

Estimation Formula for Noise Variance of Motion Blurred Images

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Abstract. The key problem in restoration of motion blurred images is how to get the point spread function and the noise information. It is commonly assumed as the white Gaussian noise. The paper corrects mistakes in literature [1] and puts forward an accurate estimation formula for noise power of linear motion-blurred images and provides the relative error formula. The relative error is very small. Experiments prove that the noise power can be computed accurately by utilizing this method.

Keywords: Image Restoration • White Gaussian Noise • Motion-blurred Images

1 Introduction

The key problem in motion-blurred image restoration is how to get the point spread function (PSF) and the noise information. If the PSF and the power spectrum of noise are given, a lot of methods for restoring images are discussed^[2-4]. The noise is commonly assumed as the white Gaussian noise with zero mean. The variance of noise in smooth area is regarded as noise power. The common method to estimate the variance of noise is to compute the variance of smooth area in the noisy-blurred image^[2]. Sometimes, the smooth area does not exist and the result is rough in this way. Suppose the motion direction is horizontal, the length of motion blur can be estimated by utilizing these methods in literature, such as Spectrum, Cepstrum, and Whitening and so on^[5-8]. The size of image is assumed as $M \times N$ and σ^2 denotes the variance of noise and the blurred length is a pixels. [1] Proposed a method for estimating noise power that a superposition operator on the difference of the motion-blurred image is used to produce a noise-dominated image. The variance of the noise-dominated image

is approximately equal to $\frac{1}{3} \frac{N}{a} \sigma^2$. But there is a serious error in the literature. The

estimation formula was not accurate and the relative error was overestimated. The paper will correct mistakes and optimize reasoning and simulation. The correct formula will be proved in section two and the simulation will be done in section three.

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2 The algorithm on estimating the noise variance

For convenience, assume that the direction of movement is horizontal. In a very short period of time, variable motion can usually be approximated as uniform motion. The uniform motion is assumed along x direction, the speed $x_0(t) = \frac{at}{T}$, where a is constant and T is the exposed interval. Let $f(x, y)$ be the original image. Let $n(x, y)$ be the Gaussian white noise with zero mean and variance σ^2 . $g(x, y)$ denotes the motion blurred image with noise. $g(x, y)$ can be obtained as follows:

$$\begin{aligned} g(x, y) &= \int_0^T f[x - x_0(t), y] dt + n(x, y) = \int_0^T f[x - \frac{at}{T}, y] dt + n(x, y) \\ &= \int_{x-a}^a f(\tau, y) d\tau + n(x, y) \end{aligned} \quad (1)$$

with $\tau = x - \frac{at}{T}$.

Since motion is along x direction, y can be omitted. Eq. (1) can be rewritten

$$g(x) = \int_{x-a}^a f(\tau) d\tau + n(x) \quad (2)$$

Taking the derivative of $g(x)$ with respect to x in Eq. (2) gives

$$g'(x) = f(x) - f(x-a) + n'(x) \quad (3)$$

Given that the size of image is $M \times N$. Let

$$K = \left\lfloor \frac{N}{a} \right\rfloor, \quad m = \left\lfloor \frac{x}{a} \right\rfloor, \quad x = z + m a, \quad \text{where } z \in [0, a) \quad (4)$$

K is the integer part of (N/a) , m is the integer part of (x/a) .

Inserting Eq. (4) into Eq.(3) gives

$$f(z + ma) = g'(z + ma) + f(z + (m-1)a) - n'(z + ma) \quad (5)$$

Let $\phi(z) = f(z - a)$, we can rewrite Eq. (5) as

$$f(z + ma) = \sum_{j=0}^m g'(z + ja) + \phi(z) - \sum_{k=0}^m n'(z + ja) \quad (6)$$

Eq. (4) and Eq. (6) imply that

$$f(x) = \sum_{j=0}^m g'(x - ja) + \phi(x - ma) - \sum_{j=0}^m n'(x - ja) \quad (7)$$

$$\phi(x - ma) = f(x) - \sum_{j=0}^m g'(x - ja) + \sum_{j=0}^m n'(x - ja) \quad (8)$$

