Newton Iterated Search Excitation Source Localization Algorithm Based on the Weighted Least Squares Method

Han ming
Communication Engineering Collage
Engineering University of CAPF
Xi'an Shanxi
18729581839@163.com

Feng lijie
Information Engineering Department
Engineering University of CAPF
Xi'an Shanxi
692474073@qq.com

Abstract—**With the development of modern science and technology, target localization technology has been widely used in many areas such as the bridge monitoring, warehouse monitoring, environmental testing and other fields. In this paper, we propose the Newton iterated localization algorithm based on the least squares method to study the target recognition algorithm. This article unifies the question of localization accuracy in many kinds of localization technology. Then, the proposed algorithm is verified by using the Newton iterated localization algorithm based on the least squares method. The experimental data shows that it gives a convincible gist and support for the study on location algorithms theoretically. The results indicate that this method is effective in improving the signal source localization accuracy.**

KEY WORDS: Least square method; Newton iteration; Localization accuracy;

I. INTRODUCTION

In recent years, target localization precision based on time difference of arrival (TDOA) measurements has been constantly improved, and a rich variety of localization algorithm has been proposed such as the Chan algorithm[1] and the Tayor algorithm [2]. This paper proposes an improved Newton iterated search excitation source localization algorithm based on the weighted least squares method, and the localization algorithm can make full use of the weighted least squares method[3] for better positioning estimate. It can effectively solve the initial valve problem and insure the convergence of the algorithm, as well as improve the convergence speed of iterative algorithm. The experimental result shows that the new algorithm has higher localization accuracy, better algorithm stability and faster convergence rate.

II. TARGET LOCALIZATION ALGORITHM PRINCIPLE

Target positioning algorithm based on the time difference of arrival (TDOA) measurements is also called the hyperbolic positioning [4]. This algorithm usually makes time delay estimation and obtains the time delay between the sensor array units, then calculates the target location combined with space geometry algorithm. This kind of method to calculate the amount is small, and it is suitable for processing immediately.

In the two-dimensional plane, the arrival time difference between emitter signals and the two points identified the hyperbola with two measurement point as the focal point. The distance of any point to two focuses is a fixed value, and we can get two hyperbolic intersections by using three stations from two hyperbolic baselines, as a result, we can determine the position of the radiation source.

Suppose that monitoring stations distribute in two-dimensional space in a certain shape, as shown in Figure 1.

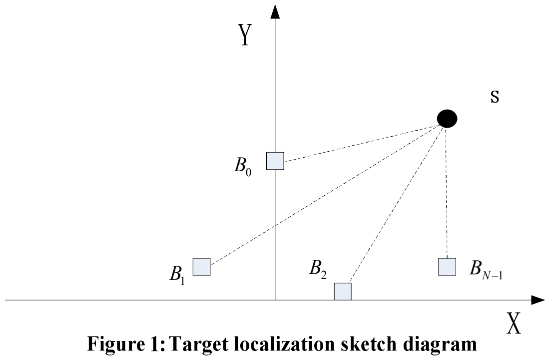

Figure 1: Target localization sketch diagram

In Figure 1, a certain basis for the origin of the two-dimensional coordinate system is established, $S(X,Y)$ is the target location, $B_i(X_i, Y_i)$ is the location of the monitoring station.

Then the mathematical model of target localization is as (1) followings:

$$\begin{cases} r_0 = \sqrt{(x-x_0)^2 + (y-y_0)^2} \\ r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} \\ r_{i,0} = c\tau_{i0} = r_i - r_0 \\ i = 1,2,\cdots N-1 \end{cases} \quad (1)$$

Where r_0 is the distance between the base station B_i to the target S, r_{i0} is the differential distance between the target to the base station B_i ($i \neq 0$) and the base station B_0, c is the propagation on velocity of electromagnetic wave, τ_{i0} is the measured value of time difference. The equation (1) indicates that the target localization in two-dimensional space is to equivalent to solve the equations of nonlinear optimization problem.

III. NEWTON ITERATION POSITIONING PRINCIPLE AND ITS IMPROVEMENT

Newton iterative method has the characteristics of fast convergence speed and the algorithm can save much time under the condition of certain location accuracy. But how to judge whether the Newton iterative method is convergent is still a problem, and the convergence speed is always associated with the choice of initial values. Solving the initial value problems is the premise guarantee of good performance of Newton iteration method. We will introduce the weighted least squares estimate value as the initial value of Newton iteration method for iteration calculation, which can realize the optimization of the algorithm.

A. Newton iteration positioning principle

According to the Newton iteration method, we select the first 2 equations from ($N-1$) equations to establish positioning equations as follows:

$$\begin{cases} F_1(x,y) = r_1 - r_0 - v\tau_{10} = 0 \\ F_2(x,y) = r_2 - r_0 - v\tau_{20} = 0 \end{cases} \quad (2)$$

The above equation (2) for jacobian matrix is:

$$F'(x,y) = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{bmatrix} \quad (3)$$

When the jacobian matrix is nonsingular matrix, there exists $\left[F'(x,y)\right]^{-1}$, it meets $\det\left(F'\right) \neq 0$, and it can be expressed as $\frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial y} \neq \frac{\partial F_2}{\partial x}\frac{\partial F_1}{\partial y}$.

And the source location coordinates using Newton iterative method can be represented as followings:

$$\begin{bmatrix} x^{(k+1)} \\ y^{(k+1)} \end{bmatrix} = \begin{bmatrix} x^{(k)} \\ y^{(k)} \end{bmatrix} + \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} \end{bmatrix}_{\substack{x=x^{(k)} \\ y=y^{(k)}}} \times \begin{bmatrix} F_1\left(x^{(k)}, y^{(k)}\right) \\ F_2\left(x^{(k)}, y^{(k)}\right) \end{bmatrix}$$

In (2), (3)and(4), (k =0,1,2…), to get the precise source location, it needs to set the right initial value $x(0), y(0)$ to guarantee the convergent speed of the algorithm.

B. Initial position choice method based on weighted least-square method

By the preceding analysis, in order to solve the problem of vibration source of the initial value, we can make full use of the least squares estimation results, which can be presented as the initial value of Newton iteration method. As a result, according to the literature 1, we can get the following equations:

$$\begin{cases}(x_i - x_0)x + (y_i - y_0)y + r_{i,0}r_0 = s_i \\ s_i = \frac{1}{2}\left(x_i^2 + y_i^2 - x_0^2 - y_0^2 - r_{i,0}^2\right)\end{cases} \quad (4)$$

Transform equations into matrix form as described below:

$$G = \begin{bmatrix} x_1 - x_0 & y_1 - y_0 & r_{1,0} \\ \vdots & \vdots & \vdots \\ x_N - x_0 & y_N - y_0 & r_{N-1,0} \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ r_0 \end{bmatrix} \quad S = \begin{bmatrix} s_1 \\ \vdots \\ s_{N-1} \end{bmatrix} \quad (5)$$

The initial position by the weighted least square method:

$X = \left(G^T WG\right)^{-1} G^T WS$； Where W is the weighted matrix and the W is treated as the measurement error covariance matrix of the time difference of arrival (TDOA).

C. Source localization algorithm calculation process and flow chart

The source localization algorithm calculation process is as follows:

Step 1: Setting the source location the initial value by the weighted least squares method as $\left(x(k), y(k)\right)$, then, we calculate the values in the formula of the Newton iteration method, and the location of the iteration is $\left(x(k+1), y(k+1)\right)(k = 0,1,2,\ldots,N-1)$.

Step 2:According to step 1, the result $\left(x(k), y(k)\right)$ and $\left(x(k+1), y(k+1)\right)$ will be put into the redundancy function expression of $F_i(x,y) = \left|r_i - r_0 - v\tau_{i0}\right| = \varepsilon_i$, next, we will make use of the optimization method, and get the equations as described below

$$\begin{cases}\varepsilon_i = \min(\varepsilon_0, \varepsilon_i) = \min\left(\varepsilon_0, F_i\left(x^{(k)}, y^{(k)}\right)\right) \\ \Delta_{k+1} = \max\left(\left|x^{(k+1)} - x^{(k)}\right|, \left|y^{(k+1)} - y^{(k)}\right|\right)\end{cases} \quad (7),$$

where ε_0 is the given value according to measurement accuracy requirements, and Δ is the precision given by the requirements.

Step 3: Making a judgment whether it meets the conditions $F_i\left(x^{(k+1)}, y^{(k+1)}\right) < \varepsilon$ or $\Delta_{k+1} < \Delta$, if it feats one of the two conditions, we will end the iterative process, the result $\left(x^{(k+1)}, y^{(k+1)}\right)$ will be output, otherwise, it will be calculated as the initial value to continue the steps above. In the process of the calculation above, we can effectively avoid unnecessary iterative process and guarantee the algorithm convergence by setting the redundant function expressions and the end of iteration precision range.

Specific algorithm flow chart is shown in the figure 2 below:

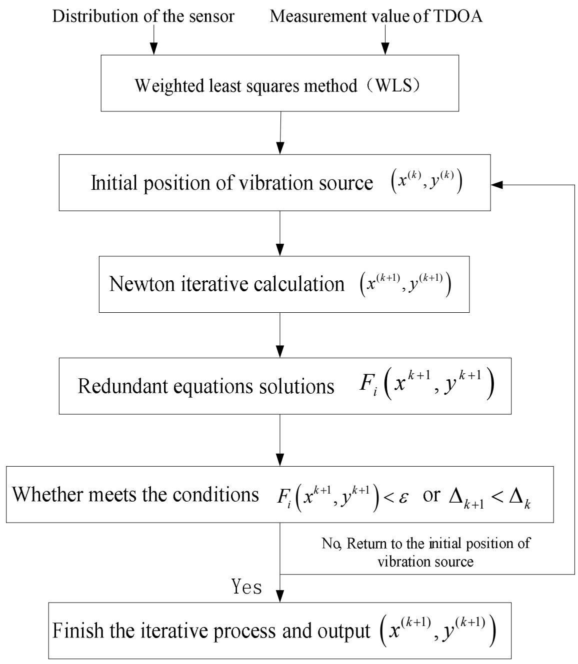

Figure 2: Algorithm flow chart

IV. EXPERIMENTAL RESULT ANALYSES

To test and compare the positioning performance of the algorithm, we take the walk vibration signals for example, and carry on the contrast experiment of the algorithm. The positioning system has shown in figure 3. There are five sensors components composed in the cross array, the coordinates of the receiving sensor are $B_0(0,0), B_1(0,10), B_2(-10,0), B_3(0, -10), B_4(10,0)$.

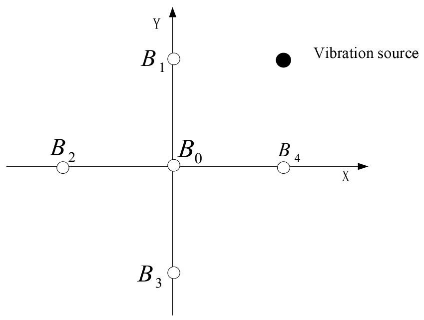

Figure3 : Distribution of the sensor

Set the vibration source move along the track of $Y=20$, the X value range is from 0 to 60 meters, and the interval value is 10 meters. Positioning accuracy is an important feature for algorithm performance, so we use the mean square error of positioning results $\text{RMSE}=\sqrt{(x-x')^2+(y-y')^2}$ to illustrate the positioning accuracy, where (x',y') is the real coordinates of vibration source, (x,y) is the positioning results of the algorithm. In order to compare, we respectively use the least square method and the proposed Newton iterative localization algorithm based on weighted least squares to experiment. The mean square error σ caused by the distance measurement error is $0.5 meter$. Suppose A is the curve of the least squares, and B is the curve of Newton iteration algorithm of weighted least squares. The trajectory we adopt respectively by the two algorithms above is in the following graph.

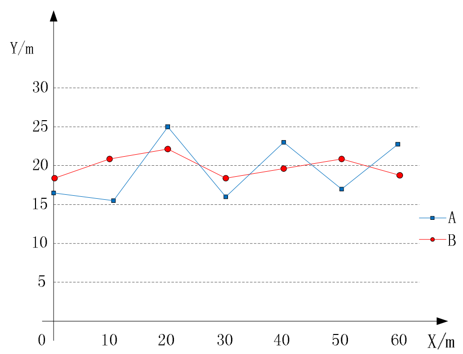

Figure4:Trajectory chart of the two algorithms

From the point of the approximation degree, the graph indicates that the curve of algorithm B is closer to the real trajectory. In combination with the results above, we can see that the proposed Newton iterative localization algorithm based on weighted least squares is of higher precision than the least squares algorithm.

V. CONCLUSION

For a limited time measurement, the conventional method is to use the arithmetic mean method. Although this method can improve the measurement results, to some extent, it's not the ideal method. Application of least square method can eliminate the measurement uncertainty of the conventional methods and obtain more reliable real-time measurement results. The general estimation of the least square method is not so precise, and the reason is the least-square method uses the measured values.

The use of the weighted method can contribute to analyze different sensors data. The higher precision sensors will get larger advised weight; on the contrary, the lower accuracy of sensors will obtain the smaller advised weights. In this paper, the experimental results and data analysis indicates that the Newton iteration algorithm based on weighted least squares method has a good convergence performance, and the positioning accuracy is significantly higher than the least squares algorithm. In addition, this paper solves the initial value problem of the Newton iterative method, and realizes the optimization of the algorithm.

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