Construction of ZCZ Sequence Pair Set with Flexible Parameters Based on Interleaving Technique*

Mingyang Li, Weidong Peng, Peng Bai and Qianzhe Wang

Synthetic Electronic Information System and Electronic Countermeasure Technology Research Department Air Force Engineering University, Xi'an 710051 China

wo.lmy@163.com, pengweidong@126.com, Bpeng@163.com, afeu_wqz@163.com

Abstract - Sequences with some desired correlation properties are useful in communication and radar systems for applications in identification, synchronization, ranging and interference mitigation. This paper put forwards a construction of zero correlation zone (ZCZ) sequence sets with flexible ZCZ based on interleaving technique. Different to traditional interleaving, it designs shift sequences according to the mathematical relations between the length of initial sequence, the length of shift sequence and the ZCZ of the resultant sequence set rather than only considering the length of the initial sequence and the length of the shift sequence. This paper analyses and proves the two shift sequences are equal when they simultaneously satisfy both of the different mathematical relations. This construction enriches quantity of ZCZ sequence sets and is meaningful in the application of quasi-synchronous code division multiple address (QS CDMA) communications.

Index Terms - Zero correlation zone (ZCZ), Interleaving technique, Shift sequence

1. Introduction

Zero correlation zone (ZCZ) sequence pair set[1] is a class of good code for quasi-synchronous code division multiple address (QS-CDMA) communications[2] and radar systems, therefore study of construction of ZCZ sequence pair set is meaningful practically and theoretically in communications and radar [3]. Many construction methods have been derived in [4-7], of which interleaving technique [8] is an effective means to easily to achieve the aimed sequence sets. The concept of shift distinction is firstly introduced and an optimal sequence set is constructed in [9], but the parameters of the novel sequence set are strictly limited. Shift sequence [10] is the core issue of interleaving construction, reference [11] generates ZCZ sequence pair set with certain volume by interleaving the initial sequence of shift according to the length of it and the shift sequence. Optimal or suboptimal sequence pair sets are proposed in [12, 13] based on perfect sequence. However, these researches concentrate mainly on the optimality of the sequence sets, rarely on the flexibility of sequence sets.

ZCZ sequence pair sets construction employing arbitrary initial sequence pair and shift sequence are investigated and ZCZ sequence pair sets with flexible ZCZ are proposed in this paper. Shift sequences are designed by comprehensively considering lengths of the initial sequence, the shift sequence and ZCZ of the aimed sequence, and feasibility of the construction is validated. Unrestricted to the limitation of lengths of the initial sequence and the shift sequence, more generally, this construction can generate ZCZ sequence pair sets with various parameters.

2. Preliminaries

Definition 1 Let $\boldsymbol{a}=\{\boldsymbol{a}_x,\boldsymbol{a}_y\}$, $\boldsymbol{b}=\{\boldsymbol{b}_x,\boldsymbol{b}_y\}$ be sequence pairs with length of L. Where, $\boldsymbol{a}_x=(a_{x,0},a_{x,1},\cdots,a_{x,L-1})$, $\boldsymbol{a}_y=(a_{y,0},a_{y,1},\cdots,a_{y,L-1})$, $\boldsymbol{b}_x=(b_{x,0},b_{x,1},\cdots,b_{x,L-1})$, $\boldsymbol{b}_y=(b_{y,0},b_{y,1},\cdots,b_{y,L-1})$. The cross correlation function (CCF) of $\boldsymbol{a}$ and $\boldsymbol{b}$ is defined as

$$R_{\boldsymbol{a},\boldsymbol{b}}(t)=\sum_{i=0}^{L-1}a_{x,i}b^*_{y,(i+t)\bmod L} \quad \text{(1-a)}$$

$$R_{\boldsymbol{b},\boldsymbol{a}}(t)=\sum_{i=0}^{L-1}b_{x,i}a^*_{y,(i+t)\bmod L} \quad \text{(1-b)}$$

Definition 2 Let $\boldsymbol{A}=\{\boldsymbol{a}^0,\boldsymbol{a}^1,\cdots,\boldsymbol{a}^{M-1}\}$ be a sequence set with M sequences of Length L, if the CCF of sequences of $\boldsymbol{A}$ satisfies Eq. (2), then $\boldsymbol{A}$ is called a $ZCZ(L,Z,M)$ sequence set.

