# An Anti-interfering Reconstruction Algorithm Based on Compressed Sensing

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*Abstract*—**When images compressed by traditional transformation-based compression algorithms are transmitted over wireless channels, if the gaussian random interference causes the loss of the crucial transformation coefficients, the contents of the reconstructed images will be lost obviously and this will reduce the accuracy of the subsequent detection and recognition results greatly. In order to solve this problem, this paper proposed an anti-interfering image reconstruction algorithm based on compressed sensing. This algorithm first confirmed the new compressed sensing signals and the new reconstruction matrix based on the locations of the compressed sensing signal components corresponding to the gaussian-interfered bit stream, and then reconstructed the original images employing the iterative threshold algorithm. The simulation results demonstrated that the new algorithm reconstructed exact images at low bit error rates, and reconstructed inexact images whose qualities were slightly lowered without loss of local contents at high bit error rates. As a result, our algorithm is able to overcome the deficiencies of compression algorithms based on diverse transformations and the iterative threshold algorithm, thus proposes a feasible solution scheme for the anti-interfering problem that arises in wireless image transmission.**

*Keywords- anti-interference; compressed sensing; image reconstruction; Gaussian random interference*

## I. INTRODUCTION

In the informational battlefield environment, images acquired by air-based or space-based sensors need to be transmitted over wireless channels. Because of the bad environment that causes the noise of wireless channels much larger than that of cable channels, and multi-path and shadow fading, the bit error rate of wireless channels is very high, seriously reducing the quality of the decoded image. Thus whether the image encoding and decoding scheme has strong anti-interfering ability rises to the key to ensure the performance of the wireless image transmission system. The traditional image encoding and decoding scheme, named transformation-based image compression, causes obvious image content lack after image reconstruction under the situation of lacking important transformation coefficients caused by bit error, which seriously influences subsequent detection and identification results [1]. Therefore, the realization of high-class image codec on wireless channel is a challenging task.

Compressed sensing (CS) is a new technology in recent years. CS permits, under certain conditions, signals to be sampled at sub-Nyquist rates via linear projection onto a random basis while still enabling exact reconstruction of the original signal[2,3]. Because the measured CS data has excellent anti-interference characteristic[4], this paper proposes an anti-interfering reconstruction algorithm of image compression based on CS. This algorithm reconstructed the images employing the undisturbed part of the whole CS data. Compared with the images reconstructed with complete CS data, the overall quality of the images reconstructed by our algorithm decreased slightly. Compared with the images reconstructed using the data of wavelet transform compression, our reconstructed images had no obvious local image loss, so the influence on the subsequent detection and recognition process was much smaller. Thus the algorithm is applied to wireless image transmission under different levels of bit error rate.

## II. THEORETICAL ANALYSIS OF ANTI-INTERFERING RECONSTRUCTION ALGORITHM BASED ON CS

Wherever Times is specified, Times Roman or Times New Roman may be used. If neither is available on your word processor, please use the font closest in appearance to Times. Avoid using bit-mapped fonts if possible. True-Type 1 or Open Type fonts are preferred. Please embed symbol fonts, as well, for math, etc.

### A. Derivation of the anti-interfering reconstruction algorithm based on CS

CS signals transmitted through the wireless channels will be disturbed by gauss random noise, resulting in distortion of the reconstructed images in different degree, eventually leads to decreasing the probability of subsequent detection and recognition. Therefore, it is necessary to adopt certain anti-interference measures to minimize the impact of gauss random noise on the quality of the reconstructed images.

The theory of compressed sensing is proposed by E. J. Candes, J. Romberg, T. Tao and D. L. Donoho in 2004. CS measurement can be expressed as [5]:

$$y = \Phi f = \Phi \Psi x = \tilde{\Phi} x \qquad (1)$$

Among them, $\Phi$ is known as the measurement matrix, $\Psi$ is the matrix composed of sparse bases, $\tilde{\Phi} = \Phi \Psi$ is known as the sensing matrix, and $x$ is sparse signal.

