Transient Stability Detection Scheme Based on the Trajectory Convexity for Multi-Machine Power System

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*Abstract—*In this paper, a convexity index is proposed to detect the stability in a Single Machine Infinite Bus (SMIB) system. Then the criterion is extended in the multi-machine power system which can be induced to an equivalent SMIB system and a supplementary criterion is proposed. Based on the instability criterion presented, we propose a transient stability detection scheme. At last, enormous simulation in IEEE New England 10-machine 39buses power system verifies the accuracy, reliability and fast detection speed of this detection scheme. The stability of all cases are correctly detected online and most unstable cases can be detected in 0.1s after the removal of fault.

Keywords- power system; trajectory convexity; real-time transient stability detection

I. INTRODUCTION

The actual power system often suffers from disturbances such as short circuit, high power load switching and so on. The power system may become unstable after fault cleared. As a result, it is vital important to detect the system stability fast and correctly after a disturbance.

Several methods such as numerical integration method, direct method and stability analysis based on trajectory information [1] are applied to detect the transient stability of power system after a disturbance. The method based on the trajectory information has its unique advantages than others. For example, it doesn't need mass computation and the stability criterion is always simple. Recently, owing to the development of measurement techniques and advances in the study of stability theory, more and more methods have been proposed for out-of-step condition detection based on Wide-Area Measurement System (WAMS) [2] such as equal area criterion, energy function [3], topological energy function [4] and quasi-real-time online transient analysis. However, they have the same shortcoming that some threshold values used are obtained through off-line simulation [5] and some parameters of simulation models vary with different system condition [6].

Therefore, it is necessary to develop a method based on the trajectory information without offline simulation. This paper focuses on the trajectory convexity and proposes a real-time fast transient stability detection scheme.

II. TRANSIENT STABILITY DETECTION METHOD IN A SMIB SYSTEM

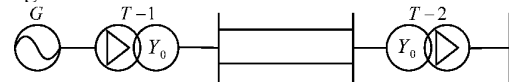A Single Machine Infinite Bus (SMIB) system is showed in Fig. 1.

Fig. 1 A SMIB system

The motion equation of this system can be described as follows.

$$\begin{cases} \dot{\delta} = \omega_0 \Delta\omega \\ M\dot{\Delta\omega} = P_m - P_e - D\omega_0\Delta\omega \end{cases} \quad (1)$$

In equation (1), $\Delta\omega$ is not the real generator's electric angular velocity but the relative angular velocity, that is $\Delta\omega = \omega / \omega_0$. ω_0 is the synchronous electric angular velocity. To simplify the analysis, the damping factor D is set as 0 and the electric power P_e has a sine relation with power angle δ. Equation (1) can be adapted to equation (2) as follows.

$$\begin{cases} \dot{\delta} = \omega_0 \Delta\omega \\ M\dot{\Delta\omega} = P_m - P_{e\max} \sin\delta \end{cases} \quad (2)$$

State variables δ and $\Delta\omega$ can constitute a two-dimensional phase plane, and the curve which is described by δ and $\Delta\omega$ in the phase plane with time t as its parameter is called phase trajectory. If the power system after a disturbance is stable, the trajectory will be convergent and the angle and the angular velocity will be finite4. On the other hand, the angle and the angular velocity will be infinite if the system is unstable after disturbed.

In order to detect the stability of the SMIB system after a disturbance, we firstly focus on the geometric feature of the trajectory. In a phase plane like Fig.2, the slope of the

trajectory is $k = \Delta\omega / \delta$. If we set the convexity index

is $l = dk/d\delta = d^2\Delta\omega/d\delta^2$, the concavity and convexity in stability analysis can be defined as follows:

(1) $l \bullet \Delta\omega < 0$ the trajectory is concave.

(2) $l \bullet \Delta\omega > 0$ the trajectory is convex.

(3) $l \bullet \Delta\omega = 0$ the point is an inflection point.