When x varies from 1 to N , m varies from 0 to $K-1$. We can derive Eq. (9) from Eq. (8)

$$\begin{aligned} K\phi(x) &= \sum_{k=0}^{K-1} f(x + ka) - \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} g'(x + ka - ja) \\ &\quad + \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} n'(x + ka - ja) \end{aligned} \quad \text{for } 0 \leq x < a \quad (9)$$

Hence

$$\begin{aligned} \phi(x - ma) &= \frac{1}{K} \sum_{k=0}^{K-1} f(x + ka - ma) - \frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} g'(x - ma + ka - ja) \\ &\quad + \frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} n'(x - ma + ka - ja) \end{aligned} \quad (10)$$

Eq. (7) and Eq. (10) imply that

$$\begin{aligned} &\frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} g'(x - ma + ka - ja) - \sum_{j=0}^m g'(x - ja) \\ &= -f(x) + \frac{1}{K} \sum_{k=0}^{K-1} f(x + ka - ma) - \sum_{j=0}^m n'(x - ja) \\ &\quad + \frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} n'(x - ma + ka - ja) \end{aligned} \quad (11)$$

Let

$$P(x) = -\sum_{j=0}^m g'(x - ja) + \frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} g'(x - ma + ka - ja) \quad (12)$$

$$R(x) = -\sum_{j=0}^m n'(x - ja) + \frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} n'(x - ma + ka - ja) \quad (13)$$

$$Q(x) = -f(x) + \frac{1}{K} \sum_{k=0}^{K-1} f(x + ka - ma) \quad (14)$$

Hence

$$P(x) = Q(x) + R(x) \quad (15)$$

$\sigma^2(R(x)), \sigma^2(f(x))$ denote respectively the variance of $R(x), f(x)$.

Inserting Eq. (4) into Eq. (13) gives

$$\begin{aligned} R(z + ma) &= -\sum_{j=0}^m n'(z + (m-j)a) + \frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^{j=k} n'(z + ka - ja) = \\ &= -\sum_{j=0}^m n'(z + ja) + \frac{1}{K} [Kn'(z) + (K-1)n'(z+a) + \dots + n(z+(K-1)a)] \\ &= \frac{1}{K} [-n'(z+a) - 2n'(z+2a) - \dots - m \times n'(z+ma) \\ &\quad + (K-(m+1))n'(z+(m+1)a) + \dots \\ &\quad + 2n'(z+(K-2)a) + n'(z+(K-1)a)] \end{aligned} \quad (16)$$

$n'(z+ia)$ for $0 \leq i \leq K-1, 0 \leq z < a$ are independent random variables with zero mean and variance $2\sigma^2$. So $R(z+ma)$ for $0 \leq m \leq K-1, 0 \leq z < a$ are independent random variables with zero mean and variance as:

$$\begin{aligned} \sigma^2(R(z+ma)) &= \frac{1}{K^2} [1^2 + 2^2 + \dots + m^2 + (K-(m+1))^2 + \dots + 2^2 + 1^2] 2\sigma^2 \end{aligned} \quad (17)$$

$$\sigma^2(R(x)) = \frac{1}{K} \sum_{m=0}^{K-1} \sigma^2(R(1+ma)) \quad (18)$$

There is an error in [1]. The error is $\sum_{m=0}^{K-1} \sigma^2(R(1+ma)) = [\frac{1}{6}K^2 + O(K)] 2\sigma^2$.

In fact, we can get the exactly result Eq. (19) from Eq. (18).

$$\begin{aligned} \sigma^2(R(x)) &= \frac{1}{K} \sum_{m=0}^{K-1} \sigma^2(R(1+ma)) \\ &= \frac{2\sigma^2}{K^3} [(K-1)*1^2 + (K-2)*2^2 + \dots + (K-(K-1))*(K-1)^2] \end{aligned}$$

$$\begin{aligned}
&= \frac{2\sigma^2}{K^3} [(K * 1^2 + K * 2^2 + \dots + K * (K-2)^2 + K * (K-1)^2) \\
&\quad - (1 * 1^2 + 2 * 2^2 + \dots + (K-2) * (K-2)^2 + (K-1) * (K-1)^2)] \\
&= \frac{2\sigma^2}{K^3} [(K * (1^2 + 2^2 + \dots + (K-1)^2) - (1^3 + 2^3 + \dots + (K-1)^3)]
\end{aligned}$$

