

# Effective Identification of the MIMO-OFDM Space-Time-Frequency Channel

Guoqiang Gong and Junqing Liu

**Abstract** An effective base expansion model (BEM) decomposition method for multi-input and multi-output (MIMO) channels is proposed. Channel parameter model is proposed to fit the time-varying space-frequency channel, which can be adopted to estimate the MIMO-OFDM channel state information. The result of channel matrix decomposition includes the main energy component and subordinate components, the subordinate components energy is decreasing with higher order decomposition. Two pilot styles are used to estimate the channel parameters; time-domain pilot block has low computational complexity. Simulations show that good performance can be obtained in MIMO space-time-frequency channels.

**Keywords** BEM · MIMO-OFDM · Time varying channel

## 1 Introduction

Multiple-input and multiple-output (MIMO) wireless systems make use of multiple transmitted and received antennas to increase the communication system capacity through spatial multiplexing. Furthermore, combination of MIMO with orthogonal-frequency-division multiplexing (OFDM), which has simple implementation and robustness against frequency selective fading channel, has been widely used in many standards. Such as IEEE 802.11n, IEEE 802.20 and

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3GPP's LTE etc. As we all know, the performance of MIMO-OFDM systems will deteriorate in time varying wireless channels, because of the channel state information would change at each instant, it is very hard to estimate the channel matrix coefficient, especially, MIMO-OFDM system has too much time-varying channel paths. Even if we adopt pilots in the whole symbol duration, the number of channel matrix coefficients have more than the one of pilots yet.

Base expansion model (BEM) used for time-varying channels is proposed by paper [1], the multi-path channels are divided into time-varying exponent factor and time-unvarying factor. Each path time-varying character is depicted by multiple parallel time-unvarying channels; the BEM is applied in OFDM systems [2] and have been received well performance. Some paper proposed that the MIMO channel has sparse characteristic [3], channels are individually sparse and at the same time share a common support set, therefore, few channel parameters are decomposed from generalization continuous channels. But the appropriate order of channels is a main problem, there are some papers adopting subspace algorithm to estimate the channels [4], it has quite complexity.

There is another attention that the inter-channel interference (ICI) [5] induced by frequency offset would influence the estimated accuracy of channels. Most MIMO-OFDM systems place discrete pilots in frequency domain; it is hard to identify the multi-path channel because of the unknowing signal interference. Some iterative algorithms are proposed, such as Turbo algorithm [6] and parallel ICI cancellation algorithm [7], etc, generally, multi-step iterations can get better performance, but compute complexity and convergence properties are always to consider in these algorithms. Additionally, if we change a style to consider ICI problem, estimation in time domain wouldn't have interference in frequency domain. Some systems adopt time-domain pilots to estimate channel [8], they have some advantages to simplify the processing, inter-symbol interference (ISI) in time domain need to cancel, by contrast, ISI elimination is easier than ICI elimination. Frequency-domain pilots MIMO-OFDM systems usually adopt cycle prefix (CP) to avoid ISI, but time-domain pilots systems have ISI and haven't ICI, we would compare the two schemes in following parts.

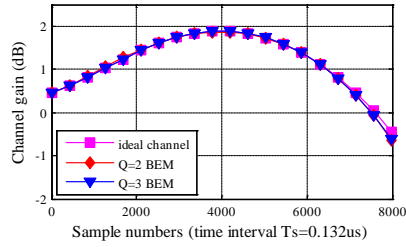
## 2 Base Expansion Channel Model

In wireless mobile channels, the receiving signals are the sum of the multiple delay duplicate induced by the remote scattering objects. Here, we call these delay paths as the separable paths, and they bring about the frequency-selective fading; simultaneity, the affection of near scattering objects is inseparable paths, they are the reason that each delay duplicate is time varying; moreover, the time-varying speed is connected with the Doppler frequency shift caused by moving. Based on the above mention, paper [1] proposed the BEM channel, which is following as:

