Study on New DOA Estimation Method based-on ULA Under Gaussian Noise

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Abstract: The DOA (Direction of Arrival) estimation problem ULA (uniform linear array) was studied, and one new LS-ICA method was proposed based on ULA (uniform linear array) under Gaussian noise, which was filtered during signals separation by ICA. With strict mathematic reasoning, steps of the new DOA estimation method were given. The simulation results showed that the proposed method was more superior to one of MUSIC (Multiple Signal Classification), especially in degree distinguishing capacity and short data length aspects. In the case of simulation parameters of paper, it was 4 times than MUSIC method in degree distinguishing capacity and one tenth in effective data length, which proved method of LS-ICA for DOA estimation was feasible and reliable. Also it provided one new thought for array signals processing and DOA problem.

Key words: DOA estimation; LS-ICA; degree distinguishing capacity; MUSIC

1.Introduction

Array signal processing, which is one data processing method by array antenna sampling space signals, has become one important branch of signal processing, applied in communication, radar, earthquake detection and so on[1]. As one of important questions of array signal processing, DOA estimation of space signals plays a role in target orientation and tracking[2]. Among DOA estimation methods, traditional ones, such as Multiple Signal Classification, Estimation of Signal Parameters via Rotational Invariance Technique[3], usually make use of orthogonal characteristic between signal subspace and noise subspace. In MUSIC method, degree distinguishing capacity is influenced by data length, which confines the improvement of time efficiency. Due to its poor performance to coherent source signals, there are some improved MUSIC methods[4], blind source separation (BSS) method based on genetic algorithm[5] by use of smoothness thinking, improving time efficiency as well as adding computing. In addition, most methods searches the whole space area so that computing is huge, thus, literature [6] combines MUSIC with FFT (Fast Fourier Transform), which reduces space searching range.

On the basis of smart array factor use, the paper solves the DOA problem by combining ICA with LS way marked as LS-ICA in short[1].

2.Model and basic algorithm of received array signal

In ULA, there are M array cells, distance of near array cells is d. Assume that incident signal is $S(t)$ with an angle of incidence of θ, distance between signal source and array antenna is more than d, thus incident signal is thought as surface wave when it enters antenna array. Take array antenna 1 as reference cell (figure 1), suppose received signal in this antenna is $x_1(t)=s(t)$, then received signal in cell 2 is[7]:

$$x_2(t)=s(t)e^{j\omega_0(t+\tau)}=s(t)e^{j(\omega_0 t+\omega_0\tau)}=s(t)e^{j(\omega_0 t+\frac{2\pi d}{\lambda}\sin\theta)} \tag{1}$$

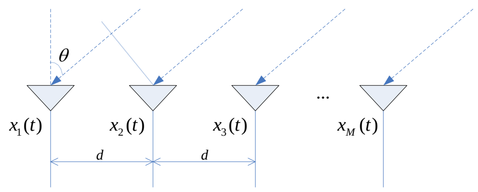

Fig. 1 system model of array antenna

Simply, input array signal vector could be described as:

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$$X(t)=s(t)e^{j\omega_0 t}[1,e^{j\frac{2\pi d}{\lambda}\sin\theta},\cdots,\ e^{j(M-1)\frac{2\pi d}{\lambda}\sin\theta}]^{\mathrm{T}}=s(t)e^{j\omega_0 t}\boldsymbol{a}(\theta) \tag{2}$$

Where $X(t)=[x_1(t),x_2(t),\cdots,\ x_{\mathrm{M}}(t)]^{\mathrm{T}}$, $\boldsymbol{a}(\theta)=[1,e^{j\frac{2\pi d}{\lambda}\sin\theta},\cdots,\ e^{j(M-1)\frac{2\pi d}{\lambda}\sin\theta}]^{\mathrm{T}}$ is direction vector. Correspondingly, the received digital sample signal's expression can be:

$$X(n)=s(n)\boldsymbol{a}(\theta),\quad n=1,2,\cdots,\ N \tag{3}$$

If there are L signals as $s_1(n),s_2(n),\cdots,\ s_L(n)$ come to ULA with the incidence angle of $\theta_1,\theta_2,\cdots,\ \theta_L$ at the same time, then received signals can be described as the following:

