

# A Kind of Adverse Selection Problem and Its Improvement

Yuanhao Zhang

LangFang Department, Hebei University of Technology  
LangFang, China

Zhigong Hao

LangFang Department, Hebei University of Technology  
LangFang, China

Jun Ren

Hebei Province Educational Testing Institute,  
Shijiazhuang, China

Guangna Yin

LangFang Department, Hebei University of Technology  
LangFang, China

**Abstract**—A kind of adverse selection problem when integrating AHP and integer 0/1 programming to aid decision-making is discussed. The causes of the problem are analyzed. A new model based on AHP range estimation and parametric programming is created to improve the old one and solve the adverse selection problem. At last, a numerical example shows the effectiveness and validity to apply this model. The methodology in this paper is easy to be expanded to other programming forms integrated with AHP to aid decision-making.

**Keywords**—adverse selection; AHP; decision-making; judgment matrix

## I. INTRODUCTION

Analytic Hierarchy Process (AHP), a kind of multi-criterion decision-making method, was created by T.L. Saaty in 1970's. As soon as presented, it is recognized as an effective decision-making technology, especially for the problems whose criteria are difficult to quantify accurately and the problems that have no enough related information (Saaty, 1980). Meanwhile, for meeting the need of practice, the studies on integrating AHP and other decision-making methods rise rapidly. The integration of AHP and mathematical programming model is an important embranchment of them [2-11].

Mostly, the integration of AHP and mathematical programming model enhances the ability to make decision. Sometimes, however, some warps might be imported because of the integration. Though, compared with the advantages of integration, the disadvantages seem to be "trivial", they are still worth to make a point of research for more scientific decision-making. For example, we encountered a case of adverse selection problem (see Section 2) when integrating AHP and integer 0/1 programming to select software components for a component-based system. In this paper, the cause of adverse selection problem is analyzed on the basis of a simple discussion about integrating AHP and mathematical programming model. Then, a new model based on AHP range estimation and parametric programming is created to improve the old one in Appendix and solve the adverse selection problem. At last, a numerical example shows the effectiveness and validity to apply this model.

## II. CASE OF ADVERSE SELECTION PROBLEM

### A. Case Background

The case introduces a process integrating AHP and integer 0/1 programming to make a decision of software component selection. A component-based software system needs four software components. Namely, there are four component requirement fields. Each requirement fields has two candidates. Make the decision in the light of the steps as follows: Firstly, evaluate the priority of each requirement field by AHP method. Secondly, with the objective function maximizing weighted benefit and decision variables denoting each candidate is selected or not, a integer 0/1 programming model is created to make the eventual decision. The similar process also can be seen in literature (Ho-Won & Byoungju, 1999).

Notation:  $w_i$  is the weight of the  $i$ th requirement field;  $c_{ij}$  is the benefit of the  $j$ th candidate of the  $i$ th requirement field;

$$\begin{aligned} \max \quad & \sum_{i=1}^4 w_i \left( \sum_{j=1}^2 c_{ij} x_{ij} \right) \\ \text{s.t.} \quad & \sum_{i=1}^4 \sum_{j=1}^2 a_{ij} x_{ij} \leq b \\ & x_{11} \leq x_{21} \\ & \sum_{j=1}^2 x_{ij} = 1 \quad i = 1, 2, 3, 4 \\ & x_{ij} = 0/1 \quad \forall i, j \end{aligned}$$

software system;  $x_{11} \leq x_{21}$  means: if the 1st candidate of the 1st requirement field is selected, then the 1st candidate of the 2nd requirement field must be selected.

The data for Model (1) is listed below.

The AHP judgment matrix indicating the relative importance of requirement fields is:

## B. Model and Data

$a_{ij}$  is the cost of the  $j$ th candidate of the  $i$ th requirement field;  $b$ , a real constant, is the limit of global cost of

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \begin{pmatrix} R_1 & R_2 & R_3 & R_4 \\ 1 & 2 & 2 & 4 \\ 1/2 & 1 & 3 & 3 \\ 1/2 & 1/3 & 1 & 4 \\ 1/4 & 1/3 & 1/4 & 1 \end{pmatrix}$$

The consistency index CI of the judgment matrix is 0.09.