$$h(n;l) = \sum_{q=0}^Q h_q(l) e^{j\omega_q n} \quad (1)$$

Where  $n \in [0, M-1]$ ,  $\omega_q = 2\pi(q-Q/2)/M$ ,  $Q = 2\lceil f_{\max} MT_s \rceil$ . In the above equation,  $l$  is the separable path,  $h(n;l)$  is the  $l$ th path gain at the  $n$  instant,  $l \in [0, L]$ ;  $q$  is the inseparable path,  $h_q(l)$  is a time-unvarying const in each path,  $T_s$  is the received signal sampling interval,  $f_{\max}$  is the maximum Doppler frequency shift. From the BEM, the time-varying channel is divided into the multiplication between the time varying exponent factor and time unvarying factor, if we have  $(Q+1)$  sample points, we can estimate the  $h_q(l)$ , the all  $[0, K-1]$  instant channel gain can be fitted by equation (1), it is showed in the following Fig. 1. Considering the range of Doppler frequency shift is  $[-f_{\max}, f_{\max}]$ , uniform sampling must be adopted in this range. We modified the exponent factor to fit channel varying better, as the following equation, the more order has the more accurate the channel is.

$$\omega_q = \frac{f_{\max} MT_s}{2} \times 2\pi(q-Q/2)/M \quad (2)$$



**Fig. 1** The curve of time varying channel gains as  $f_{\max} MT_s = 0.2$

Generally, MIMO channel is wide sense stationary uncorrelated scattering (WSSUS) model, any transmitted and received antenna obey this assumption. Moreover, the seldom channel path gains don't equal zero, it has reflected in the most of channel models in typical mobile circumstance. If we can identify the sparse non-zero-gain path, the channel parameters will decrease and the estimated precision will improve.

### 3 MIMO-OFDM Systems and Channel Identification

Suppose that MIMO-OFDM is constituted by  $N$  sub-carriers with  $K$  transmit and  $J$  receive antennas.  $\mathbf{s}_k(i)$  denotes the  $i$ th frequency-domain OFDM signal in the  $k$ th

transmit antennas  $\mathbf{s}_k(i) = [s_k(i,1), s_k(i,2), \dots, s_k(i,N)]^T$ ,  $\mathbf{s}_k(i)$  is transformed to a  $N \times 1$  time domain vector  $\tilde{\mathbf{s}}_k(i)$  after a  $N$  length inverse discrete Fourier transform (IDFT) and inserting a  $L_{cp}$  length cyclic prefix ( $L_{cp} > L$ ).  $\tilde{\mathbf{s}}_k(i) = \mathbf{F}_N \mathbf{s}_k(i)$ ,  $\mathbf{F}_N$  is  $N \times N$  DFT matrix.

The channel impulse response between the  $k_{th}$  transmit antenna and the  $j_{th}$  receive antenna is denoted as  $h^{j,k}(n,l)$ , where  $(n,l)$  denotes the moment  $n, l$  th path channel. So, each channel impulse response from the  $k_{th}$  transmit antenna to the  $j_{th}$  receive antenna in the  $i_{th}$  OFDM signal can be denote as follow.

$$h_i^{(j,k)}(n,l) = h(iN + iL_{cp} + n, l) \quad (3)$$

At the  $j_{th}$  receive antenna, the cyclic prefix is removed and an FFT is computed, let the received time-domain data vector of the  $j_{th}$  receive antenna denotes as  $\tilde{\mathbf{x}}_j(i) = [\tilde{x}_j(i,1), \tilde{x}_j(i,2), \dots, \tilde{x}_j(i,N)]^T$ , where the  $n_{th}$  received time-domain data is

$$\tilde{x}_j(i,n) = \sum_{k=1}^K \sum_{l=1}^L h_i^{(j,k)}(n,l) \tilde{s}_k(i,n-l) + \tilde{w}_j(i,n) \quad (4)$$

where  $\tilde{w}_j(i,n)$  is the Gaussian noise with mean 0 and variance  $\sigma^2$ . After removing CP and a FFT being computed, whereby the  $N$ -dimensional column vector of the observation takes the form as followings.