$$X(n)=\begin{bmatrix}x_1(n)\\x_2(n)\\\cdots\\x_M(n)\end{bmatrix}=\begin{bmatrix}1&1&\cdots&1\\e^{j\frac{2\pi d}{\lambda}\sin\theta_1}&e^{j\frac{2\pi d}{\lambda}\sin\theta_2}&\cdots&e^{j\frac{2\pi d}{\lambda}\sin\theta_L}\\\vdots&\vdots&&\vdots\\e^{j(M-1)\frac{2\pi d}{\lambda}\sin\theta_1}&e^{j(M-1)\frac{2\pi d}{\lambda}\sin\theta_2}&\cdots&e^{j(M-1)\frac{2\pi d}{\lambda}\sin\theta_L}\end{bmatrix}\begin{bmatrix}s_1(n)\\s_2(n)\\\vdots\\s_L(n)\end{bmatrix}=\boldsymbol{AS}(n) \tag{4}$$

In above formula, $\boldsymbol{S}(n)$ is the input signal vector, $\boldsymbol{A}$ is antenna array factor. When there exists noise, it can be mended as:

$$X(n)=\boldsymbol{AS}(n)+\boldsymbol{N}_t(n) \tag{5}$$

In most researches, it is confined to applicable range from different DOA estimation methods. Furthermore, if method performance is made better, it should do some filtering measurements before signal processing in received port. Algorithm proposed by the paper also follow the consideration, here give the explanation.

3. Gaussian noise processing during DOA estimation

In the paper, we consider it is Gaussian noise. Assume that L signals are influenced by additional gauss white noise (AGWN), it is reasonable to study the noise as a whole with Gaussian character. Analysis is as follows:

If, noise1~ $N(\mu_1,\sigma_1^2)$, $\cdots$, noise$L$~ $N(\mu_L,\sigma_L^2)$ (6)

Then, noise1+$\cdots$+ noise$L$~ $N(\mu_1+\cdots+\mu_L,\sigma_1^2+\cdots+\sigma_L^2)$ (7)

Mark it as: Noise~ $N(\mu_1+\cdots+\mu_L,\sigma_1^2+\cdots+\sigma_L^2)$ (8)

4. DOA estimation method based on LS-ICA

4.1. LS problem brought by ICA for analysis of ULA signals

For simple description, let $\alpha_i=\dfrac{2\pi d}{\lambda}\sin\theta_i$,i=1,2,$\cdots$,L, take out the real part of formula (5), then it is converted to:

$$\begin{bmatrix}x_1(n)\\x_2(n)\\\cdots\\x_M(n)\end{bmatrix}=\begin{bmatrix}1&1&\cdots&1\\\cos(\alpha_1)&\cos(\alpha_2)&\cdots&\cos(\alpha_L)\\\vdots&\vdots&&\vdots\\\cos((M-1)\alpha_1)&\cos((M-1)\alpha_2)&\cdots&\cos((M-1)\alpha_L)\end{bmatrix}\begin{bmatrix}s_1(n)\\s_2(n)\\\vdots\\s_L(n)\end{bmatrix} \tag{9}$$

In this expression, $n=1,2,\cdots,\ N$, and $N>L$.

If LS-ICA method is applied to DOA estimation of array signal, the intention is to solve α_i in formula (9), but the question is that $s_i(n)$ is unknown just. Whereas, blind separation technology based ICA would conquer the contradiction. Now, the focus is how to solve the two uncertain questions after BSS[9]: (1) turn of source signals has changed; (2) signal extent varies. For the first question, from formula (9) it can be seen that different location of $s_i(n)$ would not influence value of $x_i(n)$ in the left side, nothing but the solved degrees of space signal has different turn. In fact incidence angles have been known at present. For the second question, signal after BSS is $\tilde{s}_i(n)$, which extends one ratio factor than standard signal $s_i(n)$, namely $s_i(n)=k_i\tilde{s}_i(n)$. Problems would be solved if k_i could be confirmed. Take one variable replace to the first equation in formula (9):

$$k_1\tilde{s}_1(n)+k_2\tilde{s}_2(n)+\cdots+k_i\tilde{s}_i(n)+\cdots+k_L\tilde{s}_L(n)=x_1(n) \tag{10}$$

The solution to k_i in formula (13) is one typical LS problem[8]. Therefore, mark every part in formula (10) with $\tilde{\boldsymbol{S}}^{\mathrm{T}}$, $\boldsymbol{k}$, $\boldsymbol{x}_1$, it can be transformed as:

$$\tilde{\boldsymbol{S}}^{\mathrm{T}}\boldsymbol{k}=\boldsymbol{x}_1 \tag{11}$$

Further,

$$\boldsymbol{k}=(\tilde{\boldsymbol{S}}\tilde{\boldsymbol{S}}^{\mathrm{T}})^{-1}\tilde{\boldsymbol{S}}\boldsymbol{x}_1 \tag{12}$$

Up to now, $s_i(n)$ in formula (9) could be estimated exactly, α_i in the formula is also got.

4.2 method steps

Step1: Sample antenna received signal $\boldsymbol{X}(t)$ to discrete signal sequence $\boldsymbol{X}(n)$, which is one matrix with the size of $M\times N$ in fact.

Step2: because of $L<M$, choose $L+1$ signals randomly from sequence $\boldsymbol{X}(n)=[x_1(n),x_2(n),\cdots, x_M(n)]^{\mathrm{T}}$, namely $\boldsymbol{X}=[x_1'(n),x_2'(n),\cdots, x_q'(n)]^{\mathrm{T}}$, $x_q'(n)$ is one random signal sequence from M antenna arrays, and $x_q'(n)\subset\boldsymbol{X}(n)$, $q=1,2,\cdots,L$. Then, do ICA processing for the mixed signals including original signals and one Gaussian noise, get L blind estimation signal sequences $\tilde{s}_1(n),\tilde{s}_2(n),\cdots, \tilde{s}_L(n)$. The paper adopts typical ICA method as FastICA[9].

Step3: To solve value of $\boldsymbol{k}$ according to formula (10) to (12). By the transformation of $s_i(n)=k_i\tilde{s}_i(n)$, L original signals can be got exactly.

Step4: Crude estimation of DOA. Specify formula (9) again:

$$\begin{cases} s_1(n)+s_2(n)+\cdots+s_L(n)=x_1(n) \\ \cos(\alpha_1)s_1(n)+\cos(\alpha_2)s_2(n)+\cdots+\cos(\alpha_L)s_L(n)=x_2(n) \\ \cos(2\alpha_1)s_1(n)+\cos(2\alpha_2)s_2(n)+\cdots+\cos(2\alpha_L)s_L(n)=x_3(n) \\ \vdots \\ \cos((M-1)\alpha_1)s_1(n)+\cos((M-1)\alpha_2)s_2(n)+\cdots+\cos((M-1)\alpha_L)s_L(n)=x_M(n) \end{cases} \tag{13}$$

In above formula, let $p_i^{(m)}=\cos((M-1)\alpha_i)$, here $i=1,2,\cdots,L$, $m=2,3,\cdots,M$, formula (13) is converted to:

$$\begin{cases} s_1(n)+s_2(n)+\cdots+s_L(n)=x_1(n) \\ p_1^{(2)}s_1(n)+p_2^{(2)}s_2(n)+\cdots+p_L^{(2)}s_L(n)=x_2(n) \\ p_1^{(3)}s_1(n)+p_2^{(3)}s_2(n)+\cdots+p_L^{(3)}s_L(n)=x_3(n) \\ \vdots \\ p_1^{(M)}s_1(n)+p_2^{(M)}s_2(n)+\cdots+p_L^{(M)}s_L(n)=x_M(n) \end{cases} \tag{14}$$

Obviously, it can be seen that, every solution of α_i, from the 2nd equation to M_{th} equation in formula (14), is to solve LS problem with the shape of formula (10). So we could choose one or several from the M-1 equations to DOA.

When it is crude estimation, LS estimation is done by the second equation in formula (14), then $\boldsymbol{p}^{(2)} = [p_1^{(2)}, p_2^{(2)}, \cdots, \ p_L^{(2)}]$ will be got as:

$$\boldsymbol{p}^{(2)} = (\boldsymbol{S}\boldsymbol{S}^{\mathrm{T}})^{-1}\boldsymbol{S}\boldsymbol{x}_2 \tag{15}$$

In this equation, $\boldsymbol{S}$ and $\boldsymbol{x}_2$ have the same meaning with that in formula (11). And, $\boldsymbol{S}$ is $\boldsymbol{S}(n)$ appeared in the former.With that, by $\alpha_i = \dfrac{2\pi d}{\lambda}\sin\theta_i$ and $p_i^{(2)} = \cos(\alpha_i)$, estimation value of θ_i is:

$$\theta_i = \arcsin\left(\frac{\lambda}{2\pi d}\arccos\left(p_i^{(2)}\right)\right) \tag{16}$$

In literature [10], it is discussed that how array cell internal d could influence performance of DOA estimation, so here only let $d = \lambda / 2$, next, estimation value of θ_i could be simply showed:

$$\theta_i = \arcsin\left(\arccos\left(p_i^{(2)}\right)/\pi\right) \tag{17}$$

Step5: Refined estimation of DOA. It is known from formula (18) that the more sample data LS estimation uses, the higher precision estimated degrees has. Hence, make every two equations or three or more in formula (17) be added in turn to generate one new LS problem, then more precise value can be got step by step by solving the LS problem. Initially, make the first three equations be added:

$$f(\cos(\alpha_1))s_1(n) + f(\cos(\alpha_2))s_2(n) + \cdots + f(\cos(\alpha_L))s_L(n) = x_1(n) + x_2(n) + x_3(n) \tag{18}$$

where, $f(\cos(\alpha_i)) = 1 + \cos(\alpha_i) + \cos(2\alpha_i)$, because of $\cos(2\alpha_i) = 2\cos^2\alpha_i - 1$, so

$$f(\cos(\alpha_i)) = \cos(\alpha_i) + 2\cos^2\alpha_i \tag{19}$$

To point it out, $\cos((M-1)\alpha_i)$ is always polynomial of M-1 times of $\cos(\alpha_i)$, for example, $\cos(3\alpha_i) = 4\cos^3(\alpha_i) - 3\cos(\alpha_i)$, $\cos(4\alpha_i) = 8\cos^4(\alpha_i) - 8\cos^2(\alpha_i) + 1$, etc. therefore, coefficient of $s_i(n)$ is always polynomial of $\cos(\alpha_i)$, namely, it is $f(\cos(\alpha_i))$.

Let $\boldsymbol{f}(\cos(\square)) = [f(\cos(\alpha_1)), f(\cos(\alpha_2)), \cdots, f(\cos(\alpha_L))]^{\mathrm{T}}$, then

$$\boldsymbol{f}(\cos(\square)) = (\boldsymbol{S}\boldsymbol{S}^{\mathrm{T}})^{-1}\boldsymbol{S}(\boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3) \tag{20}$$

For $f(\cos(\alpha_i))$ in formula (24), we can solve $\cos(\alpha_i)$ by dichotomy method, with $p_i^{(2)}$ in crude estimation course as the initial value, which makes it easier than method by solving inverse of expression of $f(\cos(\alpha_i))$. Here, we mark result by dichotomy as $f^{-1}(\cos(\alpha_i))$ all the same, thus, there is $\cos(\alpha_i) = f^{-1}(\cos(\alpha_i))$ by dichotomy, so refined estimation value of degree can be got this time:

$$\theta_i = \arcsin\left(\arccos\left(\cos(\alpha_i)\right)/\pi\right) = \arcsin\left(\alpha_i/\pi\right) \tag{21}$$

Step6: End of DOA estimation course.

5. Simulation experiment

Under MATLAB software environment, some parameters are set as: number of antenna cell is M=8, number of signal source is L=3, incidence angles are separately 30, 45 and 60 degree, three incidence signals are $s_1 = \sin(t)$, $s_2 = \sin(20t - 0.5)$, $s_3 = 2.2\sin(50t)$, array cell internal is $d = \lambda/2$, data sampling length is 1024, noise is Gaussian White Noise.

