The Method of Quadratic Combination Weighting Fuzzy Comprehensive Evaluation applied in Water Quality Assessment

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Abstract—For the shortcoming of Fuzzy Comprehensive Evaluation Method such as subjectivity and lacking of information more, in order to improve the index weight's scientific and accuracy. In this paper, based on the application of AHP and entropy weight method, fuzzy comprehensive evaluation obtained quadratic combination weight of each index method, four dry season water quality of the study area monitoring sections were evaluated. The results showed that this method avoided the subjective differences of expert score method, meeting target weight and the effective degree of credibility. It is suggested that the method of quadratic combination weighting fuzzy comprehensive evaluation is scientific, accurate, objective, it could be widely applied in water quality assessment.

Keywords-groundwater; water quality assessment; fuzzy comprehensive evaluation; quadratic combination weighting

I. Introduction

Groundwater evaluation method [1] variety, mainly fuzzy comprehensive evaluation method, gray relational analysis, artificial neural networks, fuzzy comprehensive evaluation method for its systematic, clear, you can solve the fuzzy and difficult to quantify the characteristics of the problem has been widely applied.

II. The Second Combination Weighting Fuzzy Comprehensive Evaluationmodel

A. AHP Seeking Quality Index Weights Each

AHP [2] is to be evaluated in accordance with the logic of the system to evaluate the indicators form an ordered hierarchy. Then on the same floor or in the same domain indicators pairwise comparison, according to the provisions of scale value construct comparative judgment matrix. Then calculate the maximum comparative judgment matrix eigenvalues and corresponding eigenvectors, and the feature vector normalization, and finally get the weight of each indicator.

B. Entropy Method for Each Water Quality Index Weights

Entropy value reflects the utility value of the index information, with the entropy method [3] calculated index weights. In the judgment of the evaluation matrix based on normalized treatment to obtain the normalized judgment matrix, determine the evaluation index entropy. The final calculation of the weight of each index value. Greater reliability of the method, adaptive function, suitable for samples with data.

C. Second of Each Indicator to Obtain the Optimal Portfolio Weights

Empowerment is a combination of two or combination of two kinds of methods for empowering the results obtained, the combination of empowerment can effectively improve the accuracy of the weight assignment. In this paper, A secondary combination weighting [4] approach: in AHP and entropy weight method, based on the weight of processing, depending on the target weight and the weight of the principle of minimum deviation, construct nonlinear optimization model and solution of quadratic optimal portfolio weights obtained W *.The fuzzy comprehensive evaluation with fuzzy relation matrix for computing, and finally get to see the whole thing been assessed on the quality levels of membership.

D. Quadratic Combination Weighting Fuzzy Comprehensive Evaluation of calculation steps

Step 1: Determine the factors evaluated, that evaluation of m $U = \{u\_1, u\_2, u\_3 ... u\_m\}$;

Step2: Determine the evaluation level, $V = \{v\_1, v\_2, v\_3 ... v\_n\}$;

Step3: Perform single factor evaluation, fuzzy relationship matrix R;

In the constructor of the class fuzzy subset, the assessment should be things one by one from each factors to quantify, determined from the single factor was rated things look fuzzy subset for each class of membership (R/ui), then get fuzzy relationship matrix.

Step 4: by AHP and entropy method to get the weight according to the following formula for computing;

$$\begin{cases} \text{wi} = \dfrac{w1i*w2i}{\sum_{i=1}^{n} w1i*w2i} \\ \text{wi} = \dfrac{1}{2}(w1i + w2i) \end{cases} \quad (1)$$

Get a second combination weight vector W1= (w1$^{(1)}$, w2$^{(1)}$, w3$^{(1)}$…wn$^{(1)}$)and W2=(w1$^{(2)}$,w2$^{(2)}$, w3$^{(2)}$…wn$^{(2)}$)

Step 5: Construct the optimal combination of weight vector;

$$W^* = (w_1 {}^* w_2 {}^* w_3 {}^* \ldots\ldots w_n {}^*)$$

$$\sum_{i=1}^{n} wi=1(i=1,2,3……n)$$

$$W^*-W^1=(w_1^*-w1^{(1)},w_2^*-w2^{(1)},\\ w_3^*-w3^{(1)}…w_n^*-wn^{(1)})$$

$$W^*-W^2=(w_1^*-w1^{(2)},w_2^*-w1^{(2)},\\ w_3^*-w1^{(2)}…w_n^*-wn^{(2)})$$

Step 6: According to goal weight and the weight of the principle of minimum deviation, nonlinear optimization model is constructed as follows(2)

$$\begin{cases} \min\sum_{k=1}^{2}\left\| W*-W^k \right\| = \min\sum_{k=1}^{2}\sum_{i=1}^{n}\left(wi*-wi^k\right) \\ s.t.\sum_{i=1}^{n}wi=1 \end{cases} \quad (2)$$