The numerical values of other coefficients is shown as follows:

$$c_{11}=9.75, c_{12}=9, c_{21}=8, c_{22}=9, c_{31}=8, c_{32}=7, c_{41}=9, c_{42}=8, a_{11}=8.3, a_{12}=8.2, a_{21}=8.5, a_{22}=8.8, a_{31}=8.5, a_{32}=7.9, a_{41}=9, a_{42}=8.8, b=34.$$

Applying EVM method, the result of AHP weights is  $w_1=0.412, w_2=0.316, w_3=0.194, w_4=0.079$ . The solution of model (1) is  $x_{12}=x_{22}=x_{32}=x_{41}=1$ , with the objective value 8.621. Applying LLSM method, the result of AHP weights is  $w_1=0.422, w_2=0.307, w_3=0.191, w_4=0.080$ . The solution of model (1) is  $x_{11}=x_{21}=x_{32}=x_{41}=1$ , with the objective value 8.6275. Obviously, there is a contradiction in candidate selection between the 1st requirement field and the 2nd requirement field because of the different algorithms for AHP priority vectors.

## III. CAUSES OF PROBLEM

Each of AHP and mathematical programming model has its own advantages. AHP adapts to the more subjective decision-making problems. mathematical programming model is appropriate for the more objective problems with accurately quantified criteria. In practice, most of decision-making problems are the mix of subjectivity and objectivity. Hence, integrating AHP and mathematical programming model can combine their advantages and make more reasonable decision. Through literature review, it can be concluded that there are three sorts of integration of AHP and mathematical programming model: ① Each of AHP and mathematical programming model is a relatively independent period of the whole multi-period optimization. In one period, AHP is used to evaluate the priority of criteria. In subsequent period, mathematical programming model accepts the AHP priority as cost or benefit coefficients (Lei & Jinmin, 2003; Ghodsypour & O'Brien, 1998; Ho-Won & Byoungju, 1999; Peisong D., Chuijie Y., Jianguo, 2003). ② Apply one method to improve another. For example, apply AHP to improve mathematical programming model [6] or apply mathematical programming model to improve AHP [7-10]. ③ Study unifying theory between AHP and mathematical programming model, and explore their common theory basis [11].

The adverse selection problem in Appendix can be categorized into the first sort mentioned above. Namely, AHP and integer 0/1 programming are two periods of a multi-period optimization. Both AHP and integer 0/1 programming are broadly applied in practice. The integration of them enhances the capability of decision-making. Because of the complexity of practice, however, some warps might occur when integrating them.

It can be found out that the direct reason of adverse selection problem is the difference of algorithms for AHP priority vectors. Analyzing more, three causes can be deduced:

### A. When AHP Judgment Matrix is Inconsistent, Different Algorithms for AHP Priority Vectors Lead to Various Outcomes.

Since Satty used a principal Eigen Value Method (EVM) to derive AHP priority vectors, some other algorithms, such as Least Squares Method (LSM) [12], Logarithmic Least Squares Method (LLSM) [13], Least Deviation Method (LDM) [14] etc., are presented. It has been proved that when the positive reciprocal matrix is consistent, all these methods lead to the same outcome [15]. But when judgment matrix is inconsistent, even though it passes consistency check, the outcomes derived from these methods are different [16]. That leads to the final adverse selection problem.

### B. When AHP Judgment Matrix is Inconsistent, all of These Methods Including EVM, LSM, LLSM, LDM etc., Only Provide Approximate Point Estimation, and Lose Some Useful Information.

When AHP judgment matrix is consistent, it can be thought of that the judgments of people have no deviation. When AHP judgment matrix is inconsistent, the inconsistency reflects the subjective deviation caused by objective uncertainty. In real life, judgments are frequently inconsistent, and in this situation, point estimation is not enough. Some useful information embodying uncertainty is lost.

### C. Integer 0/1 Programming is Sensitive to AHP Weights

Some coefficients and variables of integer 0/1 programming are sensitive to AHP weights. That makes the solution of integer 0/1 programming different with the variety of AHP priority vectors.