Firstly, separate the received mixture signals by ICA method and estimate the source signals nearly by LS method.

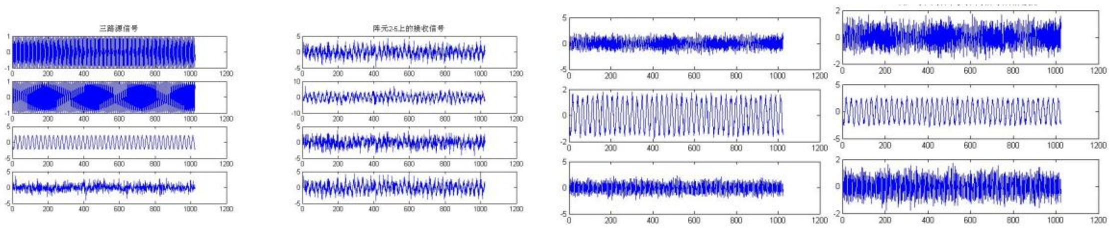


Fig. 2 kinds of signals in ICA process (a. three source signals and noise b. received signals in array 2 to array 5 c. signals after separation d. signals extended by ratio)

5.1 DOA estimation performance by LS-ICA in different SNR

DOA estimation by LS-ICA in different SNR is shown in table 1.

Table 1 DOA estimation by LS-ICA in different SNR

SNR(dB)	0	2	4	6	8	10	12	14	16	18	20
Angle1	27.65	28.71	28.34	28.34	27.65	29.02	27.65	27.65	28.34	27.65	27.65
Angle2	43.81	44.94	43.64	43.64	43.81	43.10	43.81	43.81	43.64	43.81	43.81
Angle 3	60.07	60.07	58.72	58.72	60.07	59.06	60.07	60.07	58.72	60.07	60.07

For comparison, continuous curve in figure 3 is spatial spectrum drawn by MUSIC method.. Easily being seen, in the case of low SNR, estimation value by LS-ICA can be collected near real value, which will be nearer to real value when SNR increases. In MUSIC method, on low SNR, peak location is not obvious or the DOA value has one bigger offset. Figure 4.c shows results by two method in SNR=10, which proves LS-ICA method is superior to MUSIC one.

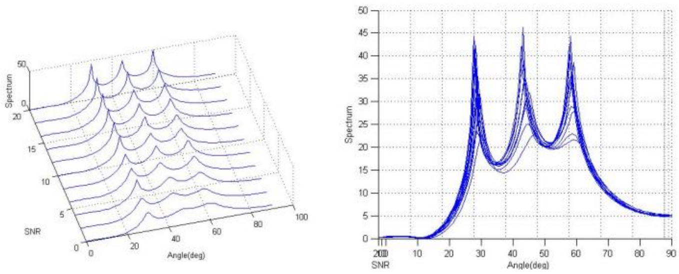


Fig 3. DOA estimation performance when SNR varies from 0 t0 20 dB(a. LS-ICA b. frontal effect of MUSIC)

5.2 degree distinguishing capacity of LS-ICA estimation

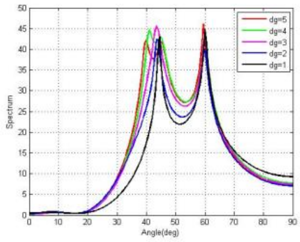
Three signals' shape keeps the same, but angle of the first ranges from 40 to 45 degree, other two angles are still, which will result that internal of two incidence angel is near. Degree distinguishing capacity is just studied in this case.

The results of two methods is as shown in table 2 and Fig 4.

