Dimension-reduced Robust Beamforming Method for MIMO Radar

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***Abstract*—A new robust beamforming algorithm for multiple-input and multiple-output (MIMO) radar is proposed in the presence of unknown mismatches phenomenon of the desired signal steering vector. Considering that the transmitted waveforms of MIMO radar are orthogonal to each other, after matched filtering MIMO radar can be thought equivalent to multiple single input and multiple output (multi-SIMO) radars. So we decomposed the full-dimensional beamforming of MIMO radar into multiple dimension-reduced beamforming for equivalent multi-SIMO radars. This kind of equivalent has benefits of reducing the number of whole sampling data and the dimensions of sampling data that each equivalent SIMO radar needs to process. Otherwise, we used particle swarm optimization (PSO) algorithm to modify the mismatch of each desired signal steering vector, and the robustness has been greatly improved because the use of PSO. Numerical experiments with the desired signal steering vector mismatches are provided to demonstrate the effectiveness of the proposed dimension-reduced robust adaptive beamforming algorithm.**

Keywords-MIMO radar, robust beamforming, dimension-reduced, equivalent multi-SIMO radars, particle swarm optimization, desired signal steering vector mismatches

I. INTRODUCTION

Multiple-input and multiple-output (MIMO) radar is a research focus in recent years [1, 2] and beamforming is an important branch of MIMO radar [3-9]. In [3, 4], transmit/receive beamforming with minimum variance distortionless response (MVDR) for MIMO radar is proposed, which performance is better than phased-array radar. A bi-Capon algorithm for MIMO radar based on a simplified correlation matrix was investigated in [5], this method decreased the computational complexity and the training samples requirement. Literature [6] gave a beamforming algorithm with two side adaptivity capability such that more active and inactive interferences can be suppressed at a low computational complexity and a low sample number requirement. The dimensions and number of training samples are huge when we use full-dimensional beamforming methods for MIMO radar. Consider the synthetic receiving steering vector is the Kronecker product of transmitting steering vector and receiving steering vector, in order to reduce the dimensions and number of training samples, the beamforming weight vector is decomposed [7-9]. Especially, Literature [7, 8] introduced an iterative procedure using Markov model for design of adaptive linear beamformers, and simulation results demonstrated high accuracy of its analytical derivations. A new beamforming method based on second-order cone programming and bi-iterative algorithm [9] was proposed when the mismatches of the desired signal steering vector existed.

Unfortunately, the non-intelligent algorithms [7-9] have a poor convergence and exactness compared with intelligent algorithm. In this paper, we proposed a new concept of equivalent multi-SIMO radars after matched filtering in order to reduce the dimensions and number of training samples, and used PSO algorithm to modify the mismatches of the desired signal steering vector, the new method has a similar performance with the full-dimensional method and lower complexity than the full-dimensional one.

II. PROBLEM FORMULATION

Consider a MIMO radar system composed of M transmitters and N receivers. The system transmits M orthogonal waveforms simultaneously, denoted by $\mathbf{S}=[\mathbf{s}(1),\mathbf{s}(2),...,\mathbf{s}(M)]^{\mathrm{T}}$, $\mathbf{s}(m)=[s(1),s(2),...,s(L)]$, m=1,2,…,M, l=1,2,…,L, where L means the number of sub-pulses in each transmitted signal pulse. Let θ denote the direction of a point target when the target is in the far filed. Then the transmitting and receiving linear steering vector can be written as

$$\begin{aligned}\mathbf{a}(\theta)&=[1,...,\exp(-j\phi_m),\mathrm{K}\ ,\exp(-j\phi_M)]^{\mathrm{T}}\\ \mathbf{b}(\theta)&=[1,...,\exp(-j\phi_n),\mathrm{K}\ ,\exp(-j\phi_N)]^{\mathrm{T}}\end{aligned} \tag{1}$$

where $\phi_m=2\pi d_t(m-1)\sin\theta/\lambda$, $\phi_n=2\pi d_r(n-1)\sin\theta/\lambda$, m=1,2,…M, n=1,2,…N, d_t and d_r respectively mean transmitting and receiving array element spacing, λ is the carrier wavelength. The echo signal can be described as follows [9]

$$\mathbf{X}=\alpha\mathbf{b}(\theta)\boldsymbol{a}^{\mathrm{T}}(\theta)\mathbf{S}+\mathbf{Z} \tag{2}$$

where α is the scattering coefficient of target, $\mathbf{Z}$ denotes the additive white Gaussian noises and interferences which are independent. The MN outputs after matched filtering are

$$\begin{aligned}\mathbf{y}&=\alpha(\mathbf{R}_{\mathrm{ss}}\otimes\mathbf{I}_N)\mathbf{c}(\theta)+vec(\mathbf{Z}\mathbf{S}^{\mathrm{H}}/L)\\ &=[\mathbf{y}_{r1t1},\mathbf{y}_{r2t1},...,\mathbf{y}_{rntm},..,\mathbf{y}_{rNtM}]^{\mathrm{T}}\end{aligned} \tag{3}$$

where $\mathbf{R}_{ss} = \mathbf{S}\mathbf{S}^{\mathrm{H}} / L$, $\otimes$ means Kronecker product operator, $\mathbf{I}_N$ is an identity matrix of size N. $vec(\cdot)$ denotes vectorized and $\mathbf{c}(\theta) = \mathbf{a}(\theta) \otimes \mathbf{b}(\theta)$. The subscript $rntm$ corresponds to the matched filter output of mth transmitted signal in nth receiving array.

The full-dimensional MVDR beamforming weight vector $\mathbf{w}$ can be obtained by the following optimization problem [9]

$$\min_{\mathbf{w}} \mathbf{w}^{\mathrm{H}} \mathbf{R}_{\mathbf{y}} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^{\mathrm{H}} \mathbf{c} = 1 \tag{4}$$

Then the well-known solution to the above problem is given by

$$\mathbf{w} = \beta \mathbf{R}_y^{-1} \mathbf{c}(\theta) \tag{5}$$

where $\beta = 1/(\mathbf{c}^{\mathrm{H}}(\theta) \mathbf{R}_y^{-1} \mathbf{c}(\theta))$, $\mathbf{R}_y$ is usually instead of the sample covariance matrix $\hat{\mathbf{R}}_y$,

$$\hat{\mathbf{R}}_y = \frac{1}{K} \sum_{k=1}^{K} \mathbf{y}_k \mathbf{y}_k^{\mathrm{H}} \tag{6}$$

where $\mathbf{y}_k$ represents the kth sampling, in order to cut down the loss of signal to noise ratio (SNR), we usually choose the number of $K \geq 2MN$ and ensure the loss of $\mathrm{SNR} < 3\mathrm{dB}$.

The full-dimensional beamforming is shown in Fig. 1.

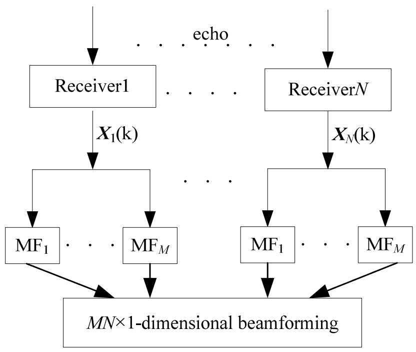

Figure 1. The full-dimensional beamforming

The full-dimensional method needs to estimate $MN \times MN$ matrix and its inverse, its computation complexity reaches to O(KM^2N^2+M^3N^3).

III. New Beamforming Method

A. MVDR based on equivalent multi-SIMO radars without mismatches

Assume the desired signals, interferences and noises are irrelevant. Ideally, the interferences and noises will be suppressed completely after matched filtering, and the desired signal is only present. Then according to formula (3), the ideal full-dimensional beamforming outputs can be expressed as

$$\mathbf{y}_{\mathrm{full}} = \mathbf{w}^{\mathrm{H}} (\mathbf{R}_{ss} \otimes \mathbf{I}_N) \mathbf{c}(\theta) \tag{7}$$

Because the M transmitted waveforms are mutually orthogonal, the desired signal is irrelevant to interference and noise. We can decompose formula (3) into

$$\begin{aligned} \mathbf{y}_1 &= [\mathbf{y}_{r1t1}, \mathbf{y}_{r2t1}, \ldots, \mathbf{y}_{rNt1}]^{\mathrm{T}} \\ \mathbf{y}_2 &= [\mathbf{y}_{r1t2}, \mathbf{y}_{r2t2}, \ldots, \mathbf{y}_{rNt2}]^{\mathrm{T}} \\ &\vdots \\ \mathbf{y}_M &= [\mathbf{y}_{r1tM}, \mathbf{y}_{r2tM}, \ldots, \mathbf{y}_{rNtM}]^{\mathrm{T}} \end{aligned} \tag{8}$$

where $\mathbf{y}_m$ (m=1,2,…,M) represents N outputs of the mth transmitted signal after mth group M matched filter.

That means there are M mutually orthogonal group N×1-dimensional vectors output after MN matched filter. Or can be considered, M virtual and incoherent SIMO radars existed after matched filtering. So the new method only need to estimate M times of N×N matrix. The equivalent beamforming process can be seen in fig. 2.

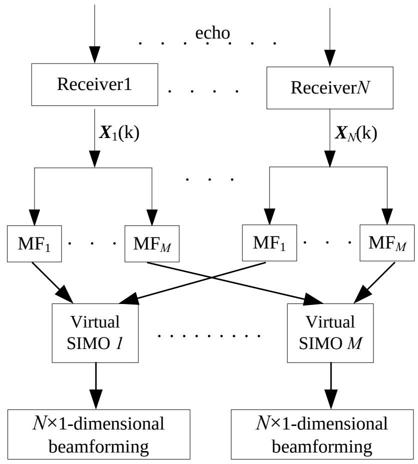

Figure 2. Equivalent beamforming

The mth (m=1,2,…,M) equivalent desired signal steering vector can be defined as

$$\boldsymbol{\rho}_m(\theta) = \exp(-j\phi_m) \mathbf{b}(\theta) \tag{9}$$

Similar to (4), the beamforming weight vector $\mathbf{w}_m$ of the mth equivalent SIMO radar can be written as

$$\mathbf{w}_m = \beta_m \hat{\mathbf{R}}_{y_m}^{-1} \boldsymbol{\rho}_m(\theta) \tag{10}$$

where $\beta_m = 1/(\boldsymbol{\rho}^{\mathrm{H}}(\theta) \hat{\mathbf{R}}_{y_m}^{-1} \boldsymbol{\rho}(\theta))$, $\hat{\mathbf{R}}_{y_m}$ means the estimate covariance matrix of mth equivalent SIMO radar output.

The beamforming output of mth (m=1,2,…,M) equivalent SIMO radar is

$$\mathbf{y}_{wm} = \mathbf{w}_m^{\mathrm{H}} \mathbf{y}_m = \mathbf{w}_m^{\mathrm{H}} (\mathbf{R}_m \otimes \mathbf{I}_N) \boldsymbol{\rho}_m(\theta) \tag{11}$$

where $\mathbf{R}_m = \mathbf{s}_m \mathbf{s}_m^{\mathrm{H}} / L$.

Then the total beamforming output can be described as

$$\mathbf{y}_{\mathrm{reduced}} = \mathbf{y}_{w1} + \mathbf{y}_{w2} + \cdots + \mathbf{y}_{wM} \tag{12}$$

Since it is assumed the desired signals are orthogonal, the desired signals, interferences and noises are irrelevant, the results of formula (7) and (12) should be consistent.

The above analysis is based on the ideal case. If such ideal conditions can not be strictly met, it is easy to understand both the results of full-dimensional and dimension-reduced method should be similar to the same.Therefore, this equivalent method is reasonable.

This new approach has the advantage of reducing the number K of training data, we can just only choose $K \geq 2\max(M, N)$. And the computation complexity of $\hat{\mathbf{R}}_{y_m}^{-1}$ (m=1,2,…,M) is only O(KN^2+N^3) , the total computation of the method of equivalent multi-SIMO radars is O(KMN^2+MN^3). Generally, the array of MIMO radar is very large, so the total computation will be much lower than the full-dimensional method.