## IV. MODEL

### A. Framework

For ameliorating the weakness that EVM etc. only provide point estimation, Gao Jie etc. presented AHP Range Estimation Method (REM) [7]. According to the judgment matrix  $A$  and the largest eigenvalue  $\lambda_{\max}$ , the polyhedral closure convex cone  $(A - \lambda_{\max} E)W \leq 0$  is constructed. Taking the polyhedral closure convex cone and AHP weights normalization rule as constraints, and maximizing or minimizing each weight as objective function, a series of linear programming models are created. By solving these models, the upper limit and low-

er limit of each weight can be calculated. The REM method provides the possible weight ranges that include more information than a point. Synthesizing REM and other algorithms can avoid losing some useful information and embody more uncertainty of real life in mathematic model. So, firstly, we synthesize REM method and other algorithms to estimate the ranges of weights. Secondly, a parametric programming model based on integer 0/1 programming is applied to analyze the sensitivity of solutions.

In conclusion, the framework of the model is shown in Figure 1.

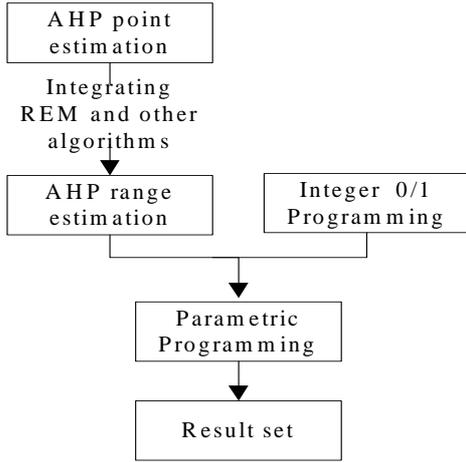


FIGURE 1. FRAMEWORK OF MODEL

### B. Model

$$\begin{aligned}
 & \text{min}(\max) \sum_{i=1}^n w_i \left( \sum_{j=1}^{m_i} c_{ij} x_{ij} \right) \\
 & \text{s.t.} \quad \sum_{i=1}^n \sum_{j=1}^{m_i} (a_{ij})_k x_{ij} \geq b_k \quad (\leq b_k; = b_k) \quad k=1, \dots, p \\
 & \quad x_{i',j'} \leq x_{i'',j''} \quad i', i''=1, \dots, n \quad i' \neq i'' \quad j', j''=1, \dots, m_i \quad (2) \\
 & \quad \sum_{j=1}^{m_i} x_{ij} = 1 \quad i=1, \dots, n \\
 & \quad x_{ij} = 0/1 \quad \forall i, j \\
 & \quad W = (w_1, w_2, \dots, w_i, \dots, w_n)^T \in S
 \end{aligned}$$

Notation:  $x_{ij}$  is decision variable;  $c_{ij}$ ,  $(a_{ij})_k$ ,  $b_k$  are real constants;  $w_i$ , an interval number, is the  $i$ th weight;  $W$  is AHP weight vector;  $S$  is given by equation (3).

$$S = \left\{ (w_1, w_2, \dots, w_i, \dots, w_n)^T \mid \underline{w}_i \leq w_i \leq \overline{w}_i \right\} \quad (3)$$

$$\underline{w}_i = \min(w_{i,REM}, w_{i,other\ method})$$

$$\overline{w}_i = \max(w_{i,REM}, w_{i,other\ method})$$

Notation:  $\underline{w}_i$  is the lower limit of  $w_i$ ;  $\overline{w}_i$  is the upper limit of  $w_i$ ;

$w_{i,REM}$  is the  $i$ th weight range from REM method;

$w_{i,other\ method}$  is the  $i$ th weight set from some other algorithms (EVM, LLSM, LSM, LDM etc.). For estimating the ranges of AHP weights, REM method should be requisite and other algorithms can be selected by need.

**Theorem 1** [15] If the AHP judgment matrix  $A$  is consistent, then EVM, LLSM, LSM, LDM give the same AHP priority vectors.

**Theorem 2** [7] If the AHP judgment matrix  $A$  is consistent, then the upper limit of  $W$  (WU) and the lower limit of  $W$  (WL) coming from REM method, and the weight WEVM coming from EVM method, meet the equation  $WU = WL = WEVM$ .

**Theorem 3** If the AHP judgment matrix  $A$  is consistent, then REM, EVM, LLSM, LSM, LDM give the same AHP priority vectors, and the equation

$w_{i,REM} = w_{i,other\ method}$  is true. Range becomes point and parametric programming becomes ordinary integer 0/1 programming. In this situation, there is no adverse selection problem.

**Proof.** Obvious, using Theorems 1 and 2.

**Corollary 1** The smaller the consistency index CI is, the smaller the interval between  $\underline{w}_i$  and  $\overline{w}_i$  is. Then, the possibility of adverse selection problem is smaller. Contrarily, The larger the consistency index CI is, the larger the interval between  $\underline{w}_i$  and  $\overline{w}_i$  is. Then, the possibility of adverse selection problem is larger.