If we derivative l over time t at the inflexion point, we get the change trend of convexity index l. From equation (2) and the definition of l, we can get following equations.

$$\frac{dl}{dt}\Big|_{l=0} = P_{e\max} M \omega_0^2 (\Delta\omega)^2 \sin\delta \qquad (3)$$

From equation (3), we can get the change direction of the trajectory at the inflection point which can be showed in Fig.2.
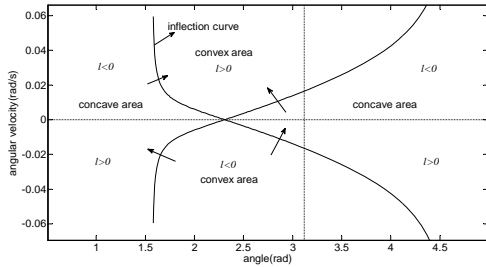

Fig.2 concave and convex area in the phase plane and the change direction of trajectory at the inflection point

In Fig.2, the inflection curve is the curve composed of inflection points. The arrows show the change direction of convexity index l at the inflection points following equation (3). Form Fig.2, we can find that if the system is stable after a disturbance, the trajectory will change in concave area and won't intersect with the inflection curve. On the other hand, if the trajectory in quadrant I intersects the inflection curve, it must come from concave area, enter the convex area and never come back when angle is in the range of $(0, \pi)$, which also means the system is unstable. In fact, trajectory won't become infinite in quadrant II or quadrant IV. As a result, what we concern is whether the trajectory enters convex area in quadrant I and quadrant III to determine the stability of power system after a disturbance.

Fig.3 illustrates the relation between stability of the system after a disturbance and convexity of the trajectory. As long as the trajectory intersects the inflection curve, which means the trajectory enters the convex area, the trajectory won't converge and the system is unstable. Thus necessary and sufficient condition of transient instability of power system can be obtained as follows [7].

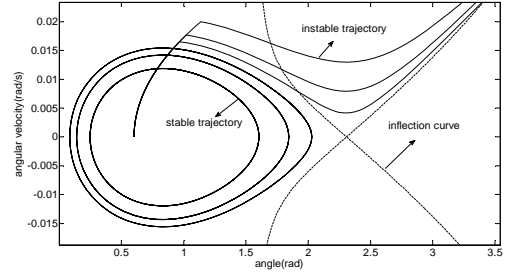$$l \bullet \Delta\omega > 0 \qquad (4)$$

Fig.3 inflection curve and trajectory

Thus we can determine the stability of SMIB system using above instability criterion. The criterion is simple and easy to obtain. What's more, we only need the current data and history data to determine the stability, so it is possible to detect the SMIB system stability by the real-time data.

III. EXTENSION OF THE INSTABILITY CRITERION TO MULTI-MACHINE POWER SYSTEM

A. The SMIB Equivalent system

We have obtained the instability criterion for SMIB system in above section, but it can't not directly be applied in a multi-machine system. The multi-machine system should be simplified to an equivalent system so as to be detected stable or unstable based on the real-time data.

The first step is to get the two-machine equivalent system of the multi-machine system. For a multi-machine system, the motion equation of each machine can be described as equation (5).

$$\begin{cases} \dot{\delta}_i = \omega_0 \Delta\omega_i \\ M_i \dot{\Delta\omega}_i = P_{mi} - P_{ei} \end{cases} \qquad i = 1, 2, \cdots, n \qquad (5)$$

Assume that the disturbed multi-machine system separates in two groups, which are defined as group A and group B respectively. Group A has a greater angle than group B. Using Center of Inertia (COI) as the reference in each group, we can follow equivalent variables.

$$\delta_A = \frac{\sum_{i \in A} M_i \delta_i}{\sum_{i \in A} M_i} \qquad \delta_B = \frac{\sum_{i \in B} M_i \delta_i}{\sum_{i \in B} M_i}$$

$$\omega_A = \frac{\sum_{i \in A} M_i \omega_i}{\sum_{i \in A} M_i} \qquad \omega_B = \frac{\sum_{i \in B} M_i \omega_i}{\sum_{i \in B} M_i} \qquad (6)$$

$$M_A = \sum_{i \in A} M_i \quad M_B = \sum_{i \in B} M_i$$

$$P_{mA} = \sum_{i \in A} P_{mi} \quad P_{eB} = \sum_{i \in B} P_{ei} \tag{7}$$

$$P_{eA} = \sum_{i \in A} P_{ei} \quad P_{mB} = \sum_{i \in B} P_{mi}$$

Therefore equation (5) can be changed to equation (8).

