The Application of Markov Chain in China's Market Economy

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Abstract

The Markov Chain, Which named after Russia mathematician A.A.Markov, has been widely used not only in other branches of mathematic,engineering technique, but also in the social science, for example: economies, insurance, financial risk, risk manage theory and technique.

Markov Chain was used to forecast the percent of market share and profit of manufactures in this article. Different models were constructed to make better decision-making so as to obtain more profit.

Keywords: Markov Chains, Matrix of Transition Probability, Stationary Distribution, Decision-Making

1. Introduction

With the consecutive development of China's socialism market economy and the update of enterprise operation system, enterprise is in need of scientific methods to analyze its production and operation activities for the sake of adjusting their production and operation timely to improve its competitiveness. The scientific prediction of enterprise's expected economic indexes for it to make correct decisions in the future needs appropriate mathematical models and methods to implement quantitative researches into enterprise's economic activities. Based on market activities' complicated, diversified and random features, in recent years, a number of scholars try to adopt Markov China in Market Economy Research.

2. Market Share Analysis

Assuming that there are 1600 residents in a certain place while a certain product is only manufactured by A, B and C. The statistics shows that, in August, the user of A, B and C are 480, 320 and 800 respectively. In September, 48 users of A change to purchase B's product, while 96 users change to purchase C's product. 32 users of B change to purchase A's product, while 64 user change to purchase C's product. 64 users of C change to purchase A's product, while 32 users change to B's product. Now, the Markov Chain is adopted to analyze and predict on the basis of above data.

Establish state and determine state probability . Market share can be divided into three kinds of states. Therefore, state space $E=\{1,2,3\}$ (State 1,2 and 3 stand for A, B and C respectively). The initial probabilitydistribution (the initial market share) is as below:

$$(p_{1,}\ p_{2,}\ p_3)=(0.3, 0.2, 0.5)$$

Establish state transition probability matrix P . Since that 48 users of A change to purchase B's product, while 96 users change to purchase C's product; 32 users of B change to purchase A's product, while 64 user change to purchase C's product; 64 users of C change to purchase A's product, while 32 users change to B's product, the frequency transition matrix is as below:

$$N = \begin{pmatrix} 336 & 48 & 96 \\ 32 & 224 & 64 \\ 64 & 32 & 704 \end{pmatrix}$$

We can get probability transition matrix:

$$P = \begin{pmatrix} 0.7 & 0.1 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.08 & 0.04 & 0.88 \end{pmatrix}$$

Assuming that $S_i(n)$ stands for the market share in nth month at the ith state probability, while the vector $S_n = (S_1(n), S_2(n), S_3(n))$ stands for the state probability vector in the nth month. As a result, we can get $S_{n+1} = S_n P$.

On the basic of initial probability distribution and transition probability transition matrix P, we can get the market share in the coming months.

Adopt transition probability to execute primary market prediction and pass the stable distribution analysis. In the below part, we adopt the stable distribution of Markov Chain to predict market share. Judged from transition probability matrix P, the Markov Chain is an irreducible, non-cycled and limited state. Therefore, it has a stable distribution. We could get from plateau equation $\pi = \pi P$ that

$$\begin{cases} \pi_1 = 0.7\pi_1 + 0.1\pi_2 + 0.08\pi_3 \\ \pi_2 = 0.1\pi_1 + 0.7\pi_2 + 0.04\pi_3 \\ \pi_3 = 0.2\pi_1 + 0.2\pi_2 + 0.88\pi_3 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

After solving, we can get:

$$\pi_1 = 0.219, \pi_2 = 0.156, \pi_3 = 0.625.$$

This result shows that, in case that transition probability matrix maintains the same, the sales state probability is $S_i(n) \to \pi_i, n \to \infty$, $(i = 1,2,3)$, indicating that a value not related to initial state has stabilized. Judged from the result, the B's market share will be lower in the future. For this manufacturer, it's recommended to update artwork, improve post-sales service, lower product price, strengthen dissemination and adopt other positive and effective measures to modify enterprise's operation for gaining more market share in the future. In fact, such modification only changes the transition probability in transition probability matrix P. in new conditions, the market shares of these three manufactures shall tend to be balance.

