Low Complexity DOA Estimation Method for MIMO Radar based on MUSIC and Compressive Sensing

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Abstract - This paper addresses the problem of two-dimensional (2D) DOA estimation in multiple-input multiple-output (MIMO) radar system. A dimensional reduction multiple signal classification algorithm is proposed to reduce the computational complexity. By introducing the compressive sensing (CS) theory, the high-dimensional matched filtering data are projected into a low-dimensional matrix, and the sparse inverse problem is solved by the MUSIC method. Simulation results show that the performance of the proposed scheme is very close to that of the traditional two-dimensional (2D) MUSIC algorithm. Since the proposed algorithm has less complexity, the proposed algorithm is more efficient than the traditional one.

Index Terms - DOA estimation, MUSIC, Compressive sensing.

1. Introduction

Two dimensional (2D) direction of arrival (DOA) Estimation is a basic problem in multiple-input multiple-output (MIMO) radar[1,2]. Super-resolution algorithms such as multiple signal classification (MUSIC)[3] and estimation of signal parameters via rotational invariance techniques (ESPRIT)[4] attached much attention due to their excellent performance. The MUSIC algorithm is effective to arbitrary arrays, which requires eigenvalue decomposition (EVD) of the signal covariance matrix, but the peak searching has high complexity. The ESPRIT algorithm exploits the invariance property of the transmit array and the receive array for DOA estimation, however, the algorithm is sensitive to the manifold of the antenna array.

As modern radar systems pursue for super performance such as high-resolution and multi-targets detection, multi-channel and wideband signals are applied to improve the detection performance. As a result, the MIMO radar system faces with the difficulty of high-dimensional data processing. Compressive sensing (CS)[5] is a newly developed signal processing method for sparse signal. The CS theory points out that if the signal is compressible or sparse in a transform domain, then it can be recovered exactly with high probability from fewer measurements via l_1-norm optimization, which is consistent with MIMO radar system, as the targets can be considered sparse in the background scenario. The CS based MIMO radar system has been investigated in [6-8].

In this paper, a CS based MUSIC algorithm is proposed. The high-dimensional matched filtering data are firstly projected into a relatively low-dimensional matrix. Then according to the similarity between sparse recovery and peak searching of MUSIC algorithm, the subspace sparse information extraction method is introduced for DOA estimation, which is more efficient than the traditional iterative approach.

The rest of the paper is organized as follows. In section II, we present the DOA estimation method for the co-located MIMO radar. Simulation results are given in section III. Finally, we provide concluding remarks in section IV

Notation: Lower case and capital letters in bold denote, respectively, vectors and matrices. The superscript $(\cdot)^T$ and $(\cdot)^H$ represents, respectively, the transpose and Hermitian transpose of a matrix.

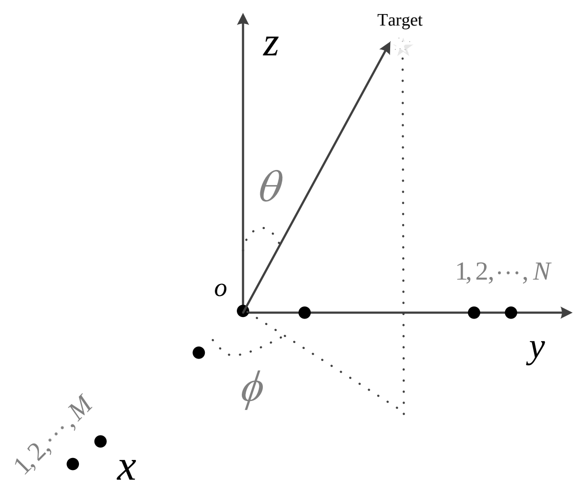

Fig. 1 Radar signal model

2. Method

1) Signal Model: Consider a monostatic L-shape MIMO radar model, as shown in Figure 1. Without loss of generality, we suppose that *M*-element transmitter and *N*-element receiver arbitrarily are distributed in the *x*-axis and *y*-axis, respectively. x_m and y_m account for the physical location of the *m*-th transmitter and the *n*-th receiver K non-coherent targets are assumed in the far-field of the antennas with the *k*-th target is at azimuth angle ϕ_k and elevation angle θ_k . It is also assumed that the transmit antennas transmit orthogonal waveforms. The direction matrix $\boldsymbol{A}(\theta,\phi)$ is defined as $\boldsymbol{A}(\theta,\phi) = [\boldsymbol{a}(\theta_1,\phi_1), \boldsymbol{a}(\theta_2,\phi_2), \ldots, \boldsymbol{a}(\theta_K,\phi_K)]$ with

$$\mathrm{a}(\theta_k,\phi_k) = \mathrm{a}_r(\theta_k,\phi_k) \otimes \mathrm{a}_t(\theta_k,\phi_k) \qquad (1)$$