$$R_{\boldsymbol{a}^i\boldsymbol{a}^j}(t)=\begin{cases}\mathrm{C}>0, i=j \ \text{ and } \ t=0,\\ 0 \ \ , i=j \ \text{ and } \ 0<t\le Z,\\ 0 \ \ , i\ne j.\end{cases} \quad (2)$$

Definition 3 Let sequence $\boldsymbol{b}=I(\boldsymbol{a}^0,\boldsymbol{a}^1,\cdots,\boldsymbol{a}^{N-1})$ of length NL is derived form N sequences $\boldsymbol{a}^0,\boldsymbol{a}^1,\cdots,\boldsymbol{a}^{N-1}$ of length L. Where, $b_{jN+i}=a^i_j$. Then $I(\boldsymbol{a}^0,\boldsymbol{a}^1,\cdots,\boldsymbol{a}^{N-1})$ is the interleaving operation of $\boldsymbol{a}^0,\boldsymbol{a}^1,\cdots,\boldsymbol{a}^{N-1}$.

Lemma 1 Let $\boldsymbol{a}^0,\boldsymbol{a}^1$ be two sequences of length L, $\boldsymbol{e}=(e_0,e_1,\cdots e_{N-1})$ and $\boldsymbol{f}=(f_0,f_1,\cdots f_{N-1})$ be two shift sequences and $e_i, f_i\ge 0$. The generated sequences $\boldsymbol{b}^0,\boldsymbol{b}^1$ are

$$\begin{aligned}\boldsymbol{b}^0&=I\left(L_{e_0}(\boldsymbol{a}^0),L_{e_1}(\boldsymbol{a}^0),\ldots,L_{e_{N-1}}(\boldsymbol{a}^0)\right),\\ \boldsymbol{b}^1&=I\left(L_{f_0}(\boldsymbol{a}^1),L_{f_1}(\boldsymbol{a}^1),\ldots,L_{f_{N-1}}(\boldsymbol{a}^1)\right).\end{aligned} \quad (3)$$

* This work is partially supported by National Science and Nature Foundation of China #61174194 to M. Y. Li

Let $\tau = N\tau_1 + \tau_2$, $0 \le \tau_2 < N$, the CCF of $\boldsymbol{b}^0, \boldsymbol{b}^1$ holds [8]:

$$R_{\boldsymbol{b}^0,\boldsymbol{b}^1}(\tau) = \sum_{i=0}^{N-\tau_2-1} R_{\boldsymbol{a}^0,\boldsymbol{a}^1}\left(f_{i+\tau_2} - e_i + \tau_1\right) + \sum_{i=N-\tau_2}^{N-1} R_{\boldsymbol{a}^0,\boldsymbol{a}^1}\left(f_{i+\tau_2-N} - e_i + \tau_1 + 1\right) \quad (4)$$

Where, $L_x(\boldsymbol{a}) = (a_x, a_{x+1}, \cdots a_{L-1}, a_0, \cdots a_{x-1})$.

3. Construct Methods

In this section, we assume N represents the length of the shift sequence, L represents the length of the initial sequence and Z represents the length of the ZCZ of the final sequence. Let $\boldsymbol{A} = \{\boldsymbol{a}^0, \boldsymbol{a}^1, \cdots, \boldsymbol{a}^{M-1}\}$ denote the initial sequence pair set with M sequence pairs of length L, and the ZCZ of $\boldsymbol{A}$ is L-1. Let $\boldsymbol{B} = \{\boldsymbol{b}^0, \boldsymbol{b}^1, \cdots, \boldsymbol{b}^{M-1}\}$ denote the resultant sequence pair set, for arbitrary i belongs to {0,1,…,M-1}, always have $\boldsymbol{b}^i = I\left(Le_0(\boldsymbol{a}^i), Le_1(\boldsymbol{a}^i), \cdots, Le_{N-1}(\boldsymbol{a}^i)\right)$. Two constructions of flexible ZCZ sequence pair sets according to the relations of N, Z and L are derived in this section.

Theorem 1 When $\gcd(N, Z+1) = 1$, design the shift sequence $\boldsymbol{e} = (e_0, e_1, \cdots e_{N-1})$ as $e_i = i/N \bmod (Z+1)$, the novel sequence set $\boldsymbol{B}$ is a $ZCZ(L, Z, M)$.

Where, $\gcd(x, y)$ denotes the great common divisor of x and y.

Proof Arbitrarily choose two sequences $\boldsymbol{a}^m$, $\boldsymbol{a}^n$ from $\boldsymbol{A}$, we discuss the CCF of the relative sequences $\boldsymbol{b}^m$, $\boldsymbol{b}^n$ in $\boldsymbol{B}$ as following.