Table 2 DOA estimation by LS-ICA when internal decreases between two signals

Internal	5	4	3	2	1
Angle1	38.4721	36.8533	40.2705	42.3607	42.2176
Angle2	45.8886	44.7394	45.371	44.0973	45.3469
Angle3	63.67	60.3521	60.709	58.7765	60.7842

It can be seen that, when incidence angle varies, estimation value by the former method is floating near real value, and when internal of the first and second signal become more and more narrow, DOA estimation generates greater floating while there are always three different angles. Meanwhile, according to figure 6.a, degree distinguishing internal of the former two estimation value is more than 1 degree. In figure 6.b, when the first incidence angle is 40 degrees, DOA estimation could be achieved by MUSIC method, but when internal between the first incidence angle and the second incidence angle, DOA estimation is not achieved (number of spectrum peak varies from 3 to 2). Thus, degree distinguishing capacity is 4 times LS-ICA than MUSIC (4/1 ≈4).



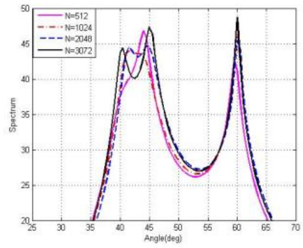


Fig 4. degree distinguishing of MUSIC methods   Fig 5 performance with different data length(MUSIC)

5.3 influence of estimation caused by data length

The results of two methods is as shown in table 3 and Fig 5.

Table 3 when data length varies, performance of DOA estimation by LS-ICA

Data length	32	64	128	256	512	1024
Angle1	33.702	34.79	39.7362	38.9741	41.1549	40.5063
Angle2	43.8717	46.6662	43.077	44.0385	46.0587	42.9492
Angle3	64.1624	60.9859	53.0567	55.9311	65.9353	59.3554

6. Conclusions

Aiming at DOA estimation problem of ULA, antenna array factor is used cutely, which helps to find new chance to study DOA estimation by LS-ICA, thus, new method of LS-ICA is proposed in paper. Strict mathematic theory foundation is given and algorithm flow is drawn, then step of realization is described in detail. With the comparison of typical MUSIC method, simulation experiment from several aspects shows good performance and high time efficiency of new method. Both algorithm flow and method step are strongly logistic to be convenient to execute by computer. To point out, the new method also doesn't deal with correlative source signals like many other methods, which is the key that next study should concern with. Nevertheless, it is true that the method has provided one new thought for studying problems of array signals' processing.

REFERENCES

[1]SHAN Z, YUM T. A Conjugate augmented approach to Direction of Arrival estimation[J]. IEEE Transactions on Signal Processing, 2005, 53 (11): 67-94.

[2] Kundu Debasis. Modified MUSIC algorithm for estimation DOA of signals[J]. Signal Processing, 1996, 48 (1): 85- 90.

[3] MAHATA K. Spectrum estimation，notch filters，and MUSIC[J]. IEEE Transactions on Signal Processing，2005，53（10）：3727-3737.

[4] FU Shujuan, JING Xiaorong, ZHANG Zufan, etc. DoA Estimation of Coherent Sources by Using Virtual Array-based Improved MUSIC Algorithm[J]. Telecommunication Engineering, 2011,51(11)：63-67.

[5] LIANG Guolong, DONG Shumin. Study on algorithms of blind separation and DOA estimation about coherent sources[J]. Journal of Harbin Engineering University, 2010,31(11)：1478-1484.

[6] JI Zhengyu, YANG Xianghua. Improved Algorithm of DOA Based on FFT and MUSIC[J]. Journal of System Simulation, 2010,22(2)：487-490.

[7] Palanisamy P, Kalyanasundaram N, Raghunandan A. A new DOA estimation algorithm for wideband signals in the presence of unknown spatially correlated no ise [J] . Signal Pro cessing, 2009, 89(10) : 1921- 1931.

[8]WANG Ding, WANG Chao, Wu Ying. Analysis of the effects of the amplitude-phase errors on spatial spectrum and resolving performance of the MUSIC algorithm[J]. Journal on Communications, 2010(4) : 55-63.

[9] Niu Dezhi, Chen Changxing, Wang Shuzhao. Research on Channel Estimation of MIMO System based on FastICA[J]. Computer Engineering and Applications, 2012,3(5):152-156.

[10] GUO Yue ,WANG Hongyuan ,ZHOU Zou. Effect of Array Elements Spacing on MUSIC Algorithm[J]. ACTA ELECTRONICA SINICA, 2007,35(9)：1675-1679.