$$\begin{cases} \dot{\delta}_A = \Delta\omega_A & \dot{\delta}_B = \Delta\omega_B \\ M_A \Delta\dot{\omega}_A = P_{mA} - P_{eA} & M_B \Delta\dot{\omega}_B = P_{mB} - P_{eB} \end{cases} \tag{8}$$

Equation (8) is the motivation equation of a two-machine system.

The second step is to reduce the two-machine to a SMIB equivalent system through the following transformation process.

$$\delta_{eq} = \delta_A - \delta_B$$

$$\omega_{eq} = \omega_A - \omega_B$$

$$P_{meq} = \frac{M_B P_{mA} - M_A P_{mB}}{M_T} \tag{9}$$

$$P_{eeq} = \frac{M_B P_{eA} - M_A P_{eB}}{M_T}$$

Where $M_{sum} = M_A + M_B$.

The motivation equation of the equivalent SMIB system can be described as:

$$\begin{cases} \dot{\delta}_{eq} = \omega_{eq} \\ M_{eq} \dot{\omega}_{eq} = P_{meq} - P_{eeq} \end{cases} \tag{10}$$

Where $M_{eq} = \frac{M_A M_B}{M_{sum}}$.

B. Supplementary criterion for the equivalent system

In above section, we have got the SMIB equivalent system to apply the instability criterion to the multi-machine system. However, the criterion is not enough to determine the stability because of the difference between original multi-machine system and the equivalent system. We should propose a supplementary criterion to eliminate the error.

We have get the conclusion that if the trajectory intersects the inflection curve, the change direction of the trajectory is pointing to the convex area from concave area. It has been proved to be right in the SMIB system. However, in the equivalent SMIB system, when trajectory intersects the inflection curve, the direction is uncertain. This is because the equivalent electric power doesn't have the same format as equation (2). As a result, equation (3) is no longer valid and the direction of trajectory at the intersect point is uncertain.

To determine the stability of the equivalent SMIB system, we should add a criterion to ensure the direction when the trajectory intersects the inflection curve. Similarly, we determine the direction through derivative of convexity

index l, which is $\frac{dl}{dt} > 0$. To simplify the analysis, we focus on the trajectory on the upper half plane, which also means that $\Delta\omega > 0$.

Through mathematical derivation, we can obtain the supplementary criterion

$$r = \frac{dl}{dt}\Big|_{l=0} = \frac{d^2(\Delta P/M)}{d\delta^2} > 0 \tag{11}$$

Considering the whole plane, the criterion can be extended as equation (12).

$$r \bullet \Delta\omega > 0 \tag{12}$$

Thereby, the discrete expression of the criterion for instability prediction of equivalent SMIB system can be described finally as

$$l\square\Delta\omega > 0 \ \& \ r\square\Delta\omega > 0 \tag{13}$$

C. Transient stability detection scheme in a multi-machine system

Based on the stability criterion proposed above, we can achieve the goal that detecting multi-machine system stability in real time. The online detection scheme can be detailed as follows.

Step 1: Data read. Input the real time data include the information of angle, angular velocity, electrical power and the mechanical power of each generator.

Step 2: System Equivalence. Simplify the multi-machine system to an equivalent SMIB system

Step 3: Instability criterion calculation. Calculate the value of two instability index τ and μ with the real-time data and equivalent system.

Step 4: Stability detection. Determine the stability of the system by the value of index. If the instability criterion is satisfied, that is, we can get the conclusion that the system is unstable. Otherwise, the system is stable, turn to step 1.