From $\boldsymbol{e}$ and Lemma 1 we have

$$R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = \sum_{i=0}^{N-1} R_{\boldsymbol{a}^m,\boldsymbol{a}^n}\left(\left(\frac{\tau_2}{N} + \tau_1\right) \bmod (Z+1)\right) = \sum_{i=0}^{N-1}\left(R_{\boldsymbol{a}^m,\boldsymbol{a}^n}\left(\frac{\tau}{N}\right) \bmod (Z+1)\right).$$

For $\gcd(N, Z+1) = 1$, if $\tau \ne 0$ we have $(\tau/N) \bmod (Z+1) \ne 0$, so $R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = 0$; when $\tau = 0$, $R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = \sum_{i=0}^{N-1} R_{\boldsymbol{a}^m,\boldsymbol{a}^n}\left(\frac{0}{N}\right)$.

Therefore the CCF of sequences in $\boldsymbol{B}$ holds:

$$R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = \begin{cases} NL, \tau = 0 \text{ and } m = n, \\ 0, 0 < \tau < Z. \end{cases} \quad (5)$$

From (5), we know $\boldsymbol{B}$ is a $ZCZ(L, Z, M)$ sequence set, we have Theorem 1 proved.

Theorem 2 When $N \mid (Z+2)$ 时, design the shift sequence $\boldsymbol{f} = (f_0, f_1, \cdots f_{N-1})$ as $f_i = i(Z+2)/N \bmod (Z+2)$, the novel sequence set $\boldsymbol{B}$ is a $ZCZ(L, Z, M)$.

Where, $x|y$ denotes that y is dividable by x.

Proof Arbitrarily choose two sequences a^m, a^n from $\boldsymbol{A}$, we discuss the CCF of the relative sequences b^m, b^n in $\boldsymbol{B}$ as following.

From $\boldsymbol{f}$ and Lemma 1 we have

$$R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = \sum_{i=0}^{N-\tau_2-1} R_{\boldsymbol{a}^m,\boldsymbol{a}^n}\left(\left(\frac{\tau_2(Z+2)}{N} + \tau_1\right) \bmod (Z+2)\right) + \sum_{i=N-\tau_2}^{N-1} R_{\boldsymbol{a}^m,\boldsymbol{a}^n}\left(\left(\frac{\tau_2(Z+2)}{N} + \tau_1 + 1\right) \bmod (Z+2)\right) \quad (6)$$

When $0 < \tau < Z+2-N$, we have $0 \le \tau_1 \le (Z+2)/N - 2$, therefore $0 < \tau_2(Z+2)/N + \tau_1 \le (\tau_2+1)(Z+2)/N - 2 \le Z-1$; when $(Z+2)/N \le \tau \le Z$, we have $0 < \tau_1 = (Z+2)/N - 1$ and $0 \le \tau_2 < N$, so $\tau_2(Z+2)/N + \tau_1 = (\tau_2+1)(Z+2)/N - 1 \le Z$. For all above when $0 < \tau_2 \le Z-1$, we can get $0 < \tau_2(Z+2)/N + \tau_1 \le (Z+2) - 2 = Z$, that is when $0 < \tau \le Z$, we always have $R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = 0$; when $\tau = 0$, $R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = NR_{\boldsymbol{a}^m,\boldsymbol{a}^n}(0)$. Therefore the CCF of sequences in $\boldsymbol{B}$ holds:

$$R_{\boldsymbol{b}^m,\boldsymbol{b}^n}(\tau) = \begin{cases} NL, \tau = 0 \text{ and } m = n, \\ 0, 0 < \tau < Z. \end{cases} \quad (7)$$

From (7), we know $\boldsymbol{B}$ is a $ZCZ(L, Z, M)$ sequence set, we have Theorem 2 proved.

Theorem 3 When $\gcd(N, Z+1) = 1$ and $N \mid (Z+2)$, if $N < Z+2$, then shift sequences designed in Theorem 1 and Theorem 2 are equal.

Proof Multiple e_1 of Theorem 1 and f_1 of Theorem 2 respectively by N, we have:

(1) According to $\boldsymbol{e}$, we can get $Ne_1 - 1 = N[1/N \bmod (Z+1)] - 1 = m(Z+1)$, so $Ne_1 = m(Z+1) + 1$, we can easily get $N \mid [m(Z+1)+1]$. For $N \mid (Z+2)$, we have $m = 1$ and $Ne_1 = Z+2$.

(2) Similar to (1) we can get $Nf_1 = N[(Z+2)/N \bmod (Z+2)]$, therefore $Nf_1 = Z+2$.

When $N < Z+2$, from $Ne_1 = Z+2$ and $(N-1)e_1 = Z+2-e_1$, we can get $e_1 > 1$, therefore $(N-1)e_1 < Z+1$. So for arbitrary $i \in (0, 1, \cdots, N-1)$, we always have $e_i = ie_1$. In a similar way, for arbitrary $i \in (0, 1, \cdots, N-1)$, we always have $f_i = if_1$. From $Ne_1 = Nf_1$ we have $e_1 = f_1$, therefore for arbitrary $i \in (0, 1, \cdots, N-1)$, we always have $f_i = e_i$, that is $\boldsymbol{f} = \boldsymbol{e}$.

When $N = Z+2$, from $Ne_1 = Nf_1 = Z+2$ and $e_1 = f_1 = 1$, the two shift sequences are

$$e_i = \begin{cases} 0, i = N-1, \\ i, \text{otherwise}. \end{cases} \quad (8)$$
$$f_i = i, 0 \le i \le N-1.$$

From Eq. (8) when $N = Z+2$, shift sequence $\boldsymbol{e}$ and $\boldsymbol{f}$ are not equal, we can generate two different sequence sets with the same parameters.

Sequence sets constructed in Theorem 1 and Theorem 2 can also be extended as [5, 14] to generate more ZCZ sequence sets with flexible parameters.

4. Examples

4.1 Comparison of Shift Sequences

Shift sequences designed in this paper comprehensively take N, Z and L into consideration; shift sequence of this paper and Ref. [14] is as Talbe 1.

TABLE I Comparison of shift sequences in this paper and Ref. [14]

	This paper		Reference [14]	
parameters	gcd(N,Z+1)=1	N|Z+2	gcd(N,L)=1	N|L
shift sequences	$e_i = \frac{i}{N} \bmod (Z+1)$	$e_i = \frac{i(Z+2)}{N} \bmod (Z+2)$	$e_i = \frac{i}{N} \bmod L$	$e_i = \frac{iL}{N} \bmod L$

When $\gcd(N, Z+1) = 1$ and $Z+1 = L$ or $N \mid (Z+2)$ and $Z+2 = L$, shift sequences of Ref. [14] are equal to shift sequences of this paper, that is construction of Ref. [14] is a special case of this paper.

4.2 Construction Examples

Simulations for distinct parameters are implemented by employing initial sequence pairs generated by [14].

Example 1 When $\gcd(N, L) = 1$ and $\gcd(N, Z+1) = 1$, $N = 4$, $L = 31$, $Z = 4$. Choose $\boldsymbol{e} = (0,4,3,2)$ according to Theorem 1 and initial sequence pair according to Ref. [14] as
$\boldsymbol{a}_x$ = (+-++++-+-+---+--+++-----++--+-+),
$\boldsymbol{a}_y$ = (0-0000-0-0---0--000-----00--0-0). The novel sequence pair is

$\boldsymbol{b}_x$ = (++++-++++-++++-++-+-++-+--+-+--+----++----+----
+-+--+++--+++--+++--++---+--------+---++---++---++ ,
+--+-+--+-+-++-+-++-+-++++-+)

$\boldsymbol{b}_y$ = (0000-0000-0000-00-0-00-0--0-0--0----00----0----0-0--
000--000--000--00---0--------0---00---00---000--0-0-- .
0-0-00-0-00-0-0000-+)

Part of the CCF of $\boldsymbol{a}$ and $\boldsymbol{b}$ are illustrated in Figure 1.

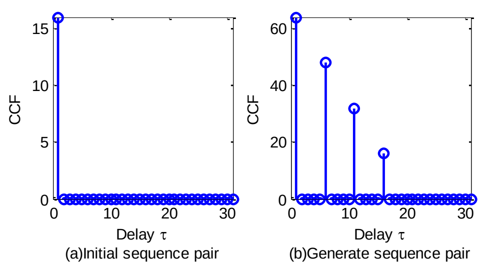

Fig 1. Correlation functions when gcd(N,Z+1)=1

Example 2 When $\gcd(N, L) = 1$ and $N \mid (Z+2)$, $N = 4$, $L = 31$, $Z = 6$. Choose $\boldsymbol{e} = (0,2,4,6)$ according to Theorem 1 and initial sequence pair same as Example 1. The novel sequence pair is

