Ant Colony Algorithm Based on Information Entropy Theory to Fuzzy Vehicle Routing Problem

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Abstract

To adapt the changing of market need, logistics providers make efforts to reduce costs and improve customer service levels to meet the customer satisfaction as individual demands. In the process of actual distribution, delivery vehicles will encounter various often uncertain external factors, resulting in delivery times of uncertainty. And it indirectly affects the normal production and operation. In this paper, the travel time based on fuzzy mathematical model of vehicle routing problem take the time window as a fuzzy variable. Information entropy and the path chosen by the use of random disturbance control strategy to the adaptive algorithm. Finally, a numerical example is given to show the effectiveness of the algorithm.

Keywords: Vehicle routing problem, Fuzzy, Ant colony algorithm, Information entropy

1. Introduction

Vehicle Routing Problem (VRP) derived from the transport, Dantzig in 1959, a typical NP-hard combinatorial optimization problem. Atlanta refineries to the gas station for the study of the transport of petrol delivery route optimization, and quickly became the forefront field of operations research and combinatorial optimization study and hot, attracting a large number of academics to carry out research. Usually graph G = (V, E) is used to describe the problem, $G = (V, E)$, V={0,1,2,…,). E={ (i, j), $i \neq j$, $i, j \in V$), node 1 denotes the depot and other nodes denote customers. Each customer's demand is q_i, while edge (i, j) corresponding to the time or distance or transportation costs as C_{ij}. Q denotes the capacity of vehicle, all vehicles from the warehouse, complete task and return to warehouse. Each customer can only be visited one time and the objective function is usually to minimize the number of vehicles or transport costs.

Ant colony algorithm is a new heuristic algorithm optimization method, suitable for vehicle routing and other combinatorial optimization problems. ACO was proposed initially for solving the traveling salesman problem by the Italian scholars Dorigo [1]. Along with the deepening of research, the ACO has been introduced into electronics, telecommunications, and shop scheduling and other engineering fields. In practical applications, due to the uncertainty of various external factors, which lead to vehicle served time uncertain, affecting the normal operation at the same time, will lead to the decline in customer satisfaction with the services. Thus, the certainty VRP of traditional theories and methods are no longer suitable for dealing with uncertain problems. There are a lot fuzzy theory research has been used to VRP [2]-[9]. Some domestic scholars have raised the Fuzzy VRP mathematical models and algorithms [10]-[12]. Because the outstanding performance of the entropy theory dealing with fuzzy issues, ant algorithm based on information entropy theory is used to the vehicle routing problem with fuzzy travel time (FTTVRP) [13].

2. Related to the Fuzzy Theory and FTTVRP Models

2.1. Fuzzy Theory

Fuzzy solutions and the process is time variable, which can be described as follows: if a complete set U, $u^* \in U$, one of the elements. If after a given process, u becomes a set of fuzzy set A, which takes U as a universe set. This particular process is known as fuzzy. Variable time forms triangular fuzzy numbers, as shown in Figure 1. Fuzzy conversion formula is as following:

$$A(u)=\begin{cases}\dfrac{u-a}{m-a}, & a\le u\le m\\ \dfrac{b-u}{b-m}, & m<u\le b\\ 0, & others\end{cases} \quad (1)$$

Defuzzify process is on the contrary to fuzzy, it is a clear value W^* fuzzy sets in the process. In this paper, we take the center dispel defuzzify. Its formula as following:

$$W^*=\frac{\sum_{j=1}^{m}\int_w wB_j(w)d_w}{\sum_{j=1}^{m}\int_w B_j(w)d_w} \quad (2)$$

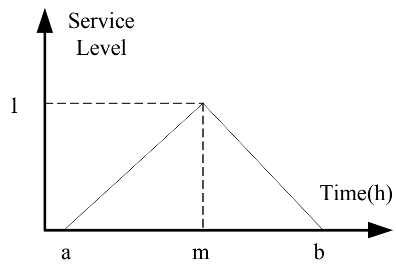

Fig. 1: Triangle fuzzy number

Time is not considered at first, after TSP calculated for the shortest path, Fuzzy time basis to determine the level of service and customer service order. We give priority to the needs of nodes in a high level of service requirements.

2.2. The Mathematical Model of FTTVRP

Zhang proposed the fuzzy vehicle routing problem based on vehicles travel time (FVRP) [11]. FVRP is described as follows: in a transport network, there is a depot and service nodes m for service, denotes with 0, l, 2,…, m. Vehicles start from the depot, returned after serviced a certain number of nodes. It is known that C denotes the vehicle transport capacity and each node has a demand d_i. Between the nodes i and j expected travel time is a fuzzy number. Minimum transportation costs to traffic routes to meet the requirements. This paper considers the shortest period of time as the objective, taking into the satisfaction of serviced time consideration.

