Hidden Markov Model for Predicting the Turning Points of GDP Fluctuation

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Abstract - At present, the methods of predicting the turning points of GDP fluctuation is difficult to choose suitable influence indexes, and emphasize static function dependency or dynamic propagation of time series so that the static and dynamic information can not be consistently combined. In this paper, hidden Markov model is introduced for predicting the turning points of GDP fluctuation. The real GDP data of China from 1990 to 2013 is used for modeling and the experimental results show that hidden Markov model has good practicability and reliability.

Index Terms – GDP, turning point, hidden Markov model, dynamic Bayesian network

1. Introduction

The business cycle is a complicated phenomenon in economical operation, and the research on business cycle is an attracting issue[1]. And gross domestic product (GDP) is the most watched economic statistics in macro economy because it is the most important index to measure the national economic development level. We can use the change of GDP to describe the cyclical fluctuation of economic growth. Therefore, the research of model for predicting the turning points of GDP fluctuation if significant in both theory and practice. There have some methods to predict the cyclical turning points of economic fluctuation such as function fitting and time series prediction, etc. But these methods emphasize static function dependency or dynamic propagation of time series[2,3] so that the static and dynamic information can not be consistently combined.

Hidden Markov model (HMM) is a representative of state space models and is also a special case of dynamic Bayesian networks. It expresses the change of internal state of the system caused by input, which make the change of output. HMM not only reflects the internal state of system but also shows the relationship of internal state and external input and output of system. In 80s of 20th century, HMM was successfully applied in speech recognition[4]. HMM is also applied in the fields of optical character recognition, bioinformatics, troubleshooting and finance, etc[5-7].

In this paper, based on the data of official website of China's national bureau of statistics and the data of relevant statistical yearbook, we use HMM to predict the turning points of GDP fluctuation. There are four sections. The second section is the theory of HMM. The third section gives the method of hidden Markov model for predicting the turning points of GDP fluctuation. And the last is conclusion.

2. Hidden Markov Model

A HMM is a statistical Markov model in which the system being modeled is assumed to be a Markov process with unobserved states. In a HMM, the state is not directly visible, but output, dependent on the state, is visible. Each state has a probability distribution over the possible output tokens. Therefore the sequence of tokens generated by a HMM gives some information about the sequence of states. A HMM can be considered a generalization of a mixture model where the hidden variables, which control the mixture component to be selected for each observation, are related through a Markov process rather than independent of each other. In HMM, the state space of the hidden variables is discrete, while the observations themselves can either be discrete or continuous. In this paper, we only study the discrete HMM.

A general mathematical expression of HMM is $\lambda = (N, M, A, B, \pi)$, abbreviated as $\lambda = (A, B, \pi)$. $S = \{S_1, S_2, \cdots, S_N\}$ is the set of hidden states and N is the number of them. Let X_t denote the state at time t. $V = \{V_1, V_2, \cdots, V_M\}$ is the set of observation and M is their number. $A = \{a_{ij}\}_{N\times N}$ is matrix of state transition probability, where $a_{ij} = P(X_{t+1} = S_j \mid X_t = S_i)$, $i, j = 1,2,\cdots,N$ is the probability that state changes from S_i to S_j, $0 \le a_{ij} \le 1$, $\sum_{j=1}^{N} a_{ij} = 1$. $B = (b_{jk})_{N\times M}$ is probability matrix of observation vector, where $b_{jk} = P(Y_t = V_k \mid X_t = S_j)$ is the observation probability that V_k is observed when the state is S_j and Y_t is the observation random variable at time t. $\pi = \{\pi_i, 1 \le i \le N\}$ is the initial probability of state, where $\pi_i = P(X_1 = S_i)$, $0 \le \pi_i \le 1$ and $\sum_{i=1}^{N} \pi_i = 1$. Let $\boldsymbol{O} = \{O_1, \ldots, O_t\}$ denote the observation sequence.

There are three classical problems in HMM.

1) Model evaluation: For given the observation sequence $\boldsymbol{O} = \{O_1, \ldots, O_t\}$ and model parameters $\lambda = (A, B, \pi)$, how do we calculate the probability $P(\boldsymbol{O} \mid \lambda)$?

2) Most probable path decoding: For given the observation sequence $\boldsymbol{O} = \{O_1, \ldots, O_t\}$ and model parameters $\lambda = (A, B, \pi)$, what is the most probable transition sequence $X^* = X_1^*, X_2^*, \ldots, X_T^*$?

3) Model training: For given the observation sequence $\boldsymbol{O}=\{O_1,\dots,O_t\}$ and number of hidden states N and the number of observations M, how do we estimate or optimize the parameters of an HMM?

In view of the above questions, there are forward-backward, viterbi and Baum-Welch algorithms to solve them respectively[4].

3. The Prediction Method of the Turning Points of GDP Fluctuation Base on HMM

A. Prediction algorithm

For a given observation sequence $\boldsymbol{O}=\{O_1,\dots,O_t\}$, we assume that this sequence can be modeled by a HMM with parameters $\lambda=(A,B,\pi)$. We want to predict the observation $\hat{O}_{t+h}$ at time $t+h$, where h is the prediction step. Concrete steps are as follows.

1) Model training: According to observation sequence $\boldsymbol{O}=\{O_1,\dots,O_t\}$, we use the initial parameters $\lambda=(A,B,\pi)$ and EM algorithm to train and obtain the model parameters of HMM $\lambda_{\text{opt}}=\underset{\lambda}{\arg\max}\{f(\boldsymbol{O}\mid\lambda)\}$, that is to find $\lambda_{\text{opt}}=(A,B,\pi)$ which maximize likelihood $f(\boldsymbol{O}\mid\lambda)$ or logarithmic likelihood $\ln f(\boldsymbol{O}\mid\lambda)$.