$\boldsymbol{b}_x$ = (+++--++++++--++++++---+++-----++-+----+-+----+-+-+--
+++-+--++--+--++--+---+--+---+--+---+--+-+-+--+-+++- ,
-+-++---++++-+-+++++-++)

$\boldsymbol{b}_y$ = (000--00000--00000---000-----00-0----0-0----0-0-0--000-
0--00--0--00--0---0--0---0--0---0--0-0-0--0-000--0-00--- .
0000-0-0000-00)

Part of the CCF of $\boldsymbol{a}$ and $\boldsymbol{b}$ are illustrated in Figure 2.

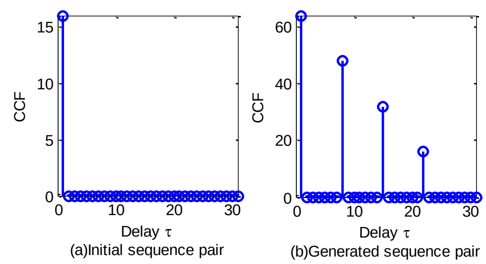

Fig 2. Correlation functions when N|(Z+2)

Example 3 When $N \mid L$ and $\gcd(N, Z+1) = 1$, $N = 3$, $L = 15$, $Z = 3$. Choose $\boldsymbol{e} = (0,3,2)$ according to Theorem 1 and initial sequence pair according to Ref. [14] as $\boldsymbol{a}_x$ = (-+-+--++-+++---+), $\boldsymbol{a}_y$ = (-0-0--00-000---0). The novel sequence pair is
$\boldsymbol{b}_x$ = (-+-+-+---++--++--+++-+++-+++-++-- +--- ---+---+),
$\boldsymbol{b}_x$ = (-0-0-0---00--00--000-000-000-00--0------0---0). Part of the CCF of $\boldsymbol{a}$ and $\boldsymbol{b}$ are illustrated in Figure 3.

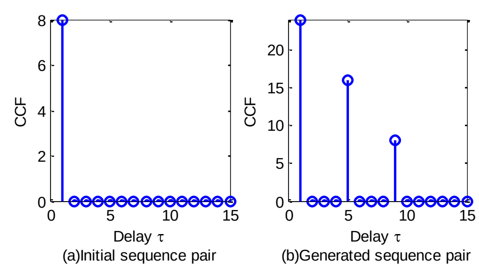

Fig 3. Correlation functions when gcd(N,Z+1)=1

Example 4 When $N|L$ and $N|(Z+2)$, $N=3$, $L=15$, $Z=4$. Choose $\boldsymbol{e}=(0,2,4)$ according to Theorem 2 and initial sequence pair same as Example 3. The novel sequence pair is $\boldsymbol{b}_x =$ (---++---++-+-+--+++-++++-+-++-+--+----+----++), $\boldsymbol{b}_y =$ (---00---00-0-0--000-0000-0-00-0--0----0----00). Part of the CCF of $\boldsymbol{a}$ and $\boldsymbol{b}$ are illustrated in Figure 4.

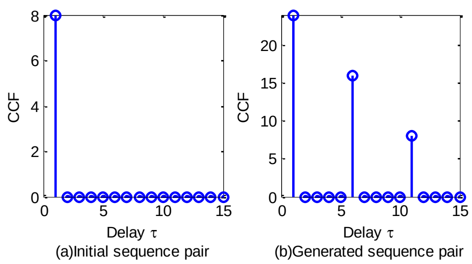

Fig 4. Correlation functions when $N|(Z+2)$

In Fig 1-4, the ZCZ of the novel sequence pair is consistent to Theorem 1 and Theorem 2, this construction is effective.

Example 5 When $\gcd(N,Z+1)=1$, $N=6$, $L=15$, $Z=6$. Choose $\boldsymbol{e}=(0,6,5,4,3,2)$ according to Theorem 1 and initial sequence pair same as Example 3. The novel sequence pair is

$\boldsymbol{b}_x =$ (-+--+-+++--+--++--++-++--++-++-+++-++-+++-+--+++
----+++----+++----+-+----+-+---- +-+----+-+),

$\boldsymbol{b}_y =$ (-0--0-000--0--00--00-00--00-00-000 -00-000-0--000---
-000----000----0-0----0-0----0-0----0-0).

Part of the CCF of $\boldsymbol{a}$ and $\boldsymbol{b}$ are illustrated in Figure 5.