Simplifying the above formula and getting Eq. 19.

$$\sigma^2(R(x)) = \left(\frac{1}{3}K - \frac{2}{K}\right)\sigma^2 \quad (19)$$

Inserting Eq. (4) into Eq. (14) gives

$$Q(z + ma) = \frac{1}{K} \sum_{k=0}^{K-1} [f(z + ka) - f(z + ma)] \quad (20)$$

$$\sigma^2(Q(x)) \approx \frac{2}{K}\sigma^2(f(x)) \quad (21)$$

Eq. (15), Eq. (19) and Eq. (22) imply that

$$\begin{aligned}
\sigma^2(P(x)) &\approx \sigma^2(R(x)) + \sigma^2(Q(x)) \approx \left(\frac{1}{3}K - \frac{2}{K}\right)\sigma^2 + \frac{2}{K}\sigma^2(f(x)) \\
&= \frac{1}{3}\sigma^2 \left[K + \frac{1}{K} \left(\frac{2\sigma^2(f(x))}{\sigma^2} - 1\right)\right] = \frac{1}{3}\sigma^2 \left[K + O\left(\frac{1}{K}\right)\right] \approx \frac{1}{3} \frac{N}{a} \sigma^2.
\end{aligned}$$

Finally, we get the estimation formula Eq. (23)

$$\begin{cases} \sigma^2 = \frac{3a}{N} \sigma^2(P(x)) \\ P(x) = -\sum_{j=0}^m g'(x - ja) + \frac{1}{K} \sum_{k=0}^{K-1} \sum_{j=0}^k g'(x - ma + ka - ja) \end{cases} \quad (23)$$

$P(x)$ is the result of superposition operation on the difference of the motion-blurred image. We compute $\sigma^2(P(x))$ and get the noise variance σ^2 from equation (23).

The relative error e_r can be obtained as follows:

$$e_r \approx \frac{1}{K^2} \left(\frac{2\sigma^2(f(x))}{\sigma^2} - 1 \right) \quad (24)$$

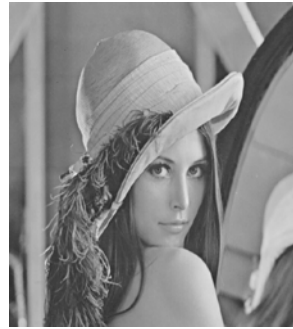
The relative error e_r is $O(\frac{1}{K^2})$. If K is large enough, e_r is nearly zero. That e_r is $O(\frac{1}{K})$ in [1] is wrong.

3. Simulation

In order to validate the algorithm, we have carried out simulation experiments and repeated the experiments in [1]. We use the images Lena (512x512), Cameraman (256x256), Peppers (384x512) and the gray value is from 0 to 1. Simulation images about Lena are shown in Fig.1. The results about noise variance estimation are shown in Table 1. Table 1 proves that the estimation method is considerably accurate.

Table 1 Variance of white Gaussian noise in images

image	Blur length	True variance	Estimation of variance	Relative error
Lena (512x512)	9	0.0010	0.00103	3%
	9	0.0050	0.00501	0%
	17	0.0050	0.00506	1%
	63	0.0050	0.00493	1%
Cameraman (256x256)	9	0.0010	0.00104	4%
	9	0.0050	0.00460	7%
	17	0.0050	0.00462	7%
	63	0.0050	0.00467	6%
Peppers (384x512)	9	0.0010	0.000987	1%
	9	0.0050	0.004970	0%
	17	0.0050	0.004916	2%
	63	0.0050	0.004891	2%



(a) Original image



(b) Blur length 9 , noise variance 0.001



(c) Blur length 9, noise variance 0.005.



(d) Blur length 17, noise variance 0.005

Fig. 1 Original image and noisy motion-blurred images of different blur length and noise variance about Lena.

4. Conclusions

The paper corrects mistakes in [1] and puts forward an accurate estimation formula for noise power of linear motion-blurred images. The estimation formula relative error is very small. Experiments prove that the noise power can be computed accurately by utilizing this method. Disadvantage of this approach is the need to known blur length.

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