3. Prediction of Enterprise Profit

A lot of scholars have noticed the significant effect of Markov Chain in economic prediction, such as Reference 4 and 5. The Reference 4 focuses on the stable distribution researches without analysis on which solution enterprise administrator could adopt. Reference 5 adopts stable state probability to analyze four enterprise decisions: long term operation, short term operation, changeable strategy and unchangeable strategy. On the basis of summary [4,5], we innovatively bring in new evaluation index $V_i(n)$, which stands for the company's expected profit after n months since from receiving the ith order $(i = 0,1,2)$. This index is adopted as the basis for discussing enterprise operation decision-making.

Assuming that a company receives maximum 2 orders monthly, X_n stands for the order quality received in the nth month, set X_n as one homogeneous Markov Chain, $E = \{0,1,2\}$ stands for state space, in which State i standard for the ith order received by the company and $i = 0,1,2$,judged from data analysis, the transition probability matrix and profit matrix of received order are as below:

$$P=\begin{pmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{pmatrix}, \quad R=\begin{pmatrix} r_{00} & r_{01} & r_{02} \\ r_{10} & r_{11} & r_{12} \\ r_{20} & r_{21} & r_{22} \end{pmatrix}$$

r_{ij} indicates the profit of j th order in the second month when totally i orders were received in the first month.

Assuming that $V_i(n)$ stands for the expected profit n month after receiving ith orders $(i=0,1,2)$, we can get:

$$V_i(1)=\sum_{j=0}^{2} r_{ij} p_{ij}$$

and

$$V_i(2)=\sum_{j=0}^{2}\left(r_{ij}+V_j(1)\right) p_{ij}$$

we can get the recursion formula

$$V_i(n)=\sum_{j=0}^{2}\left(r_{ij}+V_j(n-1)\right) p_{ij}$$

Assuming that $V_i(0)=0$ and the initial profit is 0, we can get $V_i(1), i=0,1,2$. It indicates that, under three kinds of conditions, the expected profit after 1 month.

Different from the indexes in Reference 4, in the below part, we will adopt expected value as the index to research into how decision-makers of the company modify the production and operation based on the profit prediction model and same strategy expense. Assuming that the transition probability matrix of Strategy A and Strategy B are as below:

$$P=\begin{pmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{pmatrix}, \quad P'=\begin{pmatrix} p'_{00} & p'_{01} & p'_{02} \\ p'_{10} & p'_{11} & p'_{12} \\ p'_{20} & p'_{21} & p'_{22} \end{pmatrix}$$

Assumingthat $V_i(k)$ and $V_i'(k)$ stand for the expected profit of k months since receiving the ithorder $(i=0,1,2)$ under the Strategy A and Strategy B respectively, we can analyze which strategy could gain the most optimized profit under different strategies. Take the company's 2 month short term operation with 2 existed order as example, in case that $V_2(2)<V_2'(2)$, Strategy B will help the company to gain better profit. In case that no existed order and $V_2(2)>V_2'(2)$, the Strategy A will help to company to gain better profit. Please refer for Reference 4 and 6 for more details.

4. Conclusion

By adopting Markov Chain in prediction, we can fulfill two targets: Firstly, it could predict states probability distribution after they change into other states within given time. Secondly, the transition probability matrix could help to determine long term (in stable states) probability distribution. So that it could guide enterprise to make corresponding modifications to operation and enable enterprise administrators to adopt the most optimized strategy to promote enterprise's healthy development and maximize enterprises profit. However, we need to notice that, when adopting Markov Chain in analysis, the transition probability matrix is considered to be unchanged and this situation is normally cannot be fulfilled in actual occasions. Therefore, this model still suffers some limitations. As a result, in actual economic prediction, we need to investigate into specific situations and collect data to update transition probability matrix so as to guarantee the precision of predicted state probability distribution.

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