Precise large deviation of claim surplus process in a nonstandard renewal risk model with constant premium rate

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Abstract:

In this paper, we consider a nonstandard renewal risk model in which the claim sizes and their inter-arrival times form a sequence of independent and identically distributed random variables, respectively. The claim size and corresponding inter-arrival time satisfy a certain dependence structure. In addition, the premium rate is a constant, and the number of insurance policies is described by a renewal process. When the distribution of claim sizes belongs to the consistent variation class, we obtain precise large deviation of claim surplus process.

Keywords: Precise large deviation, claim surplus process, nonstandard renewal risk model, constant premium rate, consistent variation class.

1. Introduction

We consider the following nonstandard renewal risk model: claim sizes $\{X_i, i=1,2,\cdots\}$ form a sequence of independent, identically distributed (i.i.d) and nonnegative random variables with common distribution F. The inter-claim times $\{\theta_i, i=1,2,\cdots\}$ form another sequence of i.i.d and nonnegative random variables. Let $\tau_k = \sum_{i=1}^{k}\theta_i$ denote the arrival time of the kth claim. Then $0=\tau_0 \le \tau_1 \le \cdots$, constitute a renewal counting process

$$N(t)=\sup\{n\ge 1:\tau_n \le t\},\ \ t\ge 0.$$

Write $EN(t)=\lambda t$.

The amount of aggregate claims up to t can be expressed as $S(t)=\sum_{i=1}^{N(t)}X_i$.

Denote by X and θ the generic random variables of the claim sizes and their corresponding inter-claim times. We make use of the same dependent structure as in [1]. That is to say, X and θ satisfy the following assumption.

Assumption 1: There is some $x_0>0$ such that it holds for all $x\ge x_0$ and $t>0$ that

$$P(\theta>t \mid X>x)\le P(\theta^*>t)$$

.

In addition, we assume that an insurer receives insurance policies in a discrete-time way.

The arrival times of the successive insurance policies constitute a renewal process $\{M(t), t\ge 0\}$. Let c denote the premium of each insurance policy. The amount of aggregate premiums up to time t can be represented by $cM(t)$, where $EM(t)=\lambda_1(t)$. Let $u>0$ be the initial reserve of the insurer. The risk reserve process is given by

$$R(t)=u+cM(t)-\sum_{i=1}^{N(t)}X_i \quad , \quad t\ge 0.$$

The claim surplus process can be expressed as

$$Y(t)=\sum_{i=1}^{N(t)}X_i-cM(t)\ , \qquad t\ge 0. \qquad (1)$$

The amount of aggregate claims can be denoted by

$$S(t)=\sum_{i=1}^{N(t)}X_i\ , \quad t\ge 0. \qquad (2)$$

We assume $M(t)$ and $\sum_{i=1}^{N(t)}X_i$ are mutually independent. [1] presented precise large deviation of the amount of aggregate claims $\{S(t),t\ge 0\}$. [2] showed precise large deviation for the compound Poisson risk model. In the present paper, when the claim-size distribution belongs to the consistent variation class, we give precise large deviation of the claim surplus process $\{Y(t),t\ge 0\}$ for the above nonstandard renewal risk model.

2. preliminaries

Denote $\overline{F}(x)=1-F(x)=P(X>x)$. We first introduce some related heavy-tailed distribution class, which can be found in [3] and [4].

A distribution F on $[0,\ \infty)$ is said to belong to the long-tailed class, if

$$\overline{F}(x-y)\sim\overline{F}(x),$$

for any $y\in(-\infty,\infty)$. In this case, we write $F\in L$.

In addition, we say that a distribution F belongs to the dominated variation class and write $F\in D$, if

$$\overline{F}(xy)=O(1)\overline{F}(x)\ , \quad \text{for all } 0<y<1.$$

Compared with D, the consistent variation class C is a smaller class. We say $F\in C$, if

$$\lim_{y\downarrow 1}\liminf_{x\downarrow\infty}\frac{\overline{F}(xy)}{\overline{F}(x)}=1.$$

It is well known that C belongs to $D\cap L$.

By the definition of consistent variation class C, we have the following lemma.

Lemma 1. If $F\in C$, then

$$\lim_{x\to\infty}\frac{\overline{F}(x+o(1)x)}{\overline{F}(x)}=1.$$

The following lemma is due to [5].

Lemma 2. For a distribution F on $[0,\ \infty)$, if $F\in D$, then for any $p>J_F^+$, there is some positive number c_1 and d_1 such that

$$\frac{\overline{F}(y)}{\overline{F}(x)}\le c_1\left(\frac{x}{y}\right)^p,$$

for all $x\ge y\ge d_1$.

The following is from [6].

Lemma 3. For a renewal process $\{M(t),t\ge 0\}$, the generic inter-renewal distance T has distribution F and expectation $\lambda_1 < \infty$. If $F(\infty) = 1$, then

$$\lim_{t\to\infty} \frac{\lambda_1(t)}{t} = \frac{1}{\lambda_1}.$$

The following lemma is from [1].

Lemma 4. Consider the amount of aggregate claims (2). Suppose that Assumption is satisfied, and that $F \in C$ and $E(X) = \mu \in (0,\infty)$. Then, for any fixed $\gamma > 0$, it holds uniformly for all $x \geq \gamma t$ that

$$P(S(t) - \mu\lambda t > x) \sim \lambda t\overline{F}(x),$$
$$t \to \infty.$$

The following lemma is referred to [7].

