SNR-based Adaptive Threshold Decision PN Code Acquisition Algorithm

SHANG Yao-bo[1,2], GUO Ying[1], XU Yuan-hang[2], HAN Li-feng[1,2,] TAN Rui-lian[2]

[1] Information and Navigation college, Air Force Engineering University Xi'an 710077

[2] Synthetic Electronic Information System and Electronic Countermeasure Technology Research Department, Air Force Engineering University Xi'an 710051

Abstract - This paper proposes an algorithm based on noise power and SNR (signal to noise ratio) for adaptive threshold proposed PN code acquisition problem. First, this algorithm adjusts the threshold based on SNR through setting parameters, then smoothes the threshold waveform by median filters. It reduces error acquisition caused by fluctuations. Finally, by limiting filters, it makes the threshold reduces the false alarm probability caused by glitches at a certain noise level. This paper proves that the system's detection probability and false alarm probability are obviously improved comparing improved algorithm and original one through the analysis of false alarm probability and detection probability using improved algorithm and Monte Carlo simulation.

Index Terms - adaptive threshold, signal detection, false alarm probability, acquisition

1. Introduction

In radio mobile communication systems, signal detection is a significant matter in communication systems [1]. At present, threshold is usually used in PN code acquisition and there are two methods, fixed threshold and adaptive threshold [2]. Electrical level of received signal is dynamic due to the effect of users' relative displacement and channel fading and so on in mobile communication systems. Therefore, fixed threshold can't adapt to the application environment of mobile radio.

Adaptive threshold acquisition methods are widely used due to its excellent characteristic. Among them, a series of adaptive threshold algorithm based on constant false alarm have a low detection probability in low SNR condition because of its ignorance of signal power [3-5]. A scheme of preset multistage threshold is proposed in references [6, 7], and then it is regulated according to the algorithm. But this scheme is sensitive to the threshold totality; it will make an increase in acquisition time if setting is inappropriate. Three kinds of adaptive threshold scheme based on correlative accumulation power estimation are proposed in references [8-10]. Their performance is significantly improved in comparison with fixed threshold detection methods while there are still some disadvantages. OSAP (order statistics acquisition processor) is the best among them. But its performance will drop due to high estimation in condition of narrowband interference.

Aiming to solve the problems above, According to the actual condition of radio mobile communication this paper proposes a new PN code acquisition adaptive threshold method combining with adaptive threshold algorithm and nonlinear filtering theory. By performance analysis and numerical simulation, it is proved that the algorithm has advantages of higher detection probability, lower false alarm probability, more practical in comparison with average power based adaptive algorithm.

2. System Model

The structure of PN code acquisition system is illustrated in Fig.1. Suppose the signal is BPSK modulated, PN code correlation operation is done through matched filter and signal is detected through adaptive threshold algorithm. Effect to spread spectrum system of frequency shift is left out in order to simplify the analysis. The received complex baseband signal is described as:

$$r(k) = ac(k)e^{j\theta} + n(k) \tag{1}$$

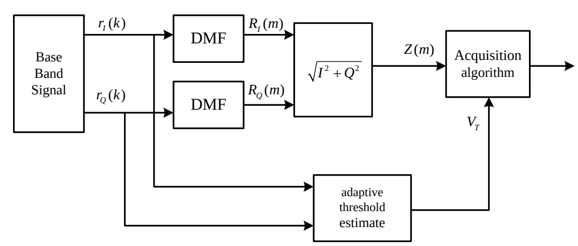

Fig.1 PN code acquisition searching system

In the equations, a and θ respectively denotes the amplitude and phase shift of received signal, $c(k)$ is the PN code sequence, $n(k)$ is the complex additive white Gaussian noises(AWGN) with single sideband power spectrum density of N_0 and they are independent. As the signal processing shown in Fig.1 the received baseband signal is sent into digital matched filter(DMF), so the I and Q two output is described as:

$$R(m) = aR_c(m)e^{j\theta} + N_c \tag{2}$$

In the equations, $R_c(m) = \sum_{k=1}^{M} c(k+m)c(k)$,

$N_c = \sum_{k=1}^{M} n(lM+k+m)c(k)$. According to the properties of AWGN, it is known that N_c are complex additive white Gaussian noises with single sideband power spectrum density of MN_0. M stands for PN period length, the length is the same as the tap coefficient of the matched filter, l Stands for the No. l PN code.

