Dynamic Analysis of High Dynamic GNSS Signal Based on PMF-FFT

Xuan Xia[1] **Haihui Long**[2] **Jiankang Zhao**[3]

Abstract. This paper focus on the GNSS(Global Navigation Satellite System) signal processing based on PMF(Partial Matching Filter)-FFT(Fast Fourier Transform), and a dynamic analysis method of high dynamic GNSS signal is proposed. PMF-FFT has a wide range of applications in GNSS signal acquisition, but no one ever apply it to signal tracking so far. Traditional tracking loop acceleration tolerance is smaller than 10g, but it can be up to 70g by this dynamic analysis method. Compared with the traditional method, dynamic analysis based on PMF-FFT make use of characteristic of high dynamic signal, receiver can get real-time Doppler shift frequency and estimate acceleration after PKF(Polynomial Kalman Filter) without external auxiliary information, and simplifies the signal tracking loop processing.

Keywords: High dynamic, GNSS, PMF, FFT, PKF.

1 Introduction

GNSS signal, like GPS and BD2 signal, belongs to Direct Sequence Spread Spectrum(DSSS) signal, which requires receivers to synchronize with the carrier and pseudo-code of the received signals. In the method based on Partial Matched Filter(PMF) and Fast Fourier Transform(FFT), PMF fix on the code phase and

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FFT estimates the Doppler shift[1]. Hence this method can simultaneously detect Doppler shift and code phase efficiently, and has a wide range of applications.

High dynamic is a kind of situation that missiles and space vehicles often met, which means high speed, high acceleration, and high jerk[2]. High performance GNSS signal acquisition and tracking is required in the military[3] and the aerospace applications[4]. PMF-FFT also can be applied to high dynamic GNSS signal acquisition[5], but no one ever apply it to signal tracking so far. Existing signal tracking technology generally use high-order loop and maximum likelihood estimator[6], or Inertial Navigation Systems(INS) information[7], but it is sensitive to noise, or increase in system cost and the complexity of filter design is high[8].

For a long time, the detailed analysis of high dynamic signals is lack, and the dynamic information that long time high dynamic signal contains is ignored. How to make use of the characteristic of high dynamic instead of eliminate its influence? This is the most important question tried to be answered in this paper.

2 High Dynamic GNSS Signal and PMF-FFT

Doppler frequency shift elimination is one of the most important problems in GNSS signal acquisition. Satellite motion causes a Doppler frequency shift and affects the performance of the acquisition and the tracking of the GNSS signal. The first step of GNSS signal reception is stripping carrier and extract frequency shift by signal despreading in order to eliminate the influence of Doppler frequency shift, then the second step is tracking this signal, fully despread the signal by frequency-lock loop(FLL) and carrier phase tracking loop(PLL).

2.1 Signal Despreading

As to the L1 signal acquisition, assume the desired down conversion to baseband IF signal is

$$\begin{aligned} I_C &= D(t)C(t)\cos[2\pi(f_0 t + f_d t) + \varphi] + n(t) \\ Q_C &= D(t)C(t)\sin[2\pi(f_0 t + f_d t) + \varphi] + n(t) \end{aligned} \tag{2.1}$$

Where, $D(t)$ is the data bit, $C(t)$ is the C/A code, f_0 is the IF carrier frequency. f_d is the Doppler frequency shift, $n(t)$ is the white Gaussian noise. Then the I-Q complex signal:

$$I_C + jQ_C = D(t)C(t)e^{j[2\pi(f_0 t + f_d t)+\varphi]} + n(t)(1 + e^{j(\pi/2)}).$$

The local carrier generator generates a conjugate complex signal $G = e^{-j2\pi f_0 t}$, and multiply it with I-Q,

$$s = G \times (I_C + jQ_C)$$
$$= D(t)C(t)e^{j(2\pi f_d t+\varphi)} + \sqrt{2}n(t)e^{j(2\pi f_0 t+\pi/4)}.$$

Then the IF carrier signal is transformed to an additive noise. If f_0 is a fixed value and $n(t)$ is a fixed white Gaussian white noise, the formula can be simplified to:

$$s = D(t)C(t)e^{j2\pi f_d t+\varphi} + N(t) \quad (2.2)$$

Then signal s goes to PMF-FFT Module.

2.2 PMF-FFT

The PMF-FFT method can be considered as an amelioration of linear correlation that the code/frequency searching are processing in parallel.

