method using singular value decomposition.

Consider the case that 10 of the 20 nodes will be selected as the accessible nodes for fault diagnosis. In Table II, $\mathbf{A}_1$ is found by the method presented in Section III, and $\mathbf{A}_2$, $\mathbf{A}_3$ as well as $\mathbf{A}_4$ are built by the vectors, which are randomly picked from **A**. The second column presents the row indexes of the vectors in **A**, and the third column presents the condition number of $\mathbf{A}_j\,(j=1,2,3,4)$. Obviously, the condition number of $\mathbf{A}_1$ is the smallest.

TABLE II MATRIX FOUND BY THE VECTORS FROM **A**

	Row Indexes in A										$K(\mathbf{A}_j)$
$\mathbf{A}_1$	18	16	20	13	8	10	9	14	12	11	18.61
$\mathbf{A}_2$	1	19	15	17	18	16	20	13	8	10	47.42
$\mathbf{A}_3$	4	7	6	5	2	1	19	15	17	18	30.39
$\mathbf{A}_4$	5	2	1	19	15	17	18	16	20	13	41.74

V. CONCLUSION

It should be noted that the matrix $\mathbf{A}_j$ found by the method toward minimal *p* is not the optimized one. But this compromised method is applicable for large scale circuit because of the small computation cost. Simulations with some typical circuits are our further work.

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