For the sake of convenience, between any two nodes i, j, we assume that the travel time $\tilde{t}_{ij}$ is a triangular fuzzy numbers $\tilde{t}_{ij}=\left(t_{ij}^1,t_{ij}^2,t_{ij}^3\right)$. t_{ij}^1 denotes the left side of fuzzy figure, t_{ij}^2 denotes the degree of membership is 1 correspond to, t_{ij}^3 denotes the right side of fuzzy figure. In order to construct the mathematical model, we define the variables x_{ijk} and y_{ik}. If the vehicles k travel from customer i to customer j, then the value of x_{ijk} equals to 1, else equals to 0; if the task of customer i is completed by the vehicle k, then the value of y_{ik} equals to 1, else equals to 0. We define the mathematical model of the vehicle routing problem with travel time as following:

$$\min \tilde{z}=\sum_i\sum_j\sum_k \tilde{t}_{ij}x_{ijk} \quad (3)$$

$$\sum_i d_i y_{ik}\le C, \quad \forall k \quad (4)$$

$$\sum_k y_{ik}=1, \quad i=1,2,\cdots,m \quad (5)$$

$$\sum_i x_{ijk}=y_{jk}, \quad j=0,1,\cdots,m; \ \forall k \quad (6)$$

$$\sum_j x_{ijk}=y_{ik}, \ i=0,1,\cdots,m; \ \forall k \quad (7)$$

$$X=\left(x_{ijk}\right)\in S \quad (8)$$

$$S=\left\{\left(x_{ijk}\right)\middle|\sum_{i\in R}\sum_{j\notin R}x_{ijk}\le|R|-1, R\in\{1,2,\cdots,m\}\right\}; \ \forall k$$

$$\Pr\{f_i(x,y,t)\in[a_i,b_i]\}\ge\beta_{\mathrm{i}}\ , i=1,2,...,n \quad (9)$$

Formula (3) is the objective function, the travel time $\tilde{t}_{ij}$ is a fuzzy figure, which denotes fuzzy travel time from i to j. Formula (4) is capacity constrain, which denotes that the task total capacity can not exceed the vehicle's capacity. Formula (5) denotes that each customer could be visited only one time. Formula (6) and (7) denote the relationship of variable y_{ik}, y_{jk} and x_{ijk}. Formula (8) is branch elimination constrain. Formula (9) denotes the probability of complete task in assignment time windows, which is used to measure the degree of customer service time satisfaction.

3. Ant Colony Algorithm Based on Information Entropy

3.1. Theory Conception of Information Entropy Theory and characteristic

The concept of Entropy came from physics, proposed by Clausius a German physicist in 1854, to describe the disorder of thermodynamic system. It was introduced to a number of other disciplines, Boltzmann entropy, information entropy, probabilistic entropy, and so on. The U.S. scholars Shannon introduced the thermodynamic entropy to information theory as an important concept of uncertainty methods, and it often was used to give a rough measure of uncertainty. In order to discrete random variables, its information entropy. $S=-k\sum_{i=1}^{n} p_i \ln p_i$. p_i denotes the probability of state. $p_i \geq 0$, $\sum_{i=1}^{n} p_i = 1$. Information entropy has the following properties: symmetry, Non-negative, Additive and minimums.

3.2. Ant Colony Algorithm Based on Information Entropy

In basic ACO, We assume that m is the ant number and d_{ij} is the distance between customer i and j. The visibility of Edge (i, j) $\eta_{ij}=1/d_{ij}$, which reflects the customer's inspiration level from customer i to j. τ_{ij} is the strength of information-track on Edge (i, j). $\Delta\tau_{ijk}$ is phenomenon quantity unit length left by ant k on arc (i, j). p_{ijk} is the state transition probability of ant k from customer i to j.

$$p_{ij}^{k}=\begin{cases}\dfrac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}{\sum_{s\in allowed_k}\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}, & j\in allowed_k \\ 0 & , otherwise\end{cases} \quad (10)$$

$allowed_k=\{0,1,\cdots, n-1\}$ denote the next customer which ant k will choice. For the two parameters α and β, they reflect the accumulated information in the course of moving and the relative importance of heuristic information when ants choice path. Each ant has a taboo table designed to record the depots ant k passed, and no ant is allowed to repeat in the current cycle.

In basic ACO the path information amount is uncertainty, so does the path ant chooses. We introduce information entropy to the algorithm, by controlling information entropy value the path chosen and the random variation in local perturbations. When entropy reaches a particular request, stop searching. Here is the definition of information entropy, $S(t)=-k\sum_{i=1}^{n} p_i(t)\ln p_i(t)$. Initially, the same information entropy in all of paths, as some selected path information increase, the entropy decrease gradually. Eventually may lead to stagnation, where local optimal solution is obtained. So $\alpha'(t)=\frac{S_{\max}-S(t)}{S_{\max}}$ and $\beta'(t)=1-\frac{S_{\max}-S(t)}{2S_{\max}}$ is introduced, $\beta'(t)$ is the probability the optimal path to be maintained. $\alpha'(t)$ is the proportion of total ant colony allowed to choose the appropriate path in small areas. Algorithms process is indicated in Figure 2. Here we define a larger entropy value stands criteria bigger than 0 and conditions as a termination.