The measurement $y = \tilde{\Phi} x$ can be extensively written as,

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_m \end{bmatrix} = \begin{bmatrix} \tilde{\phi}_1 \\ \tilde{\phi}_2 \\ \cdots \\ \tilde{\phi}_m \end{bmatrix} x = \begin{bmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & \cdots & \tilde{\phi}_{1n} \\ \tilde{\phi}_{21} & \tilde{\phi}_{22} & \cdots & \tilde{\phi}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{\phi}_{m1} & \tilde{\phi}_{m2} & \cdots & \tilde{\phi}_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ \cdots \\ \cdots \\ \cdots \\ x_n \end{bmatrix} \qquad (2)$$

Each component in $y$ can be expressed as follows,

$$y_k = \left\langle x, \tilde{\phi}_k \right\rangle \qquad k = 1, ..., M \qquad (3)$$

Among them, $\tilde{\phi}_k$ represents the k*th* measurement basis.

If we want to recover the sparse signal $x$ accurately, $y$ and $\Phi$ should satisfy certain conditions [5]. First is that the measurement matrix $\tilde{\Phi}$ must meet restricted isometry property (RIP),

$$(1 - \delta_k) \|x\|_{l_2}^2 \le \|\tilde{\Phi} x\|_{l_2}^2 \le (1 + \delta_k) \|x\|_{l_2}^2 \qquad (4)$$

In this formula $\delta_k$ represents the k*th* order restricted isometry constant, and $x$ is the k*th* order sparse vector. Second is that on the premise of original signals being $N$ dimensions and k*th*-order sparse vector, the measurement number $M$ must satisfy the lower limit,

$$M = O(k\log(N / k)) \qquad (5)$$

Now if the bit stream corresponding to the signal $y$ passes though the wireless channel full of gaussian random interference, then is decoded and directly reconstructed, we undoubtedly get the distorted images. Now that the results of directly reconstruction are not ideal, consider dealing with the interference before reconstruction. Let's view formula (3). $y_k$ represents the k*th* measured component of $y$. Assume that we can determine the location of each $y_k$ whose corresponded bits are disturbed by gaussian noise. Based on above assumption, try to remove completely distorted components in $y$ and the corresponding measurement

bases, and then reconstruct the images. Analyze whether or not this processing procedure can ensure that the reconstructed images are approximate to the optimum solution. The proof is put forward as follows from the reconstruction viewpoint.

*B. Proof*

First introduce the incoherence between the matrices. The literatures [5,6] have demonstrated that the RIP condition that the sensing matrix $\tilde{\Phi}$ should meet is equivalent to the fact that measurement matrix $\Phi$ and sparse matrix $\Psi$ satisfy the incoherence. Suppose that $\Phi$ and $\Psi$ are all $N \times N$ orthogonal matrices, then the coherence between $\Phi$ and $\Psi$ is defined as,

$$\mu(\Phi, \Psi) = \sqrt{N} \times \max_{1 \le k \le M, 1 \le j \le N} \left| \phi_k \psi_j \right| \qquad (6)$$

Among them, $\phi_k$ is in row i of $\Phi$ and $\psi_j$ is in column j of $\Psi$. Obviously $\mu$ measures the biggest cross-coherence between an orthogonal basis of $\Phi$ and an orthogonal basis of $\Psi$. According to Cauchy-Schwarz inequality,

$$\left| \left\langle x, y \right\rangle \right|^2 \le \left\langle x, x \right\rangle \times \left\langle y, y \right\rangle \qquad (7)$$

the result that $\mu(\Phi, \Psi) \le \sqrt{N}$ can be drawn. On the other hand, the constraint $\mu(\Phi, \Psi) \ge 1$ must still be satisfied, otherwise $\left| \phi_k \psi_j \right| < \dfrac{1}{\sqrt{n}}$ will hold, which means $\|\phi_k\|^2 = \sum_{j=1}^{n} \left| \phi_k \psi_j \right|^2 < 1$ and that is conflict with that $\phi_k$ is a unit vector. So the coherence $\mu$ is always in the range of [1, n]. Here we care about only the very $\left| \phi_k \psi_j \right|$ whose value of $\mu$ is small enough, for only when $\mu \approx 1$ the matrix $\tilde{\Phi}$ satisfies the RIP condition to a high probability.