2) Hidden state estimation: According to parameters $\lambda_{\text{opt}}=(A,B,\pi)$, viterbi algorithm is adopted to estimate the optimal hidden state sequence $\boldsymbol{X}^*=X_1^*,X_2^*,\dots,X_t^*$, $X_i^*\in S, i=1,\dots,t$.

3) Prediction: We assume that the hidden state is $X_t=S_i$ at time t. Because the hidden state sequence $\boldsymbol{X}=X_1,X_2,\dots,X_T$ obeys one order Markov process with matrix of transition probability A, the transfer matrix form time t to $t+h$ is $A^h=\underbrace{A\cdots A}_{h}$. Then the probability from $X_t=S_i$ at time t to $X_{t+h}=S_j$ at time $t+h$ is $A^h(i,j)$ which is the element in i th row and j th column of matrix A^h. Meanwhile, when the state is S_i at some time, the observation random variable obeys probability distribution $b_i(V)=(b_{i1},\dots,b_{iM})^{\mathrm{T}}$. Therefore, The predicted value of h steps is $\hat{O}_{t+h}=\underset{V_k}{\arg\max}\{\sum_{j=1}^{N}A^h(i,j)b_{jk}\}$.

B. Prediction Model of Turning Points of GDP Fluctuation

GDP fluctuation is the basic reflection for one country's overall economic achievements; its stable increase indicates that the economy is growing positively, various conflicts tend to be harmonized and accordingly the people will have a higher expectation for the economy; on the contrary, when GDP development is staying in a unstable and unbalanced status, temporary high outcome can not insure a better expectation and unbalanced development can lead to plenty of conflicts, which will probably trigger a deeper recession. Here, 1 and 0 are chosen to respectively represent the turning point and the unturning point in GDP development. The two points can be determined based on GDP performance in last two time-points.

As it is known to all, various factors can affect GDP development, including tax, products, national policy (whether fiscal or financial), total export, as well as the changes of people's consumption habits. The comprehensive result (market status) from the above factors decides the general GDP development, positively or negatively. An improved GDP development in a better previous market status can, in a larger extent, predict a positive trend in current status, although the probability of hindering affect can not be eliminated. It is logical to conclude that the current status can only be linked to the previous one. Based on the analysis, we can establish a HMM where market status is implicit as the determiner for GDP development and the status number is 2.

Drawn from the collected data, figure 1 shows the fluctuation trajectory of GDP development in 24 years from 1990 to 2013. In the figure, we can clearly point out the exact year as the turning point for GDP development; for example, the year 1992 is a turning point while 1991 is not.

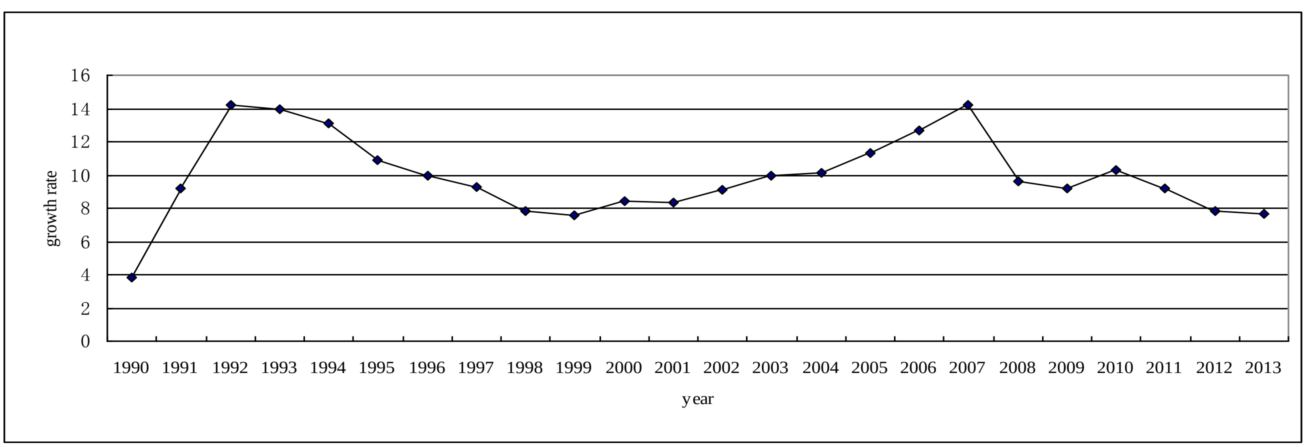

Fig. 1 The trajectory of the growth rate of GDP from 1990 to 2013

Because there are less data, we choose the prediction step $h = 5$ and use HMM to predict the turning points of GDP fluctuation from 2009 to 2013, respectively. We adopt Matlab for modeling and the initial probability of state π is defined randomly. Table I give the real values and our predictive values.

From table I, we can get that the predictive accuracy using HMM is 75%. And the results accord with the facts basically.

TABLE I The Prediction of Turning Point of GDP Fluctuation

Year	2009	2010	2011	2012	2013
Real	1	1	0	0	?
HMM	1	0	0	0	0

4. Conclusion

In this paper, HMM is introduced to predict the turning points of China's GDP. In order to build HMM, the software of Matlab is used to estimate the matrix of state transition probability and probability matrix of observation vector. Based on the HMM we estimate the future turning point of GDP fluctuation and get the better results. This method is helpful to predict the cyclical turning points of economic fluctuation.

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