B. PSO-MVDR based on equivalent multi-SIMO radars with mismatches

If each of the transmitted signal power is normalized. The output SINR can be described as

$$\mathrm{SINR} = \frac{\sum_{m=1}^{M} \left| \mathbf{w}_m^{\mathrm{H}} \boldsymbol{\rho}_m(\theta) \right|^2}{\sum_{m=1}^{M} \mathbf{w}_m^{\mathrm{H}} \hat{\mathbf{R}}_{y_m} \mathbf{w}_m - \sum_{m=1}^{M} \left| \mathbf{w}_m^{\mathrm{H}} \boldsymbol{\rho}_m(\theta) \right|^2} \tag{13}$$

If we directly use the above method when the desired signal steering vector mismatches exists, the beam can not point to the location of the desired signal with maximum power and the interferences can not obtain the maximum inhibition, so the output signal to interference plus noise ratio (SINR) will be decreased.

Set the true steering vector is $\boldsymbol{\rho}_m(\theta)$, the estimated steering vector is $\hat{\boldsymbol{\rho}}_m(\theta)$, and the estimated beamforming weight vector is $\hat{\mathbf{w}}_m$. Similar to (10) and (13), $\hat{\mathbf{w}}_m$ and the estimated output SINR are

$$\hat{\mathbf{w}}_m = \hat{\beta}_m \hat{\mathbf{R}}_{y_m}^{-1} \hat{\boldsymbol{\rho}}_m(\theta) \tag{14}$$

$$\mathrm{eSINR} = \frac{\sum_{m=1}^{M} \left| \hat{\mathbf{w}}_m^{\mathrm{H}} \hat{\boldsymbol{\rho}}_m(\theta) \right|^2}{\sum_{m=1}^{M} \hat{\mathbf{w}}_m^{\mathrm{H}} \hat{\mathbf{R}}_{y_m} \hat{\mathbf{w}}_m - \sum_{m=1}^{M} \left| \hat{\mathbf{w}}_m^{\mathrm{H}} \hat{\boldsymbol{\rho}}_m(\theta) \right|^2} \tag{15}$$

where $\hat{\beta}_m = 1/(\hat{\boldsymbol{\rho}}^{\mathrm{H}}(\theta) \hat{\mathbf{R}}_{y_m}^{-1} \hat{\boldsymbol{\rho}}(\theta))$.

From (10) and (13), $\hat{\boldsymbol{\rho}}_m(\theta)$ if the mismatches of the desired signal are only considered, we know that the variable of eSINR is. Therefore, we can be adjusted $\hat{\boldsymbol{\rho}}_m(\theta)$ to achieve maximum SINR. Interior point algorithm (IPA) is used [9] in order to tolerate the mismatch, but IPA has a poor robustness and convergence performance. Particle swarm optimization (PSO) algorithm is a kind of swarm intelligent algorithm [10, 11] and widely used in optimal network [12], resources scheduling [13], signal design [14], route planning [15, 16], et al., its robustness and convergence performance is much better than IPA. Hence we use PSO algorithm and the concept of equivalent multi-SIMO radars to modify the beamforming and reduce the computation of solving $\hat{\mathbf{R}}_{y_m}^{-1}$.

Otherwise, in order to speed up to the operating speed of PSO algorithm, during the algorithm is running we will record and skip these poor locations. This strategy can be referred to as ‘Tabu Search’.

The PSO algorithm steps are as follows.

1. Initialize the position $\mathbf{w}_0$ and velocity $\mathbf{v}_0$ of P particles, each particle corresponds to a different steering vector.
2. Calculate each particle’s eSINR using formula (9), and define the local best particle $\mathbf{p}_0$ with maximum eSINR and the global best one $\mathbf{g}_0$ is the maximum eSINR one among P particles.
3. Generate the acceleration and apply the following formula to update the swarm particle.

$$\begin{aligned} \mathbf{w}_{k+1} &= \mathbf{w}_k + \mathbf{v}_k \\ \mathbf{v}_k &= c_1 \mathbf{v}_{k-1} + c_2 \eta (\mathbf{p}_k - \mathbf{w}_k) + c_3 \xi (\mathbf{g}_k - \mathbf{w}_k) \end{aligned} \tag{16}$$

where c1, c2 and c3 are weighting coefficients, both $\eta \in [0,1]$ and $\xi \in [0,1]$ are pseudorandom numbers.