Step 7: Solving the nonlinear optimization model; Lagrangian function constructor

$$L(\lambda_1,\lambda_2,M)=\sum_{k=1}^{2}\sum_{i=1}^{n}(\lambda_1 wi+\lambda_2 wi-wik)^2\\+M(\sum_{i=1}^{n}wi-1)$$

Respectively λ_1, λ_2, M seeking an order partial derivatives equal to zero and optimized conditions expressed in the form of a matrix,

$$\begin{bmatrix} W(1)(W(1))T+ & W(1)(W(2))T \\ W(2)(W(1))T+ & W(2)(W(2))T \end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix}\\=\begin{bmatrix}W(1)(W(1))T\\W(2)(W(2))T\end{bmatrix}$$

Transformed into equations of the form

$$\begin{cases}\lambda_1 W(1)(W(1))T+\lambda_2 W(1)(W(2))T=W(1)(W(1))T\\\lambda_1 W(2)(W(1))T+\lambda_2 W(2)(W(2))T=W(2)(W(2))T\end{cases}$$

Solved λ_1, λ_2 values into the above equation, can be obtained quadratic optimal combination weight W*.

III. APPLICATION EXAMPLES

A. Assessment and Evaluation to Determine the Cross-Section

This paper selects Lalatun, Xiaobaima, Xiguan, Cishan four monitoring sections for the study According to the monitoring data of groundwater quality, select Fluoride(F^-), Total dissolved solids(TDS), Permanganate index(COD_{Mn}), The total hardness(TH), Sulfate(SO_4^{2-}), Chloride(Cl^-), Ammonia (NH_4^+) seven indicators as an evaluation factor. Taking into account seasonal variations in groundwater quality with little, therefore, the water quality of its dry season water quality assessment. Specific information in the following table:

TABLE I.　Dry Season Water Quality Test Results of the

location	Lalatun	Xiaobaima	Xiguan	Cishan
F-	0.15	0.23	0.26	0.22
TDS	691	358	1474	986
CODMn	2.57	1.8	2.37	1.11
TH	100.57	72.63	362.16	111.78
SO42-	260.11	69.93	62.25	168.86
Cl-	160.97	185.65	283.11	210.23
NH4+	0.02	0.02	0.02	0.02

TABLE II.　Groundwater Quality Grading Standards (GB/T14848-93)(mg/L)

Index	I	II	III	IV	V
F-	1	1	1	2	≥2
TDS	300	500	1000	2000	≥2000
CODMn	1.0	2.0	3.0	10	≥10
TH	150	300	450	550	≥550
SO42-	50	150	250	350	≥350
Cl-	50	150	250	350	≥350
NH4+	0.02	0.02	0.2	0.5	≥0.5

B. Afiliation Fuzzy Evaluation Matrix to Determine

According to the membership function of fuzzy comprehensive evaluation and a given quality grading standards. Determine the evaluation index for the degree of membership of each grading get affiliation matrix as follows：

Lalatun

$$R=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\0 & 0.62 & 0.38 & 0 & 0\\0 & 0.43 & 0.57 & 0 & 0\\1 & 0 & 0 & 0 & 0\\0 & 0 & 0.85 & 0.15 & 0\\0 & 0.65 & 0.35 & 0 & 0\\1 & 0 & 0 & 0 & 0\end{bmatrix}$$

Xiaobaima

$$R=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\0.27 & 0.73 & 0 & 0 & 0\\0.20 & 0.80 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0\\0.30 & 0.70 & 0 & 0 & 0\\0 & 0.84 & 0.16 & 0 & 0\\1 & 0 & 0 & 0 & 0\end{bmatrix}$$

Xiguan

$$R=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\0 & 0 & 0.53 & 0.47 & 0\\0 & 0.63 & 0.37 & 0 & 0\\0 & 0.59 & 0.41 & 0 & 0\\0.88 & 0.12 & 0 & 0 & 0\\0 & 0 & 0.67 & 0.33 & 0\\1 & 0 & 0 & 0 & 0\end{bmatrix}$$

Cishan

$$R=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\0 & 0.03 & 0.97 & 0 & 0\\0.59 & 0.41 & 0 & 0 & 0\\1 & 0 & 0 & 0 & 0\\0 & 0.61 & 0.39 & 0 & 0\\0 & 0.30 & 0.70 & 0 & 0\\1 & 0 & 0 & 0 & 0\end{bmatrix}$$

C. The Evaluation Factors to Determine the Optimal Weight Vector

Examples of each index entropy calculations to the data, for example Lara Tun dry season. According to Table 1 in the measured value of each indicator, establishing the initial concentration matrix evaluation indicators are negative indicators that the smaller the value corresponding to the measured water quality is better. Poor initial matrix normalized then using the standardized approach to treat the data in Table 1 to obtain standardized matrix. The results are as follows:

TABLE III. THE WATER QUALITY OF GROUNDWATER AND THE INFORMATION ENTROPY WEIGHTS

Index	Information entropy	Weights
F⁻	0.24	0.134
TDS	2.5×10^{-4}	0.179
COD_{Mn}	0.12	0.155
TH	1.2×10^{-3}	0.177
SO$_4^{2-}$	2.0×10^{-3}	0.174
Cl⁻	1.7×10^{-3}	0.175
NH$_4^+$	0.97	0.006

In Water Quality Assessment, To determine the index weights of evaluation results is essential. As a result of the impact on water quality evaluation is different, therefore it should be given different weights. The same data as an example to LalaTun, to a single factor pollution index method for processing the raw data.

$$Pi=Ci/Si$$

Ci-the measured concentration; Si-the standard concentration
Computer programming using MATLAB software to get the maximum discriminant matrix eigenvalue and eigenvector .In this case, the weight vector

$$(0.060 \quad 0.293 \quad 0.255 \quad 0.123 \quad 0.085 \quad 0.148 \quad 0.036)^T$$

According to equation (2) to obtain equations are

$$\begin{cases} 0.217\lambda_1 + 0.188\lambda_2 = 0.217 \\ 0.188\lambda_1 + 0.173\lambda_2 = 0.173 \end{cases}$$

Solution was $\lambda_1 = 0.643$ $\lambda_2 = 0.403$
To enable$\lambda_1 + \lambda_2 = 1$, Were weighted to obtain

$$\lambda_1^* = 0.615 \qquad \lambda_2^* = 0.385$$

Then get the optimal weights evaluation The results as follows:

D. Evaluation Results

The resulting optimal weight and fuzzy relationship matrix synthesis calculations, geting the water quality objectives fuzzy comprehensive evaluation result vector B, As follows.

TABLE IV. THE EVALUATION OF THE OPTIMAL WEIGH

Index	Optimal weight
F⁻	0.067
TDS	0.290
COD_{Mn}	0.208
TH	0.141
SO$_4^{2-}$	0.180
Cl⁻	0.105
NH$_4^+$	0.009

TABLE V. THE EVALUATION RESULTS OF WATER QUALITY MONITORING SECTIONS

Membership	Lala-tun	Xiao-baima	Xi-guan	Ci-shan
level Ⅰ	0.217	0.349	0.195	0.277
level Ⅱ	0.337	0.62	0.215	0.253
level Ⅲ	0.419	0.03	0.412	0.47
level Ⅳ	0.027	0	0.178	0
level Ⅴ	0	0	0	0
level	Ⅲ	Ⅱ	Ⅲ	Ⅲ
Complex Index	Better	Good	Better	Better

E. Analysis

The evaluation results of the above table it can be seen, Little white Ishimura's Class II water quality, The remaining three are the three types of water monitoring sections, and comprehensive index evaluation results are compared, basically the same, This method is suitable for visible water quality assessment and has a certain rationality.

IV. CONCLUSION

Evaluation of groundwater chloride and high levels of total dissolved solids, mainly because the assessment area is located in the outskirts of agriculture Huludao City, various regional distribution of agricultural irrigation and farm. Rainfall after fertilization of agricultural residues and dung fertilizer and pesticide wastewater will seep into the aquifer with rainwater, the groundwater chloride and dissolved solids content increased.

Using of secondary combination weighting fuzzy comprehensive evaluation model to evaluate the groundwater quality, accurate and objective evaluation results. The calculation of this case, Groundwater Quality Evaluation open to new ideas, Water Quality Assessment also has certain reference significance.

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