Now if distorted components in $y$ are removed completely and the corresponding measurement bases in $\tilde{\Phi}$ are eliminated, the rest of $y$ is called $y_1$, the rest of $M$ is called $M_1$, and the rest of $\tilde{\Phi}$ is called $\tilde{\Phi}_1$. Correspondingly $\tilde{\Phi}_1 = \Phi_1 \Psi$, and $\tilde{\Phi}_1$ and $\Phi_1$ become $M_1 \times N$ matrix, then

$$\mu(\Phi_1, \Psi) = \sqrt{N} \times \max_{1 \le k \le M_1, 1 \le j \le N} \left| \phi_k \psi_j \right| \qquad (8)$$

By the definition of $\mu$ we can draw the conclusion,
$$\mu(\Phi_1, \Psi) \le \mu(\Phi, \Psi)$$

Based on above conclusion, if it is not coherent between $\Phi$ and $\Psi$, it is not coherent between $\Phi_1$ and $\Psi$. According to the equivalence relation between incoherence and RIP [5,6], $\tilde{\Phi}_1$ satisfies RIP condition.

It is known that $\tilde{\Phi}$ satisfies the $kth$-order RIP constraint, therefore, $\tilde{\Phi}_1$ is sure to satisfy $k_1th$-order RIP constraint. According to $M_1 < M$ and the relationship between measurement dimension and sparsity that $M = O(k\log(N/k))$, $k_1 < k$ is derived.

By $y_1 = \tilde{\Phi}_1 x_1$, the $k_1th$-order sparse vector $x_1$ can be got from the reconstruction algorithm, and $x_1$ is optimally approximate to $x$.

The conclusion is drawn from above proof that when $y_k$ in $y$ is seriously interfered, which results in complete distortion, the following measures can be taken that removing the distorted components $y_k$ and deleting the corresponding measurement bases in $\tilde{\Phi}$ to satisfy the RIP condition again, so that the signal of $x_1$ recovered can be optimal approximate to $x$. This idea can solve the problem effectively that if the CS signals are disturbed by the gaussian random noise the reconstruction results are distorted to varying degrees.

### C. Analysis of anti-interfering reconstruction results

After the interfered data are removed, the reconstruction action is divided into two kinds of circumstances as follows.

- If the remaining number of the dimensions of CS signals is not less than the lower limit after removing finite measured components, accurate reconstruction occurs.
- If the remaining number of the dimensions of CS signals is less than the lower limit after removing finite measured components, there are obvious quality decline in the reconstruction results. The degree of the decline depends on the quantity of remaining CS components. That is, the less the number of the residual components is, the larger the errors are.

### D. Procedure of the algorithm

Block measurement and reconstruction should be employed in order to reduce the cost of measurement and calculation when the original signals are images, the high dimensional data[7,8]. Due to the reasons of being convenient to be embedded optimization strategy in iterative shrinkage/threshold method (IST) [9] in order to eliminate blocking artifacts in the reconstructed images and the fast running speed of IST, our anti-interfering reconstruction algorithm chooses to take root in IST and is called **AntiNoise** algorithm in this paper.

The core of IST algorithm is that in the solving process, the previous estimated values are filtered by a certain threshold so that new estimated values can be obtained. So the most important operations in IST are determining the initial values and processing the threshold. The update process is shown in formula (9).

$$x_{t+1} = \Gamma_\lambda(x_t) = \Psi_\lambda^T [x_t + \Phi^T(y - \Phi x_t)] \quad (9)$$

In above formula $\Phi$ means projection operator which acts directly on the image, $\Gamma_\lambda(x_t)$ acts as threshold processing operator, and $\Psi(x)$ denotes the sparse transform.

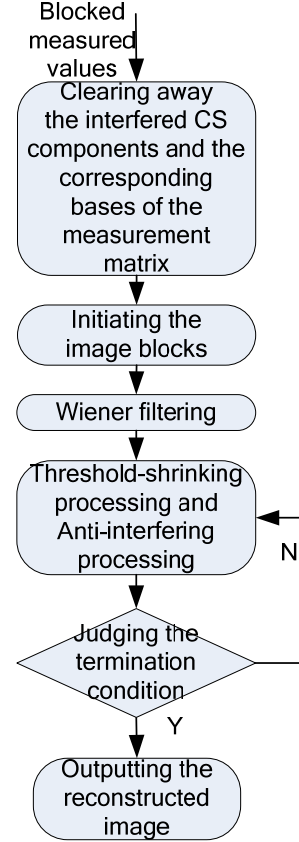The steps of AntiNoise algorithm is as Fig.1 .



Fig.1 Flow chart of AntiNoise algorithm

### III. EXPERIMENTAL RESULTS AND ANALYSIS

The image database used in the experiment is USC-SIPI Image Database, and the size of the images is $256\times256$. In the experiment AntiNoise algorithm is compared with IST. Different results are obtained by adjusting compression ratio (CR), bit error rate (BER) , and sparsity from low to high.

Two original images of the database are listed in Fig.2 (a) and Fig.3 (a). Exact reconstruction by AntiNosie algorithm is compared with the reconstruction by IST in Figure 3, and the related parameters are shown in Table 1 (a). From the parameters we can see that the actual measured dimension $M_{cr}$ is higher than the lower limit $M$, and the remainder measured dimension $M_{rec}$ after the removal of interfered measured components is higher than $M$, too. The error of PSNR is slight and there are no obvious differences visually between AntiNoise and IST by comparing Fig.2 (d) with Fig.2 (b). Inexact reconstruction by AntiNosie algorithm is compared with the reconstruction by IST in Fig. 3, and the related parameters are shown in Table 1 (b). $M_{cr}$ is higher than the lower limit $M$, and $M_{rec}$ is lower than $M$. The error of the images reconstructed by AntiNoise is beyond

a cetain range compared to those by IST, and this can be observed by comparing Fig.3 (d) with Fig.3 (b).



(a) Original   (b) Reconstructed by IST under the condition of no interference
(c) Reconstructed by IST under the condition of low bit error rate
(d)  Reconstructed by AntiNoise under the condition of low bit error rate

Fig.2 Results of exact reconstruction
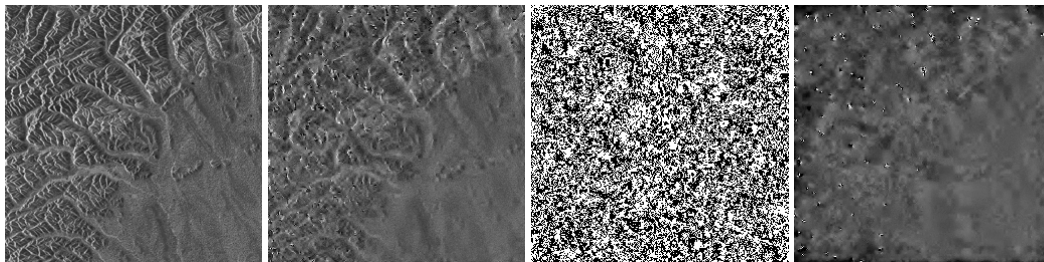


(a) Original   (b) Reconstructed by IST under the condition of no interference
(c) Reconstructed by IST under the condition of high bit error rate
(d)  Reconstructed by AntiNoise under the condition of high bit error rate