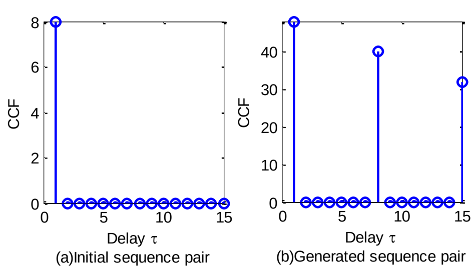

Fig 5. Correlation functions when $\gcd(N,Z+1)=1$

From $N=6$ and $L=15$, we have N and L neither relatively prime nor mutually dividable, method in Ref. [14] are not capable to deal with them; whereas, constructions in this paper overcome these limitations.

5. Conclusion

Comprehensively considering the relations of length of the initial sequence, length of the shift sequence and ZCZ of the target sequence set, shift sequences are designed and based on which and interleaving technique novel ZCZ sequence pair sets are derived in this paper. Without limited by the length of the initial sequence and the length of the shift sequence, sequence sets generated in this paper can have various parameters; thereby meet different requirements in radar and communications, et al.

Acknowledgment

The authors would like to thank the anonymous reviewers for their valuable suggestions.

References

[1] X. H. Tang, P. Z. Fan and D. B. Li, "Binary array set with zero correlation zone", *Electronics Letters*, vol. 37, no. 13, pp. 841-842, 2001.

[2] L. Hao and P. Z. Fan, "Performance evaluation for a new quasi-synchronous CDMA system employing generalized orthogonal sequences", *IEICE Transaction on Information and systems*, vol. E86-D, no. 9, pp. 1513-1524, 2003.

[3] L. Xu and Q. Liang, "Zero Correlation Zone Sequence Pair Sets for MIMO Radar", *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 3, pp. 2100-2113, 2012.

[4] X. H. Tang, P. Z. Fan and J. Lindner, "Multiple binary ZCZ sequence sets with good cross-correlation property based on complementary sequence sets", *IEEE Transaction on Information Theory*, vol.56, no. 8, pp. 4038-4045, 2010.

[5] P. H. Ke and S. Y. Zhang, "Study on the recursive constructions of zero correlation zone array pairs set", *Journal of Electronics & Information Technology*, vol. 33, no. 5, pp. 1257-1260, 2011.

[6] L. Y. Wang and X. Wang, "Design of zero correlation zone sequences via interleaving perfect sequence", *Journal of Southwest Jiaotong University*, vol.47, no. 1, pp. 47-49, 2012.

[7] T. Matsumoto, S. Matsufuji and T. Kojima, et al, "Orthogonal and ZCZ Sets of Real-Valued Periodic Orthogonal Sequences from Huffman Sequences", *IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and computer Sciences*, vol. E94, no. 12, pp. 2728-2736, 2011.

[8] G. Gong, "New designs for signal sets with cross correlation, balance property, and large linear span: GF(p) case", *IEEE Transactions on Inform Theory*, vol. 48, no. 11, pp. 2847-2867,2002.

[9] Z. C. Zhou, Z. Pan and X. H. Tang, "A new family of optimal zero correlation zone sequences from perfect sequences based on interleaved technique", 3rd *International Workshop on Signal Design and its Applications in Communications(IWSDA)*, Chengdu: IEEE Press, 2007, pp. 195-199.

[10] X. Y. Chen, C. Q. Xu and Y. B. Li, et al, "Construction of Shift Distinct Sequence Sets with Zero or Low Correlation Zone", *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol.96, no. 1, pp. 377-382, 2013.

[11] X. Y. Chen and C. Q. Xu, "Construction of shift distinct sequence sets with lower zero correlation zone based on interleaving technique", *Acta Electronica Sinica*, vol. 41, no. 5, pp. 890-896, 2013.

[12] S. Matsufiji, "Two types polyphase sequence sets for approximately synchronzied CDMA systems" *IEICE Transactions on Fundamentals*, vol. E86-A, no. 1, pp. 229-234, 2002.

[13] T. Hayashi, "Zero-correlation zone sequence set constructed from a perfect sequence", *IEICE Transactions on Fundamentals*, vol. E90-A, no. 5, pp. 1107-1111, 2007.

[14] Z. B. Li, T. Jiang, Z. Zhou, "Study on ZCZ punctured array pairs set", *Acta Electronica Sinica*, vol. 37, no. 3, pp. 489-493, 2009.