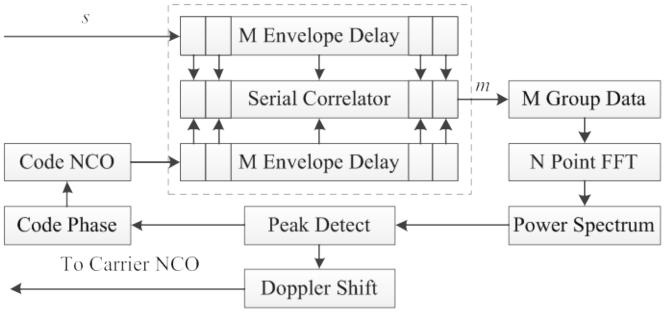

Fig. 2.1 Basic principle of signal acquisition based on PMF-FFT

As examples shown in Fig. 2.1, the main idea of PMF-FFT is:

a.Gained M ($M = L/X$, L represents the length of the code) results during the course of serial linear correlation by summing up X chips after signal s and local code are multiplied.

b.Estimate the power spectrum after calculating N($N \geq M$, N is an integer power of 2) points FFT.

c.Detect the peak of power spectrum, compare with predefined threshold value.

d.Consider local code has been synchronized with received code if the peak of power spectrum exceeds the threshold value; Readjust the code phase and Doppler shift simultaneously.

Once the local code has been synchronized with received code, ignoring the navigation message data temporarily, the output of the k-th ($1 \leq k \leq M$) PMF is

$$m(k) = \int_{(k-1)\Delta t}^{k\Delta t} [\Delta C e^{j2\pi f_d t} + N(t)]\Delta C_L \,\mathrm{d}t$$
$$= \frac{1}{2\pi f_d}(\mathrm{e}^{j2\pi f_d k\Delta t - \pi/2} - \mathrm{e}^{j2\pi f_d (k-1)\Delta t - \pi/2}) + N(k)$$

Where, Δt is the length of PMF, ΔC and ΔC_L are received C/A code and local C/A code in length of time for Δt, and $\Delta C \cdot C_L \equiv 1$ ("·" means serial correlation), $N(k) = \int_{(k-1)\Delta t}^{k\Delta t} N(t)\Delta C_L \mathrm{d}t$.

According to equation (2.2), the integral is a filtering process to white Gaussian noise and also effective both to code stream signal and trigonometric functions signal, so the PMF can eliminate noise $N(k)$ effectively. So the function of PMF is the Doppler frequency shift signal reproducibility with filter noise. After that the Doppler shift frequency can be detected by N point FFT after enlarging M to N by adding zeros. The M point group data can be approximately written as

$$M(k) = K\mathrm{e}^{j2\pi f_d k\Delta t} + N(k) \tag{2.3}$$

Where, K is the filtering gain of PMF. The performance of PMF has been proven effectively in the application of C/A code acquisition without considering $D(t)$. But unlike ordinary C/A code acquisition, $D(t)$ is almost inevitable during P code acquisition or weak GNSS signal acquisition. The positive and negative changes in signal will lead to errors in PMF calculation. A method called power PMF can be used to solve this problem.

Because $D^2(t)$=1, power PMF is to calculate the power of every k-th PMF output and then get a new M points group data $[KD(k)\mathrm{e}^{j2\pi f_d k\Delta t} + N(k)]^2$, it can be written as:

$$M'(k) = K^2\mathrm{e}^{j4\pi f_d k\Delta t} + 2KD(k)N(k)\mathrm{e}^{j2\pi f_d k\Delta t} + N(k)^2 \tag{2.4}$$

There are some disadvantages in the application of power PMF, on the one hand, SNR is decreased, on the other hand, new noise $2KD(k)N(k)\mathrm{e}^{j2\pi f_d k\Delta t}$ is added to output. By the way, now the detected frequency is $2f_d$.

2.3 High Dynamic GNSS Signal

Using Equation $f_d = fv/c$, the Doppler frequency shift can be calculated, where f is the carrier frequency, c is the speed of light, and v is the relative velocity of the satellite and the aircraft. If the relative acceleration is a, and the relative jerk is b, then when at time t, the Doppler shift frequency of carrier is given by $f_d(t) = f_{d0} + fv/c = f_{d0} + f\int(a+bt)\mathrm{d}t/c = f_{d0} + f(at+bt^2/2)/c$, where f_{d0} is initial Doppler shift frequency. Define $h = f/c$ (As to the C/A code acquisition, $h = 5.2514\mathrm{Hz}\cdot\mathrm{s/m}$), the value of h changes with carrier frequency change. Hence,

$$f_d(t) = f_{d0} + hat + hbt^2/2 \quad (2.5)$$

The influence of high dynamic can hardly be reflected in FFT output when time is short, so it is generally ignored in rapid signal acquisition. But the time must be extended when the frequency resolution need to be increased or it is required to capture a week signal, in such a case, the influence of high dynamic will gradually appear.

Now substitute formula (2.5) into formula (2.2), calculate the CFT (Continuous Fourier Transform) output of long time high dynamic signal. For convenience, Ignore the jerk, then $s = \mathrm{e}^{j[2\pi\int_0^{+\infty}(f_{d0}+hat)\mathrm{d}t+\varphi_0]} = \mathrm{e}^{j(2\pi f_{d0}t+\pi hat^2+\varphi_0)}$, and

$$S = \frac{1}{\sqrt{ha}}\mathrm{e}^{-j[\frac{\pi(f-f_{d0})^2}{ha}-\varphi_0-\frac{\pi}{4}]}. \quad (2.6)$$

Where, φ_0 is the initial phase. Decomposition of the real and imaginary parts according to Euler's formula:

$$\begin{aligned} S_I &= \frac{1}{\sqrt{ha}}\cos\left(\frac{\pi(f-f_{d0})^2}{ha}-\varphi_0-\frac{\pi}{4}\right). \\ S_Q &= -\frac{1}{\sqrt{ha}}\sin\left(\frac{\pi(f-f_{d0})^2}{ha}-\varphi_0-\frac{\pi}{4}\right). \end{aligned} \quad (2.7)$$

The square of the modulus (power spectrum) of S is

$$S^2 = S_I^2 + S_Q^2 = \frac{2}{ha}. \quad (2.8)$$

This means that a continuous and constant frequency variation will produce a continuous and constant value in power spectrum.

3 GNSS Signal Dynamic Analysis

The architecture shown in Fig. 3.1 demonstrates how to analyze the dynamic of GNSS signal. It consists two parts, the left part is responsible for the signal acquisition by short time FFT, and the right part is responsible for the dynamic analysis by long time FFT.

The purpose of signal acquisition is to get the preliminary Doppler shift frequency and code phase by the first despreading, after that, increase the number of PMF for the long time high dynamic signal processing. At this point, PMF-FFT continuous output the dynamic information so that we can detect frequency of the current moment. With this information, Doppler shift frequency can be eliminated by the second despreading, and the influence of dynamic can be predicted in tracking loop by acceleration estimation. So dynamic analysis is a kind of tracking loop preprocessing in fact, it includes two aspects: frequency detection and acceleration estimation. Its purpose is to detect the current frequency and predict the acceleration tendency in order to guarantee normal operation of FLL and PLL.

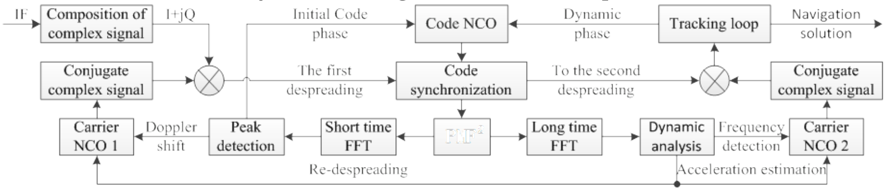

Fig. 3.1 The architecture of dynamic analysis

3.1 Frequency Detection

Doppler frequency shift information can be detected by analysis of long time PMF-FFT as show in Fig. 3.2. By the way, the power spectrum will be distorted because of truncation effect[9] and will be mirror symmetrical.

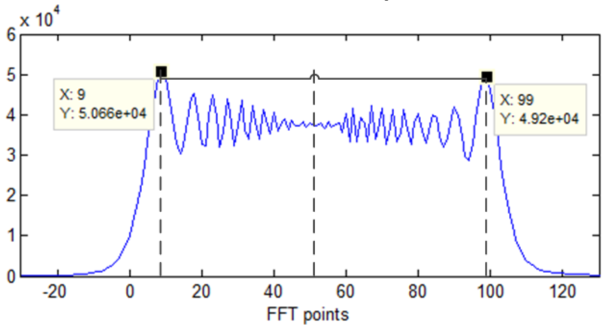

Fig. 3.2 Doppler frequency shift can be detected by long time PMF-FFT

In Fig. 3.2, Δt =0.2ms, f_{d0} =0Hz, f_s =10MHz, a=100g, b=0, M=500, N=512, C/A code, used power PMF. So the frequency resolution is r=1/ (512×0.0002ms) =9.765625Hz. After search from both sides to the middle, we can find the frequency response exist range, and the current Doppler shift frequency can be determined by symmetrical property. If the point corresponding to the peak on the left is p_a, and the point corresponding to the peak on the right is p_b, then the current Doppler shift frequency f_c is

$$f_c = r(p_a + p_b)/2 \tag{3.1}$$

Hence we can get the f_c=9.765625Hz×(9+99)/2≈527.34Hz from Fig. 3.2, and the actual frequency is 500×0.0002ms×5.2514Hzs/m×1000m/s2=5 25.14Hz, with an error of 0.42%. Frequency detection achieved enough accuracy.

As mentioned above, Doppler shift frequency can be detected continuously by this real-time detection method, to achieve the purpose of locking frequency.

3.2 Acceleration estimation

The influence of high dynamic to tracking loop mainly has two aspects: FLL lock-lose due to frequency error and PLL lock-lose due to phase error. The former effect the precision of frequency tracking, the latter affect pseudo range measurement accuracy. Frequency detection can only be used for FLL, we need to know accurate acceleration information if we want to PLL to work properly. This means that acceleration needs to be estimated.

Considering the formula (2.5) is a second-order polynomial, we can use second-order PKF (Polynomial Kalman Filter)[10], Kalman filter can not only be used to estimate the acceleration, but also can improve the accuracy of frequency detection. The discrete PKF equation is

$$\begin{aligned}
\hat{\mathbf{x}}_k &= \mathbf{\Phi}_k \hat{\mathbf{x}}_{k\text{-}1} + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}\mathbf{\Phi}\hat{\mathbf{x}}_{k\text{-}1}) \\
\mathbf{M}_k &= \mathbf{\Phi}_k \mathbf{P}_{k\text{-}1} \mathbf{\Phi}_k^T + \mathbf{Q}_k \\
\mathbf{K}_k &= \mathbf{M}_k \mathbf{H}^T (\mathbf{H}\mathbf{M}_k \mathbf{H}^T + \mathbf{R}_k)^{-1} \\
\mathbf{P}_k &= (\mathbf{I} - \mathbf{K}_k \mathbf{H})\mathbf{M}_k
\end{aligned} \tag{3.2}$$

Where, $\mathbf{z}$ is the observed value, $\hat{\mathbf{x}}$ is the estimated value. In formula (2.5), a and b is not constant values in actual situation, so it is a nonlinear time-varying signal. To accommodate this parameter changes, add process noise $\mathbf{Q}$, then the change of the acceleration can be predicted.

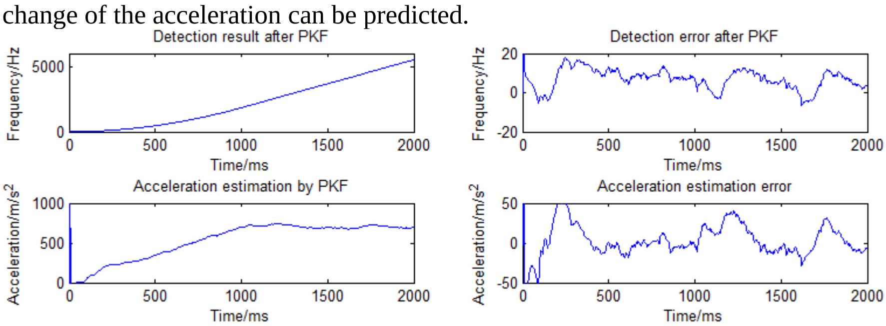

Fig. 3.3 Frequency Detection and dynamic estimation after PKF

As shown in Fig.3.3, in one of NASA's JPL(Jet propulsion Laboratory) defined high dynamic environments, 70g/s jerk step for one second starting from the Doppler shift frequency 0Hz, then acceleration maintains 70g for one second, SNR=-20dB, PKF improved the frequency detection precision, before PKF, the ME (mean error) and RMSE (root mean square error) are -4.1021Hz, 9.1885Hz respectively, they are -3.6054Hz, 5.163Hz respectively after PKF. Acceleration estimation achieved acceptable precision, estimation precision is smaller than 5g after the PKF result is convergent (ME is 1.9477m/s2 and RMSE is 21.1977m/s2). This means that equivalent acceleration in tracking loop can be reduced to less than 5g by acceleration compensation in NCO after PKF, hence tracking loop can be simplified as conventional signal processing mode.

4 Conclusions

The properties of high dynamic GNSS signal are analyzed and a method for the dynamic analysis of high dynamic GNSS signal is proposed in this paper. Receivers can get real-time dynamic information after PKF by this method. The most important significance of this paper is the making use of characteristic of high dynamic signal, analyzed the dynamic without external auxiliary information. The next further work is continued to improve the noise tolerance, improve the dynamic analysis precision, redesign and simplified the signal tracking loop.

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