3.3. Pheromone Update Strategy and Improved

Pheromone updated strategy is the key step in ant algorithm, rapid updating leads to stagnation or even fall into local optimal result, while too slow will not search the optimal result. When ants find a feasible solution, pheromone of all sections (i, j) should be global updated.

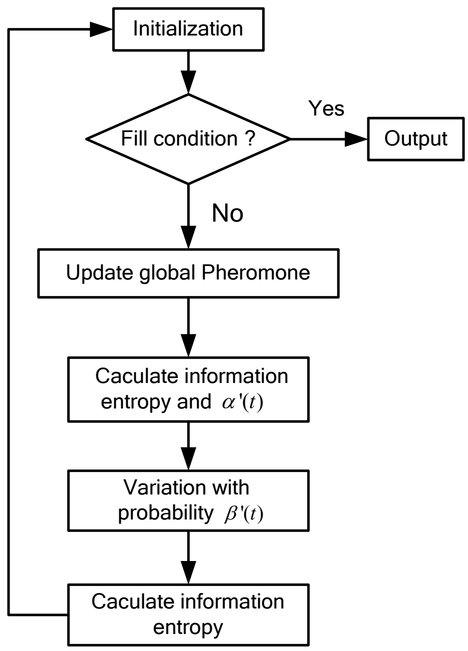

Fig. 1: Algorithm flow chart.

$$\tau_{ij}(t)=(1-\rho)\tau_{ij}(t-1)+\rho\tau_{ij}(\mathrm{t}) \ , \rho\in(0,1) \quad (11)$$

L_{gb} is the global optimum length of the line at present. Q is constant. ρ is pheromone volatile factor. τ_0 is constant, $\tau_0=1/(nl_{min})$. l_{min} is the length of the shortest line in current cycle. We set a lower limit of pheromone, reserve the value lower to improve the stability of algorithm.

4. Numerical Experiments

Here we will give an example to show models that we have just discussed and how the ant colony algorithm based on information entropy works. Let us consider a fuzzy travel time vehicle routing problem shown in Fig. 1. We assume that there are 7 customers labeled "1, 2,…, 7" and one depot labeled "0". We also assume that the travel times between customers are all triangular fuzzy variables, the time windows and demands of customers are given in Table 1 and Table 2. We assume that the unloading times are all 0 minutes and the capacities of the four vehicles are all 10. We also assume that the service level is 0.85.

The program was coded in C++ language and simulations were performed on a personal computer with a 2700 MHz Pentium 4 processor and 256MB of RAM, the runtime are about 12 s. The total minimum

customer	1	2	3	4	5	6	7	8
demand	5	5	3	2	2	8	1	8

Table 1: Customer demand table

	0	1	2	3	4	5	6	7	8
0	(0,0,0)								
1	(5,6,8)	(0,0,0)							
2	(2,2,7)	(3,4,9)	(0,0,0)						
3	(1,4,9)	(4,7,7)	(5,7,9)	(0,0,0)					
4	(1,3,5)	(1,6,9)	(2,9,9)	(2,5,9)	(0,0,0)				
5	(3,9,9)	(2,2,7)	(2,6,9)	(3,5,9)	(1,5,7)	(0,0,0)			
6	(4,4,6)	(2,2,7)	(3,8,9)	(7,7,8)	(5,5,6)	(3,5,6)	(0,0,0)		
7	(1,2,8)	(4,8,9)	(6,7,7)	(3,4,5)	(2,6,6)	(6,8,8)	(1,6,8)	(0,0,0)	
8	(5,5,5)	(1,2,8)	(1,3,4)	(2,5,6)	(2,7,8)	(8,8,9)	(1,1,7)	(1,1,8)	(0,0,0)

Table 2: Travel time matrix

ant	α	β	τ	η	ρ
10	1	4	1	3	0.1

Table 3: Basic parameter setting of ACO

time traveled by the four vehicles is 41. Furthermore, when the operational plan is performed, the run of the ant colony algorithm shows that the best operational plan is

Vehicle 1: 0—1—5—3—0
Vehicle 2: 0—7—8—0
Vehicle 3: 0—4—6—0
Vehicle 4: 0—2—0

5. Conclusions

This paper contributed to a fuzzy travel time vehicle routing problem with ant colony algorithm based on information entropy in the following respects: (a) a FTTVRP model was proposed for finding the optimal solutions of fuzzy travel time vehicle routing problems; (b) a ant colony algorithm based on information entropy to solve the fuzzy travel time vehicle routing problem was presented, focusing on total travel time minimization; (c) the effectiveness of the ant colony algorithm based on information entropy was shown by some numerical examples.

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