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Where $\boldsymbol{a_t}(\theta_k,\phi_k) = [a_1(\theta_k,\phi_k),\ldots,a_M(\theta_k,\phi_k)]^T$ denotes the transmit steering vector and $\boldsymbol{a_r}(\theta_k,\phi_k) = [a_1(\theta_k,\phi_k),\ldots,a_N(\theta_k,\phi_k)]^T$ denotes the receive steering vector for the *k*-th target, $a_m(\theta_k,\phi_k) = e^{-j2\pi x_m cos\phi_k sin\theta_k/\lambda}$, $b_n(\theta_k,\phi_k) = e^{-j2\pi y_n sin\phi_k sin\theta_k/\lambda}$, with λ denotes the wave length of the carrier. The output of the matched filters at the receiver takes the form

$$\mathrm{X} = \mathrm{A}(\theta,\phi)\mathrm{S} \tag{2}$$

Where $\boldsymbol{S} = [\boldsymbol{S_1},\boldsymbol{S_2},\ldots\boldsymbol{S_K}]$ with $\boldsymbol{S_K} \in C^{1\times L}(k=1,2,\ldots K)$ being the samples contain the amplitude and phase of all the *k*-th target, and ⊗ represents the Kronecker product. The signal covariance matrix can be decomposed as

$$\mathrm{R_x} = \mathrm{XX^H} = \mathrm{U_s\Sigma_s U_s^H} + \mathrm{U_n\Sigma_n U_n^H} \tag{3}$$

where $\boldsymbol{\Sigma_s}$ stands for the diagonal matrix that composed of the *K* significant eigenvalues, $\boldsymbol{U_s}$ is the corresponding eigenvectors. $\boldsymbol{\Sigma_n}$ accounts for the diagonal matrix that composed of the *MN*-*K* smaller eigenvalues, and $\boldsymbol{U_n}$ stands for the noise subspace corresponding to $\boldsymbol{\Sigma_n}$.

In traditional MUSIC algorithm, the noise subspace is projected into the steering vectors $\boldsymbol{b}(\theta_k,\phi_k) = \boldsymbol{a_r}(\theta_k,\phi_k)\otimes\boldsymbol{a_t}(\theta_k,\phi_k)$ and the reciprocal taken, which results in a pseudo-spectrum with sharp peaks in the target directions (θ_k,ϕ_k).

2) Computation analysis: The computational complexity of MUSIC can be summarized as followed. The formation of covariance matrix occupied $O(M^2N^2L)$, the EVD operation takes the complexity of $O(M^3N^3)$, the searching grid number of azimuth and elevation are V and W respectively, the complexity of peak searching is $O(\mathrm{VW}(M^2N^2 + M^2N^2(MN-K)))$, the total complexity of MUSIC algorithm can be approximately $O(VW(M^2N^2))$.

3) CS based MUSIC algorithm: From the above analysis we can see that the computational complexity of MUSIC algorithm is mostly influenced by the multiplication of the transmit and receive antenna number which is *MN*, and the grid number of peak searching *PQ*. Generally the grid number is determined by the accuracy requirements of DOA estimation, if we can decrease the dimension of *MN*, then the computational complexity of EVD and spectrum searching would be dropped simultaneously. The CS method is inspired to compress the high dimensional signal. A pseudo matrix $\boldsymbol{\Phi} \in C^{P\times MN}(P < MN)$ is introduced so that the high dimensional data **X** is projected into a lower dimensional matrix

$$\mathrm{Y} = \Phi\mathrm{X} = \Phi\mathrm{A}(\theta,\phi)\mathrm{S} = \Phi\mathrm{B}(\theta,\phi)\mathrm{s} = \Psi\mathrm{s} \tag{4}$$

with $\mathbf{B}(\theta,\phi)$ represents the discretized the angle space as $\boldsymbol{B}(\theta,\phi) = [\boldsymbol{b}(\theta_1,\phi_1),\boldsymbol{b}\left(\theta_2,\phi_2),\ldots,\boldsymbol{b}(\theta_Q,\phi_Q)\right)]$ and $Q \gg K$, where

$$s_q = \begin{cases} S_k & \text{if } b(\theta_q,\phi_q) = a(\theta_k,\phi_k) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The DOA estimation problem in (5) can be solved by minimizing the following formula

$$\mathrm{argmin}\|\mathrm{Y}-\Psi\mathrm{s}\|_2^2 + \eta\sum_{\mathrm{q}=1}^{\mathrm{Q}}\|\mathrm{s_q}\|_1 \tag{6}$$