**Proof.** When judgment matrix passes consistency check but inconsistent, the smaller the consistency index CI is, the less the uncertainty of judgments is. Hence, the AHP weight ranges derived from REM method are smaller [7] and the difference among other algorithms is less. That makes the

interval between  $\underline{w}_i$  and  $\overline{w}_i$  smaller and the possibility of adverse selection problem smaller. Contrarily, the interval between  $\underline{w}_i$  and  $\overline{w}_i$  is larger and the possibility of adverse selection problem is larger.

To identify adverse selection problem more accurately, a possibility judgment rule is given by Theorem 4.

**Theorem4** If  
 $w'_1, w''_1 \in [w_{i_1}, w_{i_1}]$ ,  $w'_2, w''_2 \in [w_{i_2}, w_{i_2}]$ ,  $w'_1 \neq w''_1 \neq w'_2 \neq w''_2$   
 exist to make equation (4) true, then the probability of the adverse selection is given by.

*Notation:*

$i_1, i_2 \in \{1, 2, \dots, n\}$ ,  $i_1 \neq i_2$ ;  $j_1, j_2 \in \{1, 2, \dots, m_i\}$ ,  $j_1 \neq j_2$ .

**Proof.** If equation (4) holds, then equation (5) is true obviously.

$$\begin{cases} w'_1 c_{i_1 j_1} + w'_2 c_{i_2 j_1} > w'_1 c_{i_1 j_2} + w'_2 c_{i_2 j_2} \\ w''_1 c_{i_1 j_1} + w''_2 c_{i_2 j_1} < w''_1 c_{i_1 j_2} + w''_2 c_{i_2 j_2} \end{cases} \quad (5)$$

Then, as for local objective function of model (2):

$w_1 c_{i_1 j_1} x_{i_1 j_1} + w_1 c_{i_1 j_2} x_{i_1 j_2} + w_2 c_{i_2 j_1} x_{i_2 j_1} + w_2 c_{i_2 j_2} x_{i_2 j_2}$ , there are two states as follows.

① Minimizing objective function

According to equation (5), when AHP weights meet equation:  $w_1 = w'_1$ ,  $w_2 = w'_2$ , the solution  $x_{i_1 j_2} = x_{i_2 j_2} = 1$  is better than solution  $x_{i_1 j_1} = x_{i_2 j_1} = 1$ . when AHP weights meet equation:  $w_1 = w''_1$ ,  $w_2 = w''_2$ , the solution  $x_{i_1 j_1} = x_{i_2 j_1} = 1$  is better than solution  $x_{i_1 j_2} = x_{i_2 j_2} = 1$ . Thus, to the global objective function, there is the probability of adverse selection problem.

② Maximizing objective function

Similarly, it is easy to prove that there is the probability of adverse selection problem.

Theorem 4 only explains possibility of adverse selection problem because other possibility exists, for example,  $x_{i_1 j_1} = x_{i_2 j_2} = 1$  or  $x_{i_1 j_2} = x_{i_2 j_1} = 1$ . If, according to some special requirements in practice, one of solution  $x_{i_1 j_1} = x_{i_2 j_1} = 1$  and solution  $x_{i_1 j_2} = x_{i_2 j_2} = 1$  must be selected, then the adverse selection problem comes into being inevitably.

## V. EXAMPLE

Take the case of Appendix as example.

Synthesize REM, EVM and LLSM to get the AHP weight ranges as follows:  $w_1 \in [0.409, 0.422]$ ;  $w_2 \in [0.307, 0.321]$ ;  $w_3 \in [0.191, 0.197]$ ;  $w_4 \in [0.078, 0.081]$ .

Through model (2), the sensitivity analysis results are derived to show in Table 1.