By complex modulus operation, two relevant output of in-phase and quadrature result that $Z(m) = \left| aR_c(m)e^{j\theta} + N_c \right|$. $Z(m)$ Denotes the statistical decision variable we form which is for acquisition decision by comparing with the threshold V_T generated real timely by adaptive threshold estimator. Suppose the input signal $r(k)$ and the local reference PN code $c(k)$ align, that is $Z(m)$ is greater than threshold V_T, if PN code is synchronized it is event H_1 or it is event H_0. For H_0, detection variable Z is likely to obey a Rayleigh distribution in low SNR condition and the probability density function (PDF) is [11]:

$$P(Z/H_0) = \frac{Z}{\sigma_n^2}\exp(-\frac{Z^2}{2\sigma_n^2}) \tag{3}$$

For H_1, detection variable Z is likely to obey a Rician distribution and the probability density function is[11]:

$$P(Z/H_1) = \frac{Z}{\sigma_n^2}\exp(-\frac{Z^2 + H_1^{\ 2}}{2\sigma_n^2})I_0(\frac{ZM_1}{\sigma_n^2}) \tag{4}$$

In the equation, $I_0(\bullet)$ is first kind of modified Bessel function of zero order. $\sigma_n^2 = MN_0/2$, $M_1 = Ma$.

3. Adaptive Threshold Based PN Code Acquisition Algorithm

A. Analysis on noise power based adaptive threshold algorithm

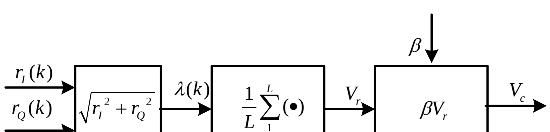

Fig.2 Adaptive threshold generating algorithm framework

The threshold of noise power based adaptive detection method is determined by mean power of received baseband signal. That is the adaptive threshold is formed through the estimation of signal power before matched filtering. As shown in Fig.2. Taking modulus operation on I and Q two baseband signals can result that the signal variable $\lambda(k)$. According to equation (1), $\lambda(k) = \sqrt{r_I^2(k) + r_Q^2(k)}$. When there is no signal received, $\lambda(k) = \sqrt{n_I^{\ 2}(k) + n_Q^{\ 2}(k)}$ obeys a Rayleigh distribution. According to the principle of mathematical statistics, the mean value of $\lambda(k)$ can be estimated through sampling points and it is[11]:

$$V_r = \frac{1}{L}\sum_{k=1}^{L}\lambda(k) \approx E[\lambda(k)] = \sqrt{\pi/2}\sigma \tag{5}$$

In the equation $\sigma_n^2 = N_0/2$. When there is signal received, $\lambda(k)$ obeys Rician distribution. Similarly it is[11]:

$$V_r = \sqrt{\pi/2}[(1+K)I_0(K/2) + KI_1(K/2)] \cdot e^{-K/2} \cdot \sigma \tag{6}$$

In the equation, $I_0(\bullet)$ is first kind of modified Bessel function of first order, L denotes the number of samples, the SNR of received signal $K = a^2/2\sigma^2$. The threshold is set to βV_r after the signal mean amplitude is known. When the false alarm probability of system is P_F, it is found that:

$$P_F = \int_{\beta V_r}^{\infty} p(Z/H_0)dZ = \exp(-\frac{(\beta V_r)^2}{2\sigma_n^2}) \tag{7}$$

Then we can get $\beta = \sqrt{-4M\ln P_F/\pi}$. Thus, β is only related with false alarm probability but not with SNR. Therefore the false alarm probability can be held constant at noise segment by setting value of β.

Similarly, it is known that $\beta = \sqrt{-4M\ln P_F e^K/\pi[(1+K)I_0(K/2) + KI_1(K/2)]^2}$ at signal segment. Thus β is both related with false alarm probability and SNR of received signal. Therefore the false alarm probability detection can't be done at signal segment by setting value of β.

B. Analysis on adaptive threshold algorithm with SNR

Taking the signal segment as object and considering adaptive threshold algorithm of detection probability, assuming that β is calculated at noise segment and applied to signal segment, we can get the threshold as:

$$V_c \approx \sigma\sqrt{-2M\ln P_F}\left[(1+K)I_0(K/2) + KI_1(K/2)\right] \cdot e^{-K/2} \tag{8}$$

At the receive terminal, if the interference is out of consideration the noise power of receiver is likely to be constant and threshold V_c is mainly effected by the SNR of received signal. According to equation (4) the detection probability is:

$$P_D = Q(\sqrt{2MK}, \sqrt{-2\ln P_F}\left[(1+K)I_0(K/2) + KI_1(K/2)\right] \cdot e^{-K/2}) \tag{9}$$

In the equation, $Q(a,b)$ is a Marcum Q function which is defined as: $Q(a,b) = \int_b^{\infty} xe^{-\frac{x^2+a^2}{2}} \cdot I_0(ax)dx$. M is the length of PN code period and K is SNR of received signal.

According to equation (3), the false alarm probability is:

$$P_F = \exp\left(\ln P_F\left[(1+K)I_0(K/2) + KI_1(K/2)\right]^2 \cdot e^{-K}\right) \tag{10}$$

This adaptive threshold algorithm improves the performance of PN code acquisition in comparison with fixed threshold. But the detection probability is low in low SNR condition for the signal is submerged in noise while the false alarm cannot be forbidden completely in high SNR condition. In addition, missing alarm will spring up due to a too high threshold in condition of interference or great noise power. Therefore this algorithm needs improvement.

C. Improved adaptive threshold algorithm

Based on the shortage of the algorithm above, the improved algorithm is shown as Fig.3. The threshold is set as $V_c' = \sqrt{\beta^2 V_r' - C}$. $\beta = \alpha M$ is in direct ratio with M and α is a constant of adjust coefficient. Generally we make it $0 < \alpha < 1$. In condition of low SNR and the signal segment can remove the positive affect to threshold of noise to make the threshold change with signal power in the most degree. Next is the determination of C. According to Fig.3, statistical properties of threshold and statistical decision variable at signal segment is investigated:

$$V_r' = \frac{1}{L}\sum_{1}^{L}\left[r_I^2(k) + r_Q^2(k)\right] \quad (11)$$

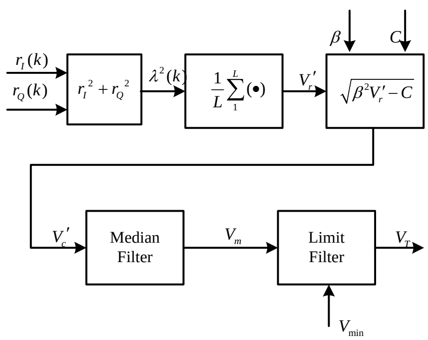

Fig.3 Framework of improved adaptive threshold generating algorithm

Put it into equation (1) to result the expectation. $E[V_r'] = E_c + N_0$ and $E_c = a^2$, $N_0 = 2\sigma^2$. Therefore it result that:

$$E[V_c'^2] = \alpha^2 M^2 (E_c + N_0) - C \quad (12)$$

In condition of H_1 is satisfied, the mean value of correlation peak square $Z^2(m)$ is:

$$E\left[Z^2(m)/H_1\right] = M^2 E_c + MN_0 \quad (13)$$

In condition of H_0 is satisfied at signal segment, the mean value of correlation peak square $Z^2(m)$ is:

$$E\left[Z^2(m)/H_0\right] = E_c + MN_0 \quad (14)$$

Therefore according to decision criterion in the range of $K \in (0, \infty)$, equations below is:

$$\begin{cases} M^2 K + M \ge \alpha^2 M^2 (K+1) - C/N_0 \\ K + M \le \alpha^2 M^2 (K+1) - C/N_0 \\ 0 < \alpha < 1 \end{cases} \quad (15)$$

According to equation (15), it results that $C = (\alpha^2 M^2 - M) N_0$. Therefore substituting it into equation (11) results that:

$$E\left[V_c'^2\right] = \alpha^2 M^2 E_c + MN_0 \quad (16)$$

Through the inference above, the adaptive threshold V_c' is obtained in condition of α and C is set. But in real systems, received sampling points are taken as the estimation samples of the threshold estimation. Assuming there are L samples for threshold estimation in all. The great fluctuation of threshold caused by the finiteness of samples will make the detection probability decrease. According to Bernoulli's law of large numbers, to make the estimation value approach the expectation, increase on number of samples is required which is usually impractical in real systems[12]. Therefore, to make as steady possible, the median filter is utilized to smooth the fluctuation of threshold. Suppose the size of filter is L_0 and L_0 is a positive integer. If the sampling points of the input sample sequence during the filter taps' delay are $V_r'(m - L_0 + 1)$, $V_r'(m - L_0 + 2)$, …, $V_r'(m)$ at the m-th time, then the output of the filter is[13]:

$$V_m(l) = \text{med}(V_r'(l - L_0 + 1), V_r'(l - L_0 + 2), \cdots, V_r'(l)) \quad (17)$$

In the equation, $\text{med}(\bullet)$ represents the operation of picking out the middle variable in the array of all the samples during the filter taps' delay which is in a ascending order. The threshold will be more steady after median filtering.

The algorithm above deals with the threshold estimation only at the signal segment. But at the noise segment, due to no signal arrive and according to equation (11), the expectation V_c' is obtained as: $E[V_r'] = N_0$ and $E\left[Z^2(m)/H_0\right] = E[V_m^2]$, the mean value of correlation peak $Z^2(m)$ is: $E\left[Z^2(m)/H_0\right] = MN_0$ so $E\left[Z^2(m)/H_0\right] = E[V_m^2]$. For $Z^2(m)$ and $V_c'^2$ are stochastic, the threshold requires a further revise due to the large numbers of false alarm in condition of a certain adaptive threshold decision V_m. The revision method is to put in a lowest threshold $V_{\min}$ to do limiting filter with input threshold V_m. The operation is: $V_T = \max(V_m, V_{\min})$ and $V_{\min}$ is obtained as $V_{\min} = \sqrt{-2MN_0 \ln P_F}$ in equation (4) in condition of constant false alarm probability. The limiting filter is mainly to make the false alarm probability of receiver with an ideal state constant in condition of a large false alarm probability at the noise segment.

4. Capability Analysis and Simulation

In the simulation the noise power is kept invariant and the SNR is adjusted by regulating the power of received signal. This paper forms the model according to the system model. Shown in Fig.1 and the threshold shown in Fig.2 and

Fig.3 and simulating the capability of improved algorithm in environment of Gaussian channel with SNR ranging [-l5dB~5dB]. Monte Carlo simulation is take to verify the validation of the method in this paper. In the simulation, a $M = 31$ m sequence is chosen as PN code and the modulation is BPSK. Related coefficients in Fig.3 are as follow: $a = \sqrt{2} \times 10^{SNR/20}$, $\sigma^2 = 1$, $\alpha = 0.22$, and $\beta = 3.4$, $C = 15.1$. The false alarm probability P_F is set separately 0.001 and 0.0001. Summation samples is $L = M = 31$ and size of median filter is $L_0 = 11$.

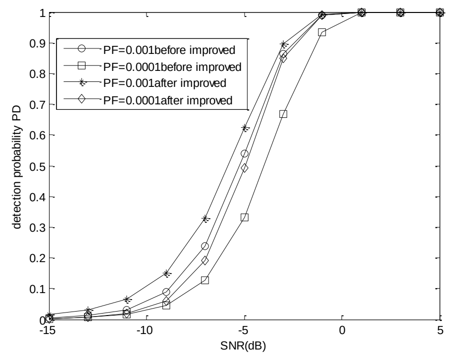

Fig.4 Comparison of detection probability in Gaussian channel

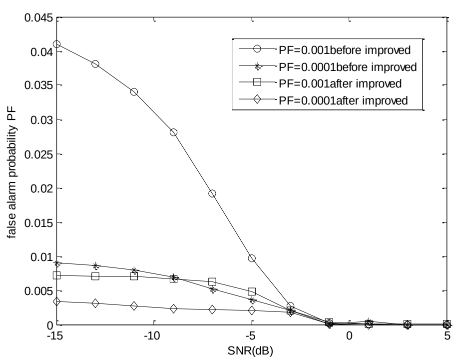

Fig.5 Comparison of false alarm probability in Gaussian channel

Fig.4 and Fig.5 separately represents the comparison of detection probability and false alarm probability at signal segment with improved algorithm and unimproved one in a Gaussian channel with SNR of [-15dB~5dB]. As shown in Fig.4, detection probability will increase with the SNR increasing. The false alarm probability is approximately constant with improved algorithm at a low SNR. This is because SNR is in consideration and noise power is the dominant contribution to the total at a low SNR. While at a high SNR, the false alarm probability will change with the SNR due to the signal's dominant contribution to total power. This is identical with the algorithm analysis. Another conclusion from the figuration is that the capability of improved algorithm is superior to unimproved one in environment of Gaussian channel.

5. Conclusion

This paper propose an improved algorithm which is able to take both noise power and SNR into consideration and make the threshold approximately smooth by median filter and limit filter. And this decreases the false acquisition caused by fluctuation. Through the analysis and numerical simulation of detection probability and false alarm probability, it is proved that the capability of system's detection probability and false alarm probability is obviously improved with this algorithm in comparison with the original adaptive threshold detection scheme which is based on noise power. Meanwhile the numerical simulation shows that this algorithm has a powerful capability of narrowband interference suppression.

References

[1] Eric Brigant, Aarne Mammela. Adaptive Threshold Control Scheme for Packet Acquisition. IEEE Trans. Comm. (S0090–6778), 1998, 46(12): 1580-1582.

[2] VITERBI A J. CDMA: Principles of spread spectrum communication. [S.1.]: Addison-Wesley, 1995.

[3] Kim E S. Park S K. Hybrid synchronization scheme for multi carrier communication systems. Journal of Electromagnetic Engineering and Sciences, 2012, 12(3):223-225,

[4] Xu Jian, Wang Zhigong, Wang Huan, et al. Design of adaptive threshold comparator. Journal of Circuits and Systems. 2011, 16(6), 6-11. (in Chinese)

[5] Hong Li, Xiaowei Cui, Mingquan Lu, and Zheruning Feng. Dual-Folding Based Rapid Search Method for Long PN-Code Acquisition. IEEE Trans. on Commun. 2008, 7(12): 5286-5296

[6] Liu Zhenkun, Huang Shunji. A improved algorithm of adaptive threshold in acquisition of PN code. Signal Processing. 2006, 22(4):458-461. (in Chinese)

[7] Kwonhue Choi, Kyungwhoon Cheun and Taejin Jung. Adapt five PN Code Acquisition Using Instantaneous Power Scaled Detection Threshold Under Rayleigh Fading and Pulsed Gaussian Noise Jamming. IEEE Trans. Common. 2002, 50(8): 867-876.

[8] Yu Dongxu. Adaptive Acquisition Threshold Algorithm Based on Mean Energy. Journal of Data Acquisition & Processing, 2010:25(S):10-13.(in Chinese)

[9] Kim C J. Adaptive acquisition of PN code in multipath fading mobile channels. Electronics Letters, 2002, 38(2):135-137.

[10] KIM C J, LEE H J, LEE H S. Adaptive acquisition of PN DSSS sequences communications. IEEE Trans for Commun, 1998, 46(8): 993-996.

[11] Joseph Boccuzzi. Signal Processing for Wireless Communications. Liu Zujun, Tian Bin, Yi Kechu. trans. Beijing: Publishing House of Electronics Industry,2010:105-111. (in Chinese)

[12] Zhao Shujie, Zhao Jianxun. Signal Detection and Estimation. Beijing: Publishing House of Tsinghua University. (in Chinese)

[13] Zhu Bin, Zeng Xiaoping, Zeng Fanxin, et al. New Adaptive Threshold Acquisition Scheme for PN Code. Journal of University of Electronic Science and Technology of China, 2010, 39(4):490-494. (in Chinese)