4. Recalculate each new particle’s eSINR after step 3 and update the local best particle and the global best value.
5. Record the location of the poor particle, those poor location will be skipped when the next update.
6. Judge whether the maximum number of iterations is met, go to step 3 if it is not satisfied, otherwise, out of the loop

IV. Numerical Experiments

Consider a MIMO radar system with M=8 transmitters and N=8 receivers, 8 sets of 4-phases encoded signals with L=128 are transmitted. Assume that the direction of the desired signal is 0^0 in the far field and the mismatch is 5^0 , three non-coherent interferences are located in $[-15^0, 15^0, 50^0]$ with different interference to noise ratio (INR), where INR=[70dB, 80dB, 30dB]. The independent Gaussian noise with mean zero and covariance $\sigma_n^2 = \mathbf{I}$ existed in receiver. The output SINRs versus the number of training samples or the input SNR are given.

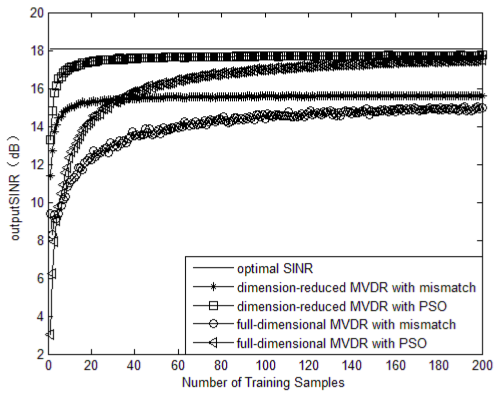

Figure 3. The output SINR versus the number of training samples (SNR=0 dB)

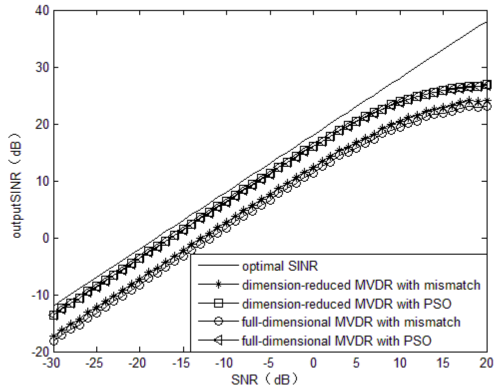

Figure 4. The output SINR versus the input SNR (L=128)

Fig. 4 shows the beamforming output SINR versus the number of training samples with the input SNR= 0dB and the beamforming output SINR versus the input SNR with $K = 128$ is given in Fig. 5, the optimal SINR was also given in both the two figures. From fig. 4 we can see that both the dimension-reduced beamforming method and the full-dimensional beamforming method have a loss of SINR if we directly use the conventional MVDR beamforming method without using PSO algorithm under the condition of the steering vector mismatches phenomenon exists; on the contrary, if we use PSO algorithm to adjust the mismatches phenomenon, the beamforming output SINR has an obvious enhancement. Fig. 4 also shows that the dimension-reduced beamforming method has a better performance than the full-dimensional beamforming method when the number of training samples is less than $K = 2MN = 128$ and more than $K > 2\max(M, N) = 16$, and if K is greater than $2MN = 128$, the performance of both methods are basically the same and good. Fig. 5 confirms that if we use PSO algorithm the performance degradation caused by the steering vector mismatches can be decreased, simulation results in fig. 5 also prove that the dimension-reduced beamforming method is almost consistent with the full-dimensional beamforming method when K=128.

V. Conclusions

This paper firstly proposed a new concept of equivalent multi-SIMO radars after matched filtering. Consider the condition of the desired signal steering vector mismatches, a new dimension-reduced MVDR beamforming method with PSO algorithm based on equivalent multi-SIMO radars was proposed. The new method needs lower number of training samples and computation than the full-dimensional method. The new algorithm has a similar performance compared with the full-dimensional when the training samples number is large enough and also keeps a perfect performance under the condition of lower number of training samples.

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