IV. SIMULATION

The proposed transient detection scheme is tested in the IEEE New England 10- machine 39-bus system. We chooses PSASP as the simulation platform [8]. The real-time data is obtained through numerical integration. The transient stability detection scheme reads the real-time data with time interval of 0.01s. Three phase short-circuit disturbance occur in the transmission line between generators buses and the load buses. Both severe and mild faults have been considered by changing the fault-clearing time. The disturbance occurs at the moment of , and is cleared at moment of . The critical clearing time is . We can verify the validity of our results by compare the value of with that of . The instability detection results of using the proposed instability criterion were compared with the results obtained by digital simulation. Table I displays the time taken to detect the stability and the swing equivalent angle at this detected moment.

TABLE I RESULT OF THE PROPOSED METHOD FOR INSTABILITY DETECTION

Fault location Bus no.	Critical clearing time (s)	Practical clearing time (s)	Stability detection	Time taken to detect instability (s)	Equivalent angle at detection moment
1006	0.29	0.27	stable	-	-
		0.28	stable	-	-
		0.29	stable	-	-
		0.3	unstable	0.82	170.9384°
		0.33	unstable	0.20	126.879°
		0.35	unstable	0.09	113.8551°
1008	0.19	0.1	stable	-	-
		0.15	stable	-	-
		0.18	stable	-	--
		0.19	stable	-	-
		0.2	unstable	0.18	121.2645°
		0.25	unstable	0.08	124.4476°
		0.3	unstable	0.04	134.3412°
		0.33	unstable	0.04	149.0995°
		0.35	unstable	0.04	159.5529°
1030	0.29	0.2	stable	-	-
		0.25	stable	-	-
		0.26	stable	-	-
		0.27	stable	-	-
		0.28	unstable	0.09	94.2102°
		0.29	unstable	0.08	95.0361°
		0.3	unstable	0.07	95.7555°
		0.33	unstable	0.04	97.2789°

On one hand, if the fault-clearing time t_c is shorter than the critical clearing time t_{cl}, the multi-machine system will be stable. In Table I, the cases in which fault is cleared before the critical clearing time t_c are all detected stable. On the other hand, if it's too late to clear the disturbance, the system will become unstable. Cases in Table I in which fault-clearing time is greater than the critical fault clearing time t_c are detected unstable by the proposed instability criterion. Even in the cases that the fault-clearing time is close to the critical fault-clearing time (the time interval is as short as 0.01s), the transient stability detection scheme still gives the right stability detection results. The simulation results can fully illustrate the accuracy of instability criterion.

What's more, Table I also shows the proposed criterion's excellent performance on stability detection speed. Most unstable cases are detected in 0.1s after the removal of the faults. Some severe cases even can be detected in 0.04s correctly. In critical cases where fault-clearing time is close to the critical fault-clearing time, it takes more time to detect the stability, no more than 0.9s. Due to the fast detection speed, the equivalent angle is not too great to be controlled. From Table I we can find that the angle is less than 160 degree when instability detected. It leaves enough time to make an emergence decision to ensure system stability. The advantage of detection time is vital important for multi-machine system stability control.

V. CONCLUSION

In this paper, we first illustrate the relation of trajectory convexity and SMIB system stability by proposing the convexity index. Then we extend the instability criterion to the multi-machine system which can be induced to an equivalent SMIB system by proposing the supplement criterion. Finally a new online transient stability detection scheme is proposed for multi-machine power system. This scheme is based on the trajectory convexity and real-time state data of each generator.

The stability criterion has the advantages of small computation and easy-obtained because of its simple discrete form. The reliability and accuracy are also verified by the simulation. What should be pointed out is that the online transient stability detection scheme works effectively. It takes only 0.1s after the removal of fault to detect the unstable cases, some even as short as 0.04s. The fast stability detection leaves enough time for developing following control measures, which is vital important for ensuring power system stability.

Certainly, this detection scheme deserves further exploration. At least it has given a theoretical analysis on how to take advantage of real-time data for transient stability detection, and it also could be used for realizing the real-time close-loop control of power system.

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