Fig.3 Results of inexact reconstruction

TABLE I.        PSNR COMPARISON OF EXACT/INEXACT RECONSTRUCTION RESULTS

（a）Exact Reconstruction

| Parameter | cr（0.3） sparsity（3992）Mcr（16116）M（17307） | | |
|---|---|---|---|
| | Mrec（19700）ber（3E-5） | | |
| | IST | AntiNoise | IST+noise |
| PSNR | 24.19 | 24.13 | 11.24 |

（b）Inexact Reconstruction

| Parameter | cr（0.3） sparsity（4882）Mcr（19712）M（18292） | | |
|---|---|---|---|
| | Mrec（3997）ber（4E-2） | | |
| | IST | AntiNoise | IST+noise |
| PSNR | 21.39 | 17.57 | -35.13 |

The experimental results show that under different compression ratios and bit error rates, the quality of the reconstructed image by AntiNoise algorithm is improved to varying degrees compared with IST. Precise reconstruction occurred when the quantity of remaining CS components was higher than the measured lower limit, and inaccurate reconstruction occurred when the opposite happened.    The size of the reconstruction error is associated with the differences between the lower limit and the number of the remaining components, and the less the differences, the less of the size of the error.

## IV.  CONCLUSION

When the images compressed by traditional transform compression algorithm are decompressed after transmitted through the gaussian white noise wireless channels, the content of the images may be defective, thus severely degrade the performance of detection and recognition. At low bit error rates, the anti-interfering reconstruction algorithm of image compression based on compressed sensing is able to ensure the overall quality of the reconstructed image approaching that in an error free condition.  Under high bit error rates the algorithm ensures that the decrease of the overall image quality occur instead of local image deficiency, which satisfies the need of subsequent detection and recognition tasks. In a word, the algorithm provides a feasible solution for the gaussian noise problem of the wireless channel.

The next step is to select appropriate detection and recognition algorithms to detect the image reconstructed by our algorithm, and further validate the effectiveness of our algorithm according to the detection probability.

## REFERENCES

[1]  H. Zhang, Detection and recognition technology, Beijing Institute of Technology Press,2005.（in Chinese）

[2]  D. L. Donoho, "Compressed Sensing," IEEE Transactions on Information Theory,Vol. 52, pp. 1289–1306, 2006.

[3]  E. J. Candès, "Compressive sampling", Int. Congress of Mathematics, Spain,Vol. 3, pp. 1433-1452, 2006.

[4]  E. J. Candes, J. Romberg and T. Tao, "Robust uncertainty principles:Exact signal reconstruction from highly incomplete frequency information," IEEE Transactions on Information Theory, Vol. 52, pp. 489-509, 2006.

[5]  E. J. Candès and M. B. Wakin, "An introduction to compressive sampling," IEEE Signal Processing Magazine, Vol. 3, pp.  21-30,2008.

[6]  E. J. Candès and J. Romberg, "Sparsity and incoherence in compressive sampling," Inverse Problems, Vol. 23, pp. 969-985, 2007.

[7]  L. Gan, "block compressed sensing of natural images," Proc. of 15th Int. Conf. on Digital signal Processing,UK :Cardiff, Vol. 8, pp. 403 – 406, 2007.

[8]  S. Mun and J. E. Fowler, "Block Compressed Sensing of Images Using Directional Transforms," Proc. of the Int. Conf. on Image Processing, Egypt:Cairo, Vol. 12, pp. 3021-3024,2009.

[9]  T. Blumensath and M. E. Davies, "Iterative hard thresholding for compressed sensing," Applied and Computational Harmonic Analysis, Vol. 27, pp. 265-274, 2009.

[10] Q. S. She, H. Y. SU, L. D. Dong, Jian Chu. "An Adaptive Deblocking Algorithm Based on DCT Domain," Information and Control, Vol. 37, pp. 202-209, 2008. （in Chinese）