Lemma 5. If $\{M(t), t \geq 0\}$ is a renewal process, then

$$\frac{M(t)}{\lambda_1(t)} \xrightarrow{p} 1.$$

3. Main result

Theorem. Consider the claim surplus process (1). In addition to Assumption 1, suppose that $F \in C$, $E(X) = \mu \in (0,\infty)$. Then, for any fixed $\gamma > 0$, it holds uniformly for all $x \geq \gamma t$ satisfying $c \geq \gamma\lambda_1$ that

$$P(Y(t) - \mu\lambda t > x) \sim \lambda t\overline{F}(x),$$
$$t \to \infty. \quad (3)$$

Proof. $P(Y(t) - EY(t) > x)$

$$= P\big(S(t) - cM(t) - ES(t) + c\lambda_1(t) > x\big)$$
$$= \sum_{|k-\lambda_1(t)|\leq\varepsilon(t)\lambda_1(t)} (P(S(t) - ES(t) > x - c\lambda_1(t) + ck) \times P(M(t) = k))$$
$$+ \sum_{k-\lambda_1(t)<-\varepsilon(t)\lambda_1(t)} (P(S(t) - ES(t) > x - c\lambda_1(t) + ck) \times P(M(t) = k))$$
$$+ \sum_{k-\lambda_1(t)>\varepsilon(t)\lambda_1(t)} (P(S(t) - ES(t) > x - c\lambda_1(t) + ck) \times P(M(t) = k))$$
$$= I_1(t) + I_2(t) + I_3(t). \quad (4)$$

First of all, we deal with $I_1(t)$. For $x \geq \gamma t$, as $t \to \infty$, $\frac{\lambda_1(t)}{x} \leq \frac{\lambda_1(t)}{\gamma t} \to \frac{1}{\gamma\lambda_1}$. When $|k - \lambda_1(t)| \leq \varepsilon(t)\lambda_1(t)$,

$$x - c\lambda_1(t) + ck = x + o(1)\lambda_1(t) = x + o(1)x.$$

By Lemma 4, Lemma 1 and Lemma 5, it holds uniformly all $x \geq \gamma t$,

$$I_1(t) = \sum_{|k-\lambda_1(t)|\leq\varepsilon(t)\lambda_1(t)} P(S(t) - ES(t) > x - c\lambda_1(t) + ck)\, P(M(t) = k)$$
$$\sim \sum_{|k-\lambda_1(t)|\leq\varepsilon(t)\lambda_1(t)} \lambda t\overline{F}(x - c\lambda_1(t) + ck) P(M(t) = k)$$
$$= \lambda t\overline{F}(x) \sum_{|k-\lambda_1(t)|\leq\varepsilon(t)\lambda_1(t)} \Big(\frac{\overline{F}(x + o(1)x)}{\overline{F}(x)} \times P(M(t) = k)\Big)$$
$$\sim \lambda t\overline{F}(x). \quad (5)$$

Next we discuss $I_2(t)$. By Lemma 2, there is some positive number D_2 such that it holds uniformly for all $x \geq \gamma t$ satisfying $\gamma t \geq \delta\lambda_1(t)$,

$$I_2(t) = \sum_{k-\lambda_1(t)<-\varepsilon(t)\lambda_1(t)} (P(S(t)-ES(t) > x - c\lambda_1(t) + ck) \times$$

$$P(M(t)=k))$$

$$= \sum_{k-\lambda_1(t)<-\varepsilon(t)\lambda_1(t)} (P(S(t)-ES(t) > x - c\lambda_1(t)) \times$$

$$P(M(t)=k))$$

$$\sim \lambda t \overline{F}(x) \sum_{k-\lambda_1(t)<-\varepsilon(t)\lambda_1(t)} \frac{\overline{F}(x-c\lambda_1(t))}{\overline{F}(x)} P(M(t)=k)$$
$$\le D_2 \lambda t \overline{F}(x) P(M(t)-\lambda_1(t) < -\varepsilon(t)\lambda_1(t))$$
$$= o(1)\lambda t \overline{F}(x). \qquad (6)$$

Now we verify the fourth step. As t is large enough and $x \ge \gamma t$, there is some $0<\varepsilon<1$ such that

$$x - c\lambda_1(t) = x\left(1-\frac{c\lambda_1(t)}{x}\right) \ge x\left(1-\frac{c}{\gamma\lambda_1(1-\varepsilon)}\right)$$
.

Since $C \subset D$,

$$\limsup_{x\to\infty} \frac{\overline{F}\left(x\left(1-\frac{c}{\gamma\lambda_1}\right)\right)}{\overline{F}(x)} < \infty.$$

Hence as t is enough large, there is some positive number D_2 such that

$$\frac{\overline{F}(x-c\lambda_1(t))}{\overline{F}(x)} \le D_2.$$

Finally, we deal with $I_3(t)$.

For any fixed $\gamma > 0$, it holds uniformly for all $x \ge \gamma t$ that

$$I_3(t)$$
$$= \sum_{k-\lambda_1(t)>\varepsilon(t)\lambda_1(t)} (P(S(t)-ES(t) > x - c\lambda_1(t) + ck) \times$$

$$P(M(t)=k))$$

$$\sum_{k-\lambda_1(t)>\varepsilon(t)\lambda_1(t)} P(S(t)-ES(t) > x) P(M(t)=k)$$

$$\sim \lambda t \overline{F}(x) P(M(t)-\lambda_1(t) > \varepsilon(t)\lambda_1(t))$$
$$= o(1)\lambda t \overline{F}(x). \qquad (7)$$

According to (4)-(7), we obtain (3). This ends the proof of the theorem.

4.Acknowledgements

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