$$\text{Let } \tilde{\mathbf{S}}(i) = [\tilde{s}_1(i,1), \dots, \tilde{s}_K(i,1), \dots, \tilde{s}_1(i,N), \dots, \tilde{s}_K(i,N)]^T$$

$$\tilde{\mathbf{X}}(i) = [\tilde{x}_1(i,1), \dots, \tilde{x}_j(i,1), \dots, \tilde{x}_1(i,N), \dots, \tilde{x}_j(i,N)]^T$$

$$\tilde{\mathbf{W}}(i) = [\tilde{w}_1(i,1), \dots, \tilde{w}_j(i,1), \dots, \tilde{w}_1(i,N), \dots, \tilde{w}_j(i,N)]^T$$

$$\tilde{\mathbf{X}}(i) = \mathbf{H}^i \tilde{\mathbf{S}}(i) + \tilde{\mathbf{W}}(i) \quad (5)$$

Where  $\mathbf{H}^i$  represents as following:

$$\mathbf{H}^i = \begin{bmatrix} \mathbf{H}^i(1,1) & \mathbf{0} & \dots & \mathbf{0} & \mathbf{H}^i(1,L) & \mathbf{H}^i(1,L-1) & \dots & \mathbf{H}^i(1,2) \\ \mathbf{H}^i(2,2) & \mathbf{H}^i(2,1) & \mathbf{0} & \dots & \mathbf{0} & \mathbf{H}^i(2,L) & \dots & \mathbf{H}^i(2,3) \\ & & & \ddots & & & & \\ \mathbf{H}^i(L,L) & \mathbf{H}^i(L,L-1) & \dots & \mathbf{H}^i(L,1) & \ddots & \ddots & & \\ & & & & & \mathbf{H}^i(N,L) & \dots & \mathbf{H}^i(N,1) \end{bmatrix}$$

where

$$\mathbf{H}^i(n,l) = \begin{bmatrix} h_i^{(1,1)}(n,l) & \dots & h_i^{(1,K)}(n,l) \\ \vdots & & \vdots \\ h_i^{(J,1)}(n,l) & \dots & h_i^{(J,K)}(n,l) \end{bmatrix}, \quad \text{Let}$$

$\mathbf{X} = [X_1(1), \dots, X_j(1), \dots, X_1(N), \dots, X_j(N)]^T$ ,  $\mathbf{S} = [S_1(1), \dots, S_K(1), \dots, S_1(N), \dots, S_K(N)]^T$  denote frequency domain received data and transmitted signal, considering one OFDM duration and ignoring index  $i$ , it has the following

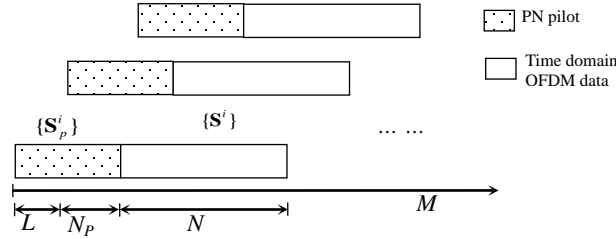
$$\mathbf{X} = (\mathbf{F}_N \otimes \mathbf{I}_J) \mathbf{H} (\mathbf{F}_N \otimes \mathbf{I}_K)^H \mathbf{S} + (\mathbf{F}_N \otimes \mathbf{I}_J) \tilde{\mathbf{W}} \quad (6)$$

Now, BEM given by equation (1) is substituted into the above equations, define  $\mathbf{b}(n) = [e^{jw_0 n}, e^{jw_1 n}, \dots, e^{jw_{Q-1} n}]$ ,  $\mathbf{0}$  is  $Q$  row vector which is all zero.