TABLE 1. SENSITIVITY ANALYSIS OF AHP WEIGHT RANGES

$w_1-w_2$	$w_3$	$w_4$	Result
[0.088, 0.102]	[0.191, 0.197]	[0.078, 0.081]	$x_{12}=x_{22}=x_{32}=x_{41}=1$
[0.102, 0.106]	[0.191, 0.197]	[0.078, 0.081]	Complex, decided by concrete values
[0.106, 0.115]	[0.191, 0.197]	[0.078, 0.081]	$x_{11}=x_{21}=x_{32}=x_{41}=1$

$$\begin{cases} w'_1 (c_{i_1 j_1} - c_{i_1 j_2}) + w'_2 (c_{i_2 j_1} - c_{i_2 j_2}) > 0 \\ w''_1 (c_{i_1 j_1} - c_{i_1 j_2}) + w''_2 (c_{i_2 j_1} - c_{i_2 j_2}) < 0 \end{cases} \quad (4)$$

Obviously, model (2) provides more information for decision-maker than model (1). Through tradeoff in result set and seeking for new evidence, people can avoid warps to a certain extent and make more reasonable decisions.

## VI. CONCLUDING REMARKS

On account of the complexity of real life, copying some methods in practice might bring errors or warps. In this paper, a kind of adverse selection problem coming from integrating AHP and integer 0/1 programming to make selection decision is presented. To solve the problem, an improvement model based on AHP range estimation and parametric programming is created. Compared with the old model, the computing complexity of the new one increases a little. However, it provides more information for decision-maker and avoids wrong decision. In addition, the methodology in this paper is easy to be expanded to other programming forms integrated with AHP.

Some aspects of the work in this paper are still insufficient. For example, how to judge the existence of adverse selection problem? What is the prerequisite of adverse selection problem? Though we present a possibility judgment rule, it is not enough. Thus, answering these questions and developing related algorithms will be the key of our work in future.

## REFERENCES

- [1] Saaty T. L.: The Analytic Hierarchy Process. McGraw-Hill, New York (1980)
- [2] Lei F., Jinmin H.: Combining the Analytic Hierarchy Process and Goal Programming for Location Model of Emergency Systems. Systems Engineering-Theory & Practice, vol. 12, (2003) 116–120
- [3] Ghodspour S. H., O'Brien C.: A Decision Support System for Supplier Selection Using an Integrated Analytic Hierarchy Process and Linear Programming. International Journal of Production Economics, vol. 56-57, (1998) 199–212
- [4] Ho-Won J., Byoungju C.: Optimization Models for Quality and Cost of Modular Software Systems. European Journal of Operational Research, vol. 112, (1999) 613–619

- [5] Peisong D., Chuijie Y., Jianguo X.: Model Evaluation With Dynamic Programming for Concept Design. *Machine Design and Research*, vol.19, August (2003) 25–27
- [6] Saaty T. L., Vargas L. G., Dellmann K.: The Allocation of Intangible Resources: The Analytic Hierarchy Process and Linear Programming. *Socio-Economic Planning Sciences*, vol. 37, (2003) 169–184
- [7] Jie G., Linyan S., Jin H.: A Range Estimation on Priority Vectors in AHP. *Systems Engineering-Theory & Practice*, vol. 3, (2004) 103–106
- [8] Xin L., Linchen L., Cheng W.: Goal Programming-Based AHP Criteria Weight Learning Algorithm. *Journal of Tsinghua University (Sci & Tech)*, vol. 44, (2004) 466–469
- [9] Zhengchao M., Sanyang L.: A Goal Programming Method for Calculating weights in Interval AHP. *Mathematics in Economics*, vol. 20, September (2003) 87–90
- [10] Junpeng G., Yuhua W., Wenhua L.: On the Analytic Hierarchy Process with Normalized Interval Weight Vector. *Systems Engineering and Electronics*, vol. 26, (2004) 900–944
- [11] Escobar M. T., Moreno-Jiménez J. M.: A Linkage Between the Analytic Hierarchy Process and the Compromise Programming Models. *Omega*, vol. 30, (2002) 359–365
- [12] Crawford G., William C. A.: A Note On the Analysis of Subjective Judgment Matrices. *Journal of Mathematical Psychology*, vol. 29, (1985) 387–405
- [13] Barzilai J., Cook W. D., Golany B.: Consistent Weights for Judgments Matrices of the Relative Importance of Alternatives. *Operations Research Letters*, vol. 6, (1987) 131–134
- [14] Baoqin C.: Two New Priority Methods in Analytic Hierarchy Process. *Journal of Systems Engineering*, vol. 5, (1990) 43–51
- [15] Wei Z., Dequan Y.: Comparison and Analysis for AHP Algorithms. *Mathematics in Practice and Theory*, vol. 1, (1995) 25–46