There is a variety of algorithms aiming to extract row sparse signal **s** from the deterministic measurements. Well known are the Orthogonal Matching Pursuit (OMP), the Compressive Sampling Matched Pursuit (CoSaMP)[9] and the Bayesian Compressive Sensing (BCS)[10]. As easy to implication, the OMP algorithm is always the best choice for (6), but it still has high computational complexity as L is relatively high in the radar application. However, we find the peak searching of MUSIC algorithm is similar to that of the atom selection process in OMP algorithm, which inspired us to extract the DOA information from the compressed data by MUSIC algorithm.

As the compressed signal **Y** already contains all the information of all the antenna array, so the the effective aperture of the MIMO radar system would not hurt by the compressive process. Because of the incoherent characteristics of $\boldsymbol{\Phi}$ and $\boldsymbol{A}(\theta,\phi)$, the K eigenvectors $\mathbf{U}_s'$ corresponding to the significant eigenvalues of $\mathbf{R_y} = \mathbf{YY^H}$ can be regarded as the compressed signal subspace, and the rest $\mathbf{U}_n'$ are treated as the compressed noise subspace, while the orthogonal properties of the compressed signal subspace and the compressed noise subspace kept. The spatial spectrum function of the compressed MUSIC algorithm can be constructed as

$$f_{cs-music} = \frac{1}{[\boldsymbol{\Phi b}(\theta,\phi)]^H\boldsymbol{U}_n'\boldsymbol{U}_n'^H[\boldsymbol{\Phi b}(\theta,\phi)]} \tag{7}$$

The CS based 2D DOA estimation algorithm is summarized as followed

Step No.	operation
Step 1	Collect data from multiple receiver and matched filtering to form the matrix **X**
Step 2.	Construct a low-dimensional matrix **Y** from **X** by left multiplication a pseudo matrix $\boldsymbol{\Phi}$.
Step 3.	Estimated the covariance matrix $\mathbf{R_y}$ of compressed signal **Y**, then separate the compressed noise subspace $\boldsymbol{U}_n'$
Step 4.	DOA estimation using (7).

4) Computation analysis of the proposed algorithm: Compared with MUSIC algorithm, the proposed CS MUSIC algorithm requires more $O(PMNL)$ multiplication operation. Accordingly, the EVD operation takes the complexity of $O(P^3)$. Given the same number angle searching grid with MUSIC, the complexity of peak searching is $O(\mathrm{VW}(PMN + P^2 + P^2(P-K)))$, the total complexity of CS MUSIC algorithm can be approximately $O(VWP^3)$. As $P < MN$, the proposed CS MUSIC algorithm is more efficient than the traditional MUSIC algorithm.

3. Simulation Results

To assess the angle estimation performance of the proposed algorithm, 1000 Monte Carlo simulations are conducted. Note that: M, N, P and K are the number of transmit antennas, the receive antennas, the total pulse and targets, respectively. The reflection coefficients of the targets are assumed to fulfill the classic Swerling Case II. The SNR is defined as the signal power to the noise power before the matched filtering. We assume there are three uncorrelated targets located at angles $(\theta_1, \phi_1) = (10°, 15°)$, $(\theta_2, \phi_2) = (20°, 25°)$, $(\theta_3, \phi_3) = (30°, 35°)$, respectively. $M = N = 10$, the transmit and receive antennas are are configured to the form of uniform linear arrays with half-wavelength spacing. The samples during a pulse is designated to $L = 128$ The compressive rate is defined as $P/MN = 0.2$, and the searching interval is 0.1.

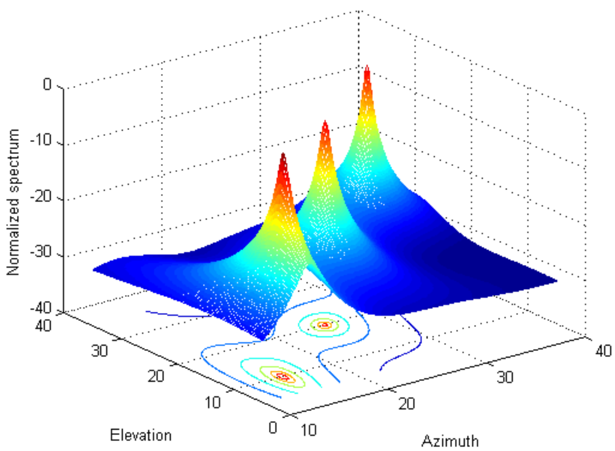

Fig. 2 Normalized spectrum of the CS MUSIC

Figure 2 presents the normalized spectrum of the proposed algorithm with $SNR = 10dB$. It is observed that the proposed scheme is able to estimate azimuth angle and elevation angle for the radar system.

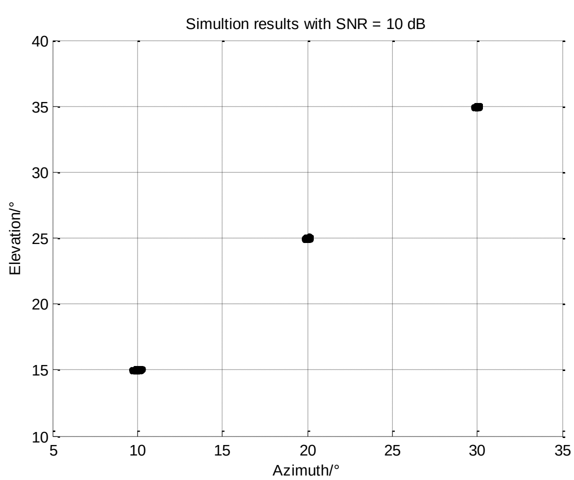

Fig. 3 Scatter of 1000 Monte Carlo simulations with SNR= 10dB.

Figure 3 depicts the scatter of 1000 Monte Carlo simulations with $SNR = 10dB$. It is obviously that the proposed algorithm is is robust to the pseudo measure matrix $\boldsymbol{\Phi}$.

The root mean squared error (RMSE) of 2D-DOD is defined as follow

$$\mathrm{RMSE} = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\sum_{i=1}^{1000}(\hat{\theta}_{i,k} - \theta_k)^2 + (\hat{\phi}_{i,k} - \phi_k)^2} \quad (8)$$

where $\hat{\theta}_{i,k}$ and $\hat{\phi}_{i,k}$ denote the estimation of azimuth and elevation angle of the k-th target in the i-th Monte Carlo trial. Figure 4 shows the comparison of the proposed CS MUSIC algorithm to MUSIC algorithm, with SNR ranges from *-5dB* to *30dB*. We find that the performance of the proposed algorithm is very close to that of the 2-D MUSIC algorithm.

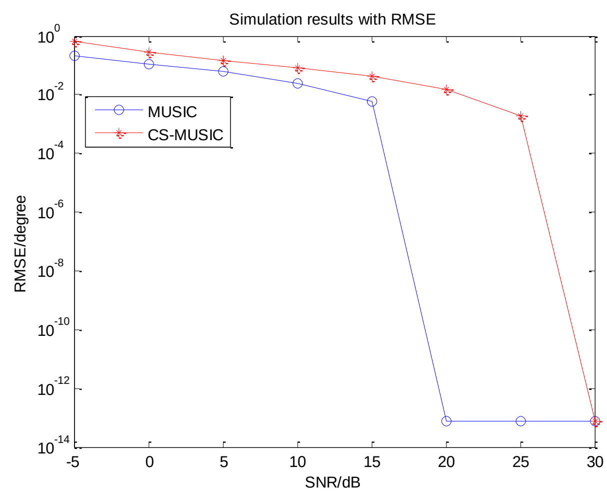

Fig. 4 RMSE comparison to MUSIC algorithm

The simulation computer is configured with Intel Xeon Quad-Core Processor W3550 (3.06 GHz, 8 MB cache, 1066 MHz memory), and with 12 GB memory. Table1 shows the average running time of the algorithms. It is obvious that the proposed algorithm process a higher operating efficiency than the traditional MUSIC algorithm, which is consistent with what we have analyzed.

Table1: Average programme time comparison

Algorithm SNR	**-5dB**	**0dB**	**5dB**	**10dB**
2-D MUSIC	19.281s	18.152s	17.924s	18.724s
CS MUSIC	7.308s	7.414s	7.374s	7.221s
Algorithm SNR	**15dB**	**20dB**	**25dB**	**30dB**
2-D MUSIC	18.553s	18.617s	16.826s	18.913s
CS MUSIC	7.164s	7.159s	7.171s	7.164s

4. Conclusion

In this paper, we propose a compressive sensing based two-dimensional DOA estimation algorithm for co-located

MIMO radar, which can reduce the complexity of traditional radar system while keeping good estimation performance. The proposed algorithm utilizes the sparsity of the targets in the background scene for DOA estimation, while the aperture of the initial antenna array are not been affected. Simulation results proved the usefulness of the proposed algorithm.

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