$$\mathbf{H}^i(n, l) = \begin{bmatrix} \mathbf{b}(n) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{b}(n) & \mathbf{0} & \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{b}(n) \end{bmatrix} \begin{bmatrix} h_0^{(1,1)}(l) & h_0^{(1,2)}(l) & \dots & h_0^{(1,K)}(l) \\ \vdots & \vdots & & \vdots \\ h_Q^{(1,1)}(l) & h_Q^{(1,2)}(l) & \dots & h_Q^{(1,K)}(l) \\ \vdots & \vdots & & \vdots \\ h_0^{(J,1)}(l) & h_0^{(J,2)}(l) & \dots & h_0^{(J,K)}(l) \\ \vdots & \vdots & & \vdots \\ h_Q^{(J,1)}(l) & h_Q^{(J,2)}(l) & \dots & h_Q^{(J,K)}(l) \end{bmatrix} = \mathbf{B}(n) \mathbf{h}(l) \quad (7)$$

### 3.1 Time Domain Pilot Block

If we adopt time domain pilots, we can reference the Chinese Digital Television Terrestrial Broadcasting standard; pseudo noise (PN) sequences are replacing the traditional CP. We design different PN in each transmitted OFDM signal head in different antenna. It shows in the following Fig. 2.



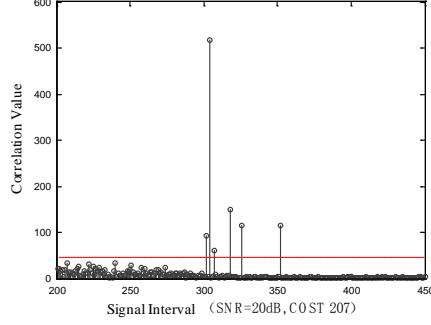
**Fig.2** MIMO-OFDM Pilot Structure

Channel identification is to compute the time unvarying coefficient channel  $\mathbf{h} = [\mathbf{h}(0), \dots, \mathbf{h}(L)]^T$ , minimum mean square estimation (MMSE) method is usually to compute them, which is following as:

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}} E\{\|\tilde{\mathbf{X}}(i) - \mathbf{H}^i \tilde{\mathbf{S}}(i)\|^2 | \tilde{\mathbf{X}}_p(i), \tilde{\mathbf{S}}_p(i)\} \quad (8)$$

It is able to see that the computing process exist in time domain, if we haven't FFT operation for OFDM signals, it can avoid ICI terms, in order to cancel ISI, there are  $L$  length prefix in the front of PN sequences. Additionally, there is another advantage that the correlation of PN sequences can be used to distinguish

the sparse paths. In fact, only seldom path gains are not zero, we can define a detecting threshold to find the main channel paths, it shows in the following Fig. 3.



**Fig. 3** Main channel paths detection

### 3.2 Frequency Domain Scattered Pilots

Frequency domain received signal is composed of two parts, which are the corresponding frequency point value and the superimposing neighbor frequency range ICI values.

Channel identification is very difficult, which shows in the following equation, many papers adopt iterative algorithm to estimate, in the initial step, ICI terms are regarded as noise interference terms; in the next steps, ICI term is subtracted while the estimated data is restored to frequency signal [2].

$$\hat{\mathbf{h}} = \arg \min_{\mathbf{h}} E\{\|\mathbf{X}(i) - (\mathbf{F}_N \otimes \mathbf{I}_J) \mathbf{H}^i (\mathbf{F}_N \otimes \mathbf{I}_K) \mathbf{S}(i)\|^2 | \mathbf{X}_p(i), \mathbf{S}_p(i)\} \quad (9)$$

It is difficult to resolve  $\mathbf{h}$  directly, we adopt the method of paper [9], what is the best linear unbiased estimator (BLUE). A linear filter  $\mathbf{F}$  is used to produce an unbiased estimate  $\hat{\mathbf{h}} = \mathbf{F}\mathbf{X}$ ,  $\mathbf{F}$  is computed by iterative algorithm, we will check performance in section IV, detail process is described in paper [9].

## 4 Simulations

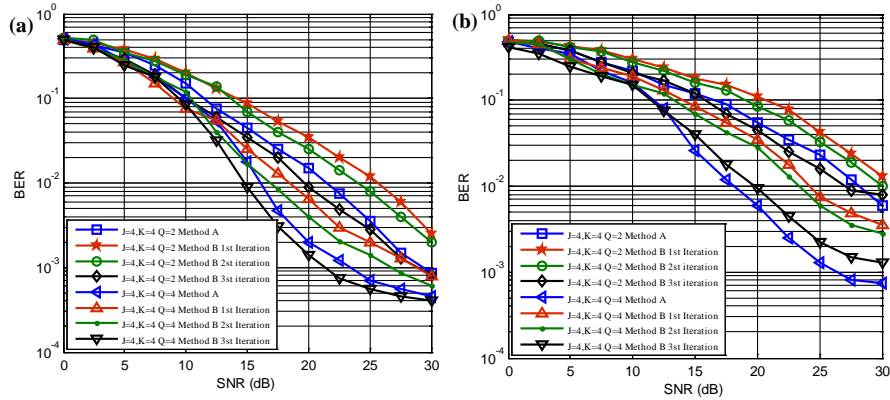
In the simulation, we adopt time varying channels conforming Jakes' Doppler profile using the spatial channel models given in paper [9]. The model generates the channel coefficients according to a set of selected parameters (e.g., delay, PAS, speed, etc.). It is a general stochastic model whose spatial channel paths are statistically independent. We assume the simulation conditions be satisfied with

following: i) the channel path gains fulfill normalize condition. ii) frequency domain pilots are distributed to insert into subcarrier of each transmitted antenna on the same position, furthermore, pilot sequences are orthogonal. iii) Each transmitted symbol power is normalized. The detail parameters are depicted by Table 1. Carrier center frequency is  $1\text{GHz}$ , one OFDM duration is  $0.5\text{ms}$ , 32 time domain PN pilots is block in the front of OFDM signal with 32 prefix sequences, frequency domain pilots is also 32.

**Table 1** MIMO Channel Parameters

Path	Delay (ns)	Gain (dB)
1	0	0
2	310	-1.0
3	710	-9.0
4	1090	-10.0
5	1730	-15.0
6	2510	-20.0
Speed (Km/h)		120,240
		Label
Carrier	$N_C$	256
Transmitter	$M_R$	2 (or 4)
Receiver	$M_T$	2 (or 4)
Pilots	$N_P$	32

Fig. 4 shows the result that two pilots' structures are used to channel identification. When mobile speed is  $120\text{Km/h}$ , the time-domain block pilots can estimate BEM parameters better than frequency-domain scattered pilots, generally, after multiple step iteration, bit error ratio would convergence, the iterative steps are more than three. With the increasing of SNR, BER attaches a floor slowly. From the simulation, we can see that frequency-domain method could attaches better performance by iteration than time-domain method, because the whole OFDM signal is regarded as pilots, furthermore, the ICI is not serious.



**Fig. 4** BER curve comparison of time-domain pilots and frequency-domain scattered pilots, (a) speed is  $120\text{km/h}$  and antenna is  $2 \times 2$ ; (b) speed is  $240\text{km/h}$  and antenna is  $4 \times 4$

When the speed is increased to 240Km/h and the antennas number is  $4 \times 4$ . It is obviously that  $Q=2$  BEM is not enough to fit the time-varying channel, it has comparatively worse BER performance. When  $Q=4$  BEM is adopted, the BER has great improvement. The frequency-domain method performance couldn't over the time-domain method though it has multiple step iteration, because the serious ICI induces the BER floor soon. Simultaneously, the antennas number is an unhelpful factor in time-varying channels.

## 5 Conclusions

In this paper, the base expansion model for multi-path time varying channel is used to identify the MIMO-OFDM channel. Two styles of pilots are used to estimate the BEM parameters, the performance of time domain method and frequency method is compared, obviously, time domain method has lower computational complexity and better bit error ratio. It is hard to overcome the ICI to obtain channel parameters in frequency domain, the numerical simulation results verify the analysis.

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