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Robust H_2/H_∞ Control of Linear Markovian Jump Systems

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Abstract-The problem of robust H_2/H_∞ control for a class of linear continuous-time uncertain systems with randomly jumping parameters is investigated. The uncertainties are assumed to be norm-bounded. The transition of the jumping parameters is governed by a finite-state Markov process. A sufficient condition is first established on the existence of the robust H_2/H_∞ controller bases on the bounded real lemma. Then the corresponding state-feedback law is given in terms of a set of linear matrix inequalities (LMIs) which can be easily solved by standard numerical software. Its solutions provide a parameterized representation of the controller. Finally, a numerical example was given to illustrate the feasibility of the proposed technique.

Keywords-robust control;H_2/H_∞;Markovian jump

I. INTRODUCTION

H_2 and H_∞ norms are two frequently-used performance measures in optimal control systems. The system controlled by H_2 optimal control has good system performance and some other features, but it has little or no robustness for the uncertainties generated by the model disturbance of control objects. Whereas, H_∞ control theory can solve the system robustness problems better at the expense of other system properties. Consequently, combining the advantages of the two design methods, the means of applying mixed H_2/H_∞ index to analysis and design system was generated and has been developing quickly. The problem of mixed H_2/H_∞ control has attracted numerous researchers. To solve the given mixed H_2/H_∞ control problem, Lagrange's multipliers method was used firstly, then an assistant H_2 index which is an upper boundary of the H_2 norm was proposed, and the solution are achieved by optimizing the auxiliary cost function as showed in [1,2].

Recently, a great deal of attention has been devoted to the linear jump system which is a special class of hybrid systems involve both time-evolving and event-driven dynamics. A number of stability conditions, controller design and filtering problems related to these systems have been studied in [3,4], and the references therein. Costa in [5] designed a dynamic controller such that the closed-loop system is mean square stable and minimizes the H_2-norm of the system by algebraic Riccati equations approach for Markovian jump system with one output. Cao and Lam in [6] studied the sufficient conditions via LMI on the existence of a stochastic stabilization and γ suboptimal H_∞ state feedback controller for linear uncertain systems with randomly jumping parameters.

However, only a little work has been addressed to the robust H_2/H_∞ control of linear Markov jump systems. In this paper, our goal is to deal with the problem of robust H_2/H_∞ control for linear Markov jump systems with randomly jumping parameters. The essential of H_2/H_∞ control is to design a controller to robustly stabilize the uncertain systems and guarantee a specific level of performance for any admissible value of the uncertainty. A stable state feedback controller is developed for this class of systems.

II. PROBLEM STATEMENT AND PRELIMINARIES

Consider a class of Markovian jump systems with uncertainties as following

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}_0(t,\boldsymbol{r}(t))\boldsymbol{x}(t)+\boldsymbol{B}_0(t,\boldsymbol{r}(t))\boldsymbol{u}(t)+\boldsymbol{B}_2(\boldsymbol{r}(t))\boldsymbol{w}(t) \\ \boldsymbol{z}_0(t) &= \boldsymbol{C}_0(\boldsymbol{r}(t))\boldsymbol{x}(t)+\boldsymbol{D}_0(\boldsymbol{r}(t))\boldsymbol{u}(t) \\ \boldsymbol{z}_1(t) &= \boldsymbol{C}_1(\boldsymbol{r}(t))\boldsymbol{x}(t)+\boldsymbol{D}_1(\boldsymbol{r}(t))\boldsymbol{u}(t) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(t)\in R^n$ is system state, $\boldsymbol{u}(t)\in R^m$ is control input, $\boldsymbol{w}(t)\in R^p$ is the disturbance input which is a square integrable and peak-bounded stochastic vector function over $[0,+\infty)$; $z_0(t)\in R^q$, $z_1(t)\in R^l$ are output to be controlled. $\boldsymbol{A}_0(t,\boldsymbol{r}(t))$, $\boldsymbol{B}_0(t,\boldsymbol{r}(t))$, $\boldsymbol{B}_2(\boldsymbol{r}(t))$, $\boldsymbol{C}_0(\boldsymbol{r}(t))$, $\boldsymbol{D}_0(\boldsymbol{r}(t))$, $\boldsymbol{C}_1(\boldsymbol{r}(t))$ and $\boldsymbol{D}_1(\boldsymbol{r}(t))$ are matrix functions of the random jumping process $\{\boldsymbol{r}(t)\}$. The parameter $\boldsymbol{r}(t)$ is a continuous-time Markovian process taking values in a finite set S=$\{1, 2, \cdots\cdots, s\}$, with transition matrix $\Pi=[\pi_{ij}]$, where $i,j=1,2,\cdots\cdots,3$ and the transition probability is given by

$$\Pr\{\boldsymbol{r}(t+\Delta)=j \mid \boldsymbol{r}(t)=i\}=\begin{cases} \pi_{ij}\Delta+o(\Delta), & i\neq j \\ 1+\pi_{ii}\Delta+o(\Delta), & i=j \end{cases}$$

where $\Delta>0$ and $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta}=0$, $\pi_{ij}\geq 0$ is the transition rate from i to j if $i\neq j$ and $\sum_{j=1,j\neq i}^{s}\pi_{ij}=-\pi_{ii}$ for each mode i, $j=1,2,\cdots\cdots,$ s. It is assumed that the jump process is accessible for every $t>0$. For each possible value $\boldsymbol{r}(t)\in S$, to simplify the notation, we denote $\boldsymbol{A}(\boldsymbol{r}(t))=\boldsymbol{A}_i$, and so on.

$x(0)=0$ and $r(0)=1$ are the initial values of the state and the mode at time $t=0$ respectively.

Time-varying uncertainties may appear in these matrices that are

$$A_{0i}(t)=A_i+\Delta A_i(t),\ B_{0i}(t)=B_i+\Delta B_i(t)$$

where A_i, B_i are governed only by the Markovian jump process and $\Delta A_i(t)$, $\Delta B_i(t)$ are real-valued functions representing time-varying parameter uncertainties. We assume that the uncertainties are norm-bounded and can be described as

$$[\Delta A_i(t) \quad \Delta B_i(t)]=H_i F_i(t)[E_{1i} \quad E_{2i}]$$

where H_i, E_{1i}, E_{2i} are known constant matrices for each $i\in S$ and $F_i(t)$ are unknown time-varying matrix function with Lebesgue measurable elements meeting

$$F_i^T(t)F_i(t)\le I,\ \forall\ t\ge 0;\ r(t)=i\in S$$

Consider a robust state-feedback control law

$$u(t)=K_i x(t) \tag{2}$$

where K_i is unknown constant matrix for each $i\in S$. Connecting the state feedback control law (2) to the original system (1), we can obtain the following closed-loop system:

$$\begin{aligned}\dot{x}(t)&=\tilde{A}_i x(t)+B_{2i}w(t)\\ z_0(t)&=\tilde{C}_{0i}x(t)\\ z_1(t)&=\tilde{C}_{1i}x(t)\end{aligned} \tag{3}$$

where $\tilde{A}_i=\bar{A}_i+H_iF_i\bar{E}_i$, $\tilde{C}_{0i}=C_{0i}+D_{0i}K_i$, $\tilde{C}_{1i}=C_{1i}+D_{1i}K_i$, $\bar{A}_i=A_i+B_iK_i$, $\bar{E}_i=E_{1i}+E_{2i}K_i$

For a given scalar $\gamma>0$, this paper is concerned with the design of a robust state-feedback law (2) such that the resultant closed-loop system (3) satisfies the following performance indexes:

a) The closed-loop system is asymptotically stable;

b) When $w(t)$ is energy bounded disturbance, the closed-loop transfer function T_{wz_1} from the exogenous input $w(t)$ to the output vector $z_1(t)$, satisfies

$$\|T_{wz_1}\|_\infty<\gamma$$

c) When $w(t)$ is white noise signal, the performance index $J(t)$ satisfies

$$J(t)=\sup\lim_{t\to\infty}\mathrm{E}\{z_0^T(t)z_0(t)\}\le\bar{J}(t)$$

where $\bar{J}(t)$ is known constant.

If the closed-loop system is asymptotically stable, $J_0(t)$ can be expressed as

$$J_0(t)=\sup_{F_i} Trace\{B_{2i}^T\tilde{P}_iB_{2i}\}$$

where $\tilde{P}_i=\tilde{P}_i^T\ge 0$ is determined by the Lyapunov equation

$$\tilde{A}_i^T\tilde{P}_i+\tilde{P}_i\tilde{A}_i+\sum_{j=1}^{s}\pi_{ij}P_j+\tilde{C}_{0i}^T\tilde{C}_{0i}^T=0 \tag{4}$$

The control law (2) is called robust H_2/H_∞ control law of system (1) if control law (2) satisfies the above design indices.

$\bar{J}(t)$ generally depends on the chosen control law (2), called robust H_2/H_∞ optimal guaranteed cost controller of system (1) if (2) can minimize $\bar{J}(t)$. Before proceeding further, we present the following lemmas which will be used in our main results.

Lemma 1. Let $D\in R^{n\times n_f}$, $E\in R^{n_f\times n}$ and $F\in R^{n_f\times n_f}$. Then, when F satisfying $\|F\|\le I$, for any scalar $\varepsilon>0$, there holds

$$DFE+E^TF^TD^T\le\varepsilon^{-1}DD^T+\varepsilon E^TE$$

Lemma 2. Given a scalar $\gamma>0$, the uncertain closed-loop system (3) is asymptotically stable and has H_∞ performance $\|T_{wz_1}\|_\infty<\gamma$, if there exist matrices $P_i>0$, $i\in S$, such that the following linear matrix inequality holds for $i,j\in S$

$$\begin{bmatrix}\tilde{A}_i^TP_i+P_i\tilde{A}_i+\sum_{j=1}^{s}\pi_{ij}P_j & P_iB_{2i} & \tilde{C}_{1i}^T\\ * & -\gamma^2I & 0\\ * & * & -I\end{bmatrix}<0 \tag{5}$$

Remark: The lemma 1 is used to deal with the uncertainty parameters included in system matrices. And the lemma 2 provides a criterion for testing the robust H_∞ performance level of the closed-loop jump system (3) in terms of coupled linear matrix inequalities.

III. Main Results

Base on the above two lemmas, a solution is given to the mixed H_2/H_∞ control problem by using linear matrix inequality approach. To this end, we first present the following result for the close-loop system (3) to be asymptotically stable with given H_2/H_∞ performance constraints which will play a key role in solving the aforementioned problem.

Theorem 1: For a given scalar $\gamma>0$, the closed-loop system (3) is asymptotically stable with $\|T_{wz_1}\|_\infty<\gamma$, if and only if there exist a constant $\alpha>0$, $\forall i\in S$, such that

$$\Upsilon_i+P_iB_{2i}(\gamma^{-2}I)B_{2i}^TP_i+\tilde{C}_{0i}^T\tilde{C}_{0i}+\alpha^{-1}\tilde{C}_{1i}^T\tilde{C}_{1i}<0 \tag{6}$$

where

$$\Upsilon_i=\tilde{A}_i^TP_i+P_i\tilde{A}_i+\sum_{j=1}^{s}\pi_{ij}p_j$$

has a positive-definite solution matrix P_i, and the solution matrix P_i satisfy

$$0\le\tilde{P}_i\le P_i$$

where $\tilde{P}_i=\tilde{P}_i^T\ge 0$ is the solution matrix of the Lyapunov equation (4).

Theorem 2: The matrix inequality (6) is establish for all admissible uncertainties $\|F_i(t)\|\le 1$. If there exist constant

$\alpha > 0$, $\beta > 0$ and matrix $X_i > 0$, satisfying the following matrix inequalities:

$$\begin{bmatrix} \Omega_i & X_i\bar{E}_i & \beta H_i & B_{2i} & X_i\tilde{C}_{0i}^T & X_i\tilde{C}_{1i}^T & X_iQ_i \\ * & -\beta I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\beta I & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\alpha I & 0 \\ * & * & * & * & * & * & -\Xi_i \end{bmatrix} < 0 \tag{7}$$

where

$\Omega_i = X_i\bar{A}_i^T + \bar{A}_iX_i + X_i\pi_{ii}$

$Q_i = \left[\sqrt{\pi_{i1}} \quad \sqrt{\pi_{i2}} \quad ...... \quad \sqrt{\pi_{i,i-1}} \quad \sqrt{\pi_{i,i+1}} \quad ...... \quad \sqrt{\pi_{is}}\right]$;

$\Xi_i = diag\{X_1 \quad X_2 \quad ...... \quad X_{i-1} \quad X_{i+1} \quad ...... \quad X_s\}$.

Theorem 3: Consider the uncertain jump linear system (1), suppose $\gamma > 0$ is given constant. The closed-loop system (3) is asymptotically stable with $\|T_{wz_1}\|_\infty < \gamma^2$, if there exist scalars $\alpha > 0$, $\beta > 0$ and matrices $X_i > 0$, W_i, such that the following linear matrix inequalities hold for

$$\begin{bmatrix} \Sigma_{1i} & \Sigma_{2i} & \beta H_i & B_{2i} & \Sigma_{3i} & \Sigma_{4i} & X_iQ_i \\ * & -\beta I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\beta I & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -\alpha I & 0 \\ * & * & * & * & * & * & -\Xi_i \end{bmatrix} < 0 \tag{8}$$

where

$\Sigma_{1i} = (A_iX_i + B_iW_i)^T + (A_iX_i + B_iW_i) + X_i\pi_{ii}$,

$\Sigma_{2i} = (E_{1i}X_i + E_{2i}W_i)^T$, $\Sigma_{3i} = (C_{0i}X_i + D_{0i}W_i)^T$,

$\Sigma_{4i} = (C_{1i}X_i + D_{1i}W_i)^T$

Furthermore, if the LMI (8) have feasible solutions α, β, X_i, W_i, then the state feedback control law

$$u(t) = W_iX_i^{-1}x(t)$$

is the control law of the system(1), and an upper H_2 guaranteed cost boundary of the closed-loop system is

$$\bar{J}(t) = Trace\{B_{2i}^TX_i^{-1}B_{2i}\}.$$

Corollary1. For a prescribed constant $\gamma > 0$ and uncertain jump system (1), if the following optimization problems

$$\min_{\alpha,\beta,X_i,W_i,N} = Trace(N) \tag{9}$$

s.t. (i) (8)

(ii) $\begin{bmatrix} -N & B_{2i}^T \\ * & -X_i \end{bmatrix} < 0$

has feasible solutions α, β, X_i, W_i, N, then the state feedback control law

$$u(t) = W_iX_i^{-1}x(t)$$

is the optimal robust H_2/H_∞ guaranteed cost control law of the system (1).

Question (9) is an optimization problem with linear matrix inequality constraints and linear objective function. Therefore we can use LMI solver to solve this problem.

IV. NUMERICAL EXAMPLE

In this section, a numerical example is present to illustrate the effectiveness of the proposed method. It is assumed that the system has two operation modes.

For mode 1, the system matrices are given by:

$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $B_{21} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $H_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $C_{01} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T$,

$C_{11} = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix}^T$, $D_{01} = 0.1$, $D_{02} = 0.5$, $E_{11} = [1 \quad 1]$, $E_{12} = [1 \quad 1]$

For mode 2, the system matrices are given by:

$A_2 = \begin{bmatrix} -1 & 0 \\ -0.8 & -1 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $B_{22} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $H_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $C_{02} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T$,

$C_{12} = \begin{bmatrix} 0.3 \\ 1 \end{bmatrix}^T$, $D_{11} = 0.1$, $D_{12} = 0.5$, $E_{21} = 0.2$, $E_{22} = 0.2$

Assume that the transition probability matrix is given by:

$$\Pi = \begin{bmatrix} -0.4 & 0.4 \\ 0.6 & -0.6 \end{bmatrix}$$

With the above data, define constant scalar $\gamma = 2$, By solving the convex optimization problem, we can obtain the following parameter solutions:

$X_1 = \begin{bmatrix} 2.4963 & -0.0893 \\ -0.0893 & 3.2317 \end{bmatrix}$, $X_2 = \begin{bmatrix} 3.3223 & -0.4473 \\ -0.4473 & 2.3828 \end{bmatrix}$,

$W_1 = [-3.1986 \quad -7.8166]$, $W_2 = [-2.0067 \quad -4.9566]$.

Corresponding to the above two operation modes $i \in \{1,2\}$, apply theorem 3, we can obtain the optimal H_2/H_∞ state feedback controller parameter matrices as

$$K_1 = [-1.3692 \quad -2.4566],\ K_2 = [-0.9070 \quad -2.2504]$$

For all admissible uncertainties, the H_2 performance of the closed-system about the controlled output $z_0(t)$ satisfies

$$J(t) \le 6.9210$$

Assume the uncertain matrices F_1(t)=0.5sin(t), F_2(t)=0.2cos(t), and the disturbance input $w(t)$ is expressed as $w(t) = \frac{5 * rand(1)}{1+t}$. Define the initial conditions are $x(0) = [1 \quad 1]^T$, $u(0) = 1$, and $i = 1$. Simulation results are shown in the following figures:

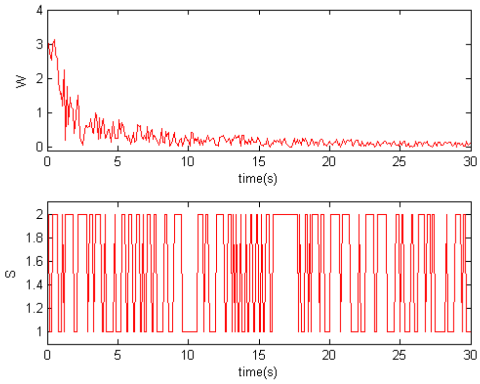

Figure1: Trajectories of $r(t)$ and $w(t)$

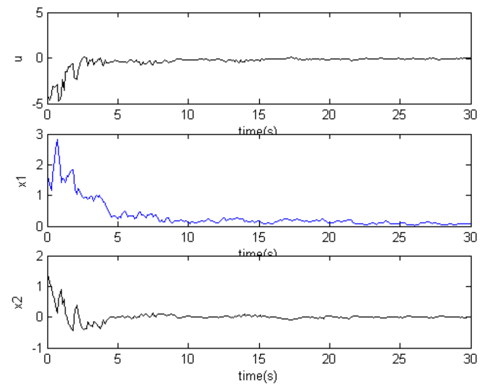

Figure2: Trajectories of $x_1(t)$, $x_2(t)$ and $u(t)$

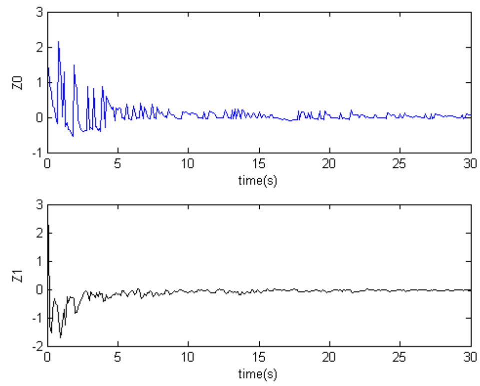

Figure3: Trajectories of $z_0(t)$, $z_1(t)$

Analyzing from Figure 1, we can conclude that the conversion between the two modes is randomly and the disturbance input $w(t)$ is energy-bounded. From Figure 2, it can be seen that the closed-loop system is asymptotic stable, and the controller has strong robustness for non-determinacy factors. Judging from the simulated curve of the Figure 3, the H_2 performance of the control output $z_0(t)$ is no more than the sub-optimum value $J(t) \leq 6.9210$ while the maximum singular value from the exogenous input $w(t)$ to the output $z_1(t)$ is less than the given constant $\gamma = 2$. More simply, the simulation results imply that the desired goal is well achieved.

V. CONCLUSION

This paper has proposed a robust H_2/H_∞ Control method for a class of Markovian jump linear systems with uncertain jumping probabilities. A sufficient condition for the existence of a robust state feedback control law is given in terms of a group of LMIs. The designed state feedback controller minimizes the H_2 norm of the closed-loop transfer matrix while satisfying the given H_∞ performance constraint simultaneously over all admissible uncertainties. An algorithm involving convex optimization was also suggested to construct such controllers effectively. The simulation results show that the proposed technique can make the system stable rapidly, and satisfy the expected performance index.

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