

A Model based on the Coupled Rules of Evidence Theory used in Multiple Objective Decisions

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Abstract - In response to a situation that it is not easy to accurately compute the indicator weight in multiple objective decisions, and it is unable to achieve effective harmonization of subjective and objective when the traditional methods were used to calculate the weights. The combination rule of evidence theory was introduced in this article, which was used to combine the weights in traditional method by using the combination weighting model effectively by setting a coupled model, and the results were not only objective but also fit the subjective preferences of decision makers. The weights computed by the combination rule were more advanced, because the objectivity of the quantitative indicators and the subjectivity of qualitative indicators were highlighted when compared with those results of linear combination. At last, the coupled model was proved that it can be effectively used in multiple objective decision issues by an example argument.

Index terms - Evidence theory, Combination rule, Information entropy theory

1. Introduction

That making multiple objective decision is a wide range of application in economy, management, military and so on, which mainly depends on some rule to scheme comparison and programs sorting. In production planning, project construction, project selection, performance appraisal and so on of the enterprise multi-objective decision, produce a large of solutions. Multi-objective decision-making problem need determine the target attribute (index) weight, so the decision problem can be solved by defining index weight. [2]The common index weighting method have subjective weighting method (such as the Delphi method, AHP), objective weighting method (including the entropy weight coefficient method, the method of fuzzy cluster analysis, the maximum deviation method) and subjective and objective weighting method. In practical application, Wang Pengfei and so on [1-5] calculate the index weight by objective weighting method, and the decision results objectivity is obvious, but ignore the subjective preferences of decision makers, so the decision results questionable. Lv Yuejin [6-7] uses AHP method as the representative of the subjective weighting method. To make the decision result is relatively accurate, make the organic unity of the index weight achieve the subjective preference and objective analysis, a large number of scholars [4,5,8,9]do research deeply in subjective and objective weighting method direction. After evidence theory (D-S theory) appears, He Jinfeng [10, 11] gradually applied it in the measure of Multi-objective decision-making index weight problems, and provides a new solution for Multi-objective decision-making problem. This paper is based on the synthesis rules of evidence

theory. The index weight results which Analytic hierarchy process (AHP) and entropy theory calculated respectively compound, and then get the combination weights. The method is applied to the weighting of multi-objective decision-making problems.

2. Related Theory and Model Building

Decision model is constructed in this paper is based on the synthesis rules of evidence theory, the object of synthesis is to determine the index weight that adopt a variety of different methods. This paper mainly selects the analytic hierarchy process (AHP) as the representative of the subjective method, and the information entropy theory as the representative of objective weighting method, then through the synthesis rules of evidence theory structure combination of subjective and objective weights model, and calculate the combination weights, finally get reasonable index weight

2.1 The analytic hierarchy process(AHP)

The analytic hierarchy process (the Analytic Hierarchy Process, abbreviated as AHP) is a combination of qualitative and quantitative decision analysis method [6], which was proposed by USA operations researcher Saaty in the 1977. The index weight determination principle is based on different indexes to cross ratio, by constructing the matrix obtained the weights of indexes. To ensure the rationality of the results of the analysis by using AHP, still need to construct judgment matrix with consistency check.

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (1)$$

In order to measure whether different order judgment matrix has the consistency of satisfactory, finally introduced to random consistency index of the judgment matrix RI . The concrete calculation method in the literature [6], its calculating steps are as follows:

Step1: Construct judgment matrix B, and calculate the maximum eigenvalue of judgment matrix B and the corresponding feature vector W1.

Step2: Measure whether different order judgment matrix has consistency of satisfied.

Step3: The results are consistent to meet is established, if not consistent with satisfied, repeat step1, Step2, till the result is consistent with satisfied

2.2 The information entropy theory

Shannon introduced the entropy in information theory and the information entropy had been proposed. For multi-objective problems of uncertainty, information entropy theory make a measurement for it, namely the greater the amount of information is, the less the uncertainty is. The smaller the amount of information is, the greater the uncertainty will be, and the greater the entropy values also the same [12]. Some scholars have made improvement to the entropy calculation process of information entropy theory, this paper selected the original expression of the entropy method to determine index weight, Calculation process can be found in the literature [13], the main steps are as follows:

Step1: To construct the multiple attribute decision making matrix M objects, n indicators, the qualitative indexes are normalized, get the judgment matrix $B = (r'_{ij})_{m \times n}$

Step2: Calculate the entropy, a set of $E = (e_1, e_2, e_3, