European Call Options on the Extremum with Risky Asset*

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Abstract – European call options are the most important with risky asset. European call and put options on the extremum of m risky assets based on the Black-Scholes assumptions of the capital markets. Note that there is a flip of sign for the correlation coefficient, attributed to the change of limits of integration when the sign of one of the integration variables is reversed.

Our research substantially reduced the corresponding extremum hedging strategies. The European put and call options on the minimum of several risky assets find their applications for a wide variety of contingent claims in corporate finance and bond pricing models. The price formulas are essentially obtained by the valuation of the discounted expectation integral defined in the paper.

Index Terms – Call option, Extremum, risky assets, pricing.

1. Introduction

The Chicago board of option exchange become the first organized exchange for trading standardized options contracts[1] in the United States. There are many types of options, and at first the terminology is confusing, with calls, puts, strap, strips, out –of-the-money options, and so on. In the paper we introduce European call options on the extremum of risky assets.

We would like to improve the original Black-Scholes formulation by relaxing some of the assumption in the model with credit risk. we consider the valuation of the chooser option ,which has the feature that the holder can choose whether the option is a call or a put after a specified period of time from the starting date of the option contract.

The European put and call options on the minimum of several risky assets find their applications for a wide variety of contingent claims in corporate finance and bond pricing models. One example is the option-bond in the Euro-bond market where payment at maturity can be made in a particular currency chosen by the holder from a list of currencies[2] (Stulz,1982). Boyle (1989) argued that the flexibilities available to the writer of a futures contract with regard to when, where, how much and what to be delivered can be modelled [3]by options on the extremum of several assets. The embedded options associated with these flexibilities are called the timing option, location option, quantity option, respectively. Also, the price of an exchange option on two assets has a simple relation with the price of the call option on the minimum of the same two assets with zero strike price. Examples of financial instruments that can be modeled by options on the extremum of several assets are given.

Risk aversion is the most important function of future market, and is also the basic reason of the developing of future market. As one of the most important species of financial futures, stock index future [4] is important to avoid the systemic risk of stock markets.

We would like to derive the pricing formulas for the European call and put options on the extremum of m risky assets based on the Black-Scholes assumptions of the capital markets. The price formulas are essentially obtained by the valuation of the discounted expectation integral defined in the paper.

2. Call Option-Pricing Model on the Extremum

Let $c_{\max}^{m}(S,T)$ and $c_{\min}^{m}(S,T)$ denote the prices of the European call options on the maximum and minimum of m risky assets, respectively, where $S=(S_1,S_2,\cdots,S_m)^T$ and T is the time to expiry.

The terminal payoffs for the above European calls are respectively

$$C_{\max}^{m}(S,0)=\max(\max(S_1,S_2,\cdots,S_m)-X,0)\,, \qquad (1)$$

$$C_{\min}^{m}(S,0)=\max(\min(S_1,S_2,\cdots,S_m)-X,0)\,. \qquad (2)$$

Where X is the strike price. For example, the terminal payoff function $C_{\max}^{2}(S_1,S_2,0)$ can be expressed explicitly as

$$C_{\max}^{2}(S_1,S_2,0)=\begin{cases} S_1-X & S_1\ge S_2 \quad and\, S_1>X \\ S_2-X & S_2\ge S_1 \quad and\, S_2>X \\ 0 & S_1\le X \quad and\, S_2\le X \end{cases}. \qquad (3)$$

The price functions, $c_{\max}^{m}(S,T)$ and $c_{\min}^{m}(S,T)$, satisfy the m-dimensional Black-scholes equation. By following the usual risk neutralized discounted expectation approach, the price of the European can on the maximum of two risky assets is given by

$$C_{\max}^{2}(z_1,z_2,t)=e^{-rT}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}C_{\max}^{2}(zt_1,zt_2,0)\phi(zt_1,zt_2,t)dzt_1dzt_2 \qquad (4)$$

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Where

$$z_i = \frac{1}{\sigma^i}[\ln S_i + (r - \frac{\sigma_i^2}{2} t], zt_i = \frac{1}{\sigma_i}\ln S_{t_i}, i = 1, 2, \quad (5)$$

and

$$\phi(zt_1, zt_2, t) = \frac{1}{2\pi t}\frac{1}{\sqrt{1-\rho_{12}^2}}$$

$$\exp(-\frac{(zt_1 - z_1)^2 - 2\rho_{12}(zt_1 - z_1)((zt_2 - z_2) + (zt_2 - z_2)^2}{2t(1-\rho_{12}^2)}) . \quad (6)$$

The functional representation[3] of the terminal payoff differs in different parts in the $(S_1, S_2)-$ plane, which are separated by the lines : $S_1 = X, S_2 = X and S_1 = S_2$ .These lines are either parallel to the coordinates axes or passing through the origin , and so one is confident to express the above price formula in terms of the bivariate normal distribution function.

Indeed, Stulz[2](1987) managed to obtain the analytic formula for $C_{\max}^2(S_1, S_2, t)$ by brute force integration of the above expectation integral . Later, Johnson [2] (1987) propose a more elegant procedure which avoids the tedious integration procedure. Also, Johnson's method can be easily generalized to deal with options on the extremum of several assets.

3. The Option Pricing with Risk Assets

The formula of $C_{\max}^2(S_1, S_2, t)$ contains the term which represents the risk neutral discounted expectation of the cash paid out conditional on the risk neutral discounted Expectation of the cash paid out conditional on the exercising of the call. For the present model, this cash payment term is

$$-Xe^{-rt}[1 - P_r(S_{t_1}, S_{t_2} < X],$$

Where $1 - P_r(S_{t_1}, S_{t_2} < X$) gives the probability that the call will be exercise in the risk neutral world. The analogy in the one-asset call option model is the term $-Xe^{-rt}N(d_2)$ in the Black-Scholes call price formula.

If we define $\gamma_i = \frac{1}{\sigma_i}\ln X, i = 1, 2, then$

$$P_r(z_{t_1}, z_{t_2} < X)$$

$$= N_2(-d_2(S_1, X, \sigma_1, \gamma); -d_2(S_2, X, \sigma_2, \gamma); \rho_{12}) . \quad (7)$$

Further, from the usual put-call parity relation: $c(S, t; X) + Xe^{-rt} = p(S, t; X) + S$, one may divide throughout by S to give

$$\frac{c(S, t; X)}{S} + \frac{Xe^{-rt}}{S} = \frac{p(S, t; X)}{S} + 1. \quad (8)$$

4. Option Pricing Model Analysis

One can deduce a similar result for the risk neutral expectation of S_{t_2} conditional on $S_{t_2} \ge S_{t_1} and\ S_{t_2} \ge X$, obtained by simply exchanging the roles of $S_1\ and\ S_2$. Collecting the three terms together, the price of a European call on the maximum of two risky assets takes the final form

$$C_{\max}^2(S_1, S_2, t) = S_1 N_2(\frac{\ln\frac{S_1}{X} + (r + \frac{\sigma_1^2}{2})r}{\sigma_1\sqrt{t}}, \frac{\ln\frac{S_1}{S_2} + \frac{\sigma_{12}^2}{2}r}{\sigma_{12}\sqrt{t}}; \frac{\sigma_1 - \rho_{12}\sigma_2}{\sigma_{12}})$$

$$+ S_2 N_2(\frac{\ln\frac{S_2}{X} + (r + \frac{\sigma_2^2}{2})r}{\sigma_1\sqrt{t}}, \frac{\ln\frac{S_2}{S_1} + \frac{\sigma_{12}^2}{2}r}{\sigma_{12}\sqrt{t}}; \frac{\sigma_2 - \rho_{12}\sigma_1}{\sigma_{12}})$$

$$-Xe^{-rt}[1 - N_2(-\frac{\ln\frac{S_1}{X} + (r - \frac{\sigma_1^2}{2})r}{\sigma_1\sqrt{t}}, \frac{\ln\frac{S_2}{X} + (r - \frac{\sigma_2^2}{2})t}{\sigma_2\sqrt{t}}; \rho_{12})] .$$

How to derive the corresponding price formula of $C_{\min}^2(S_1, S_2, r; X)$? We take advantage of the following parity relation between $C_{\max}^2(S_1, S_2, r; X)$ and $C_{\min}^2(S_1, S_2, r; X)$:

$$C_{\max}^2(S_1, S_2, r; X) + C_{\min}^2(S_1, S_2, r; X)$$

$$= C(S_1, r; X) + C(S_2, r; X) .$$

Which can be verified easily by comparing the terminal payoffs of the two sides of the above equation. Using the following identity. $C_{\min}^2(S_1, S_2, r; X)$ is obtained as follows：

$$C_{\min}^2(S_1, S_2, t) = S_1 N_2(\frac{\ln\frac{S_1}{X} + (r + \frac{\sigma_1^2}{2})r}{\sigma_1\sqrt{t}}, \frac{\ln\frac{S_1}{S_2} - \frac{\sigma_{12}^2}{2}r}{\sigma_{12}\sqrt{t}}; \frac{\rho_{12}\sigma_2 - \sigma_1}{\sigma_{12}})$$

$$+ S_2 N_2(\frac{\ln\frac{S_2}{X} + (r + \frac{\sigma_2^2}{2})r}{\sigma_2\sqrt{t}}, \frac{\ln\frac{S_1}{S_2} - \frac{\sigma_{12}^2}{2}r}{\sigma_{12}\sqrt{t}}; \frac{\rho_{12}\sigma - \sigma_2}{\sigma_{12}})$$

$$-Xe^{-rt}N_2(\frac{\ln\frac{S_1}{X} + (r - \frac{\sigma_1^2}{2})r}{\sigma_1\sqrt{t}}, \frac{\ln\frac{S_2}{X} + (r - \frac{\sigma_2^2}{2})t}{\sigma_2\sqrt{t}}; \rho_{12}) .$$

5. Conclusions

One may give the following probability interpretation of the above call price formula [5]. The last term can be identified as $-Xe^{-rt}P_r(S_{t_1}, S_{t_2} > X)$. The first ter, can be interpreted as the risk neutral expectation of S_{t_1} conditional on $S_{t_1} > X$ and , $S_{t_1} \le S_{t_2}$, that is , $\tilde{S}_{t_1} < 1 and\ \tilde{S}_{t_2} > 1$;
and the value of which is given by

$$S_1 N_2(-d_2(\frac{Xe^{-rt}}{S_1}, 1, \sigma_{11}, 0), d_2(\frac{S_2}{S_1}, 1, \sigma_{12}, 0); -\rho_{12}) .$$

The credit risk will be introduced to capped option pricing though the value of corporation value model[6]. Considering the market price of default risk of the option pricing Which is an important field of financial research. The relevance of the credit risk is a key element of options.

Application of martingale pricing and probability methods, which derives a number of special cases of the pricing formula of capped option subject to credit risk.

Note that there is a flip of sign for the correlation coefficient, attributed to the change of limits of integration when the sign of one of the integration variables is reversed. Similarly , the second term[7] can be seen to be equal to $S_2 N_2(d_2(\frac{S_1}{S_2},1,\sigma_{12},0),-d_2(\frac{Xe^{-rt}}{S_2},1,\sigma_{22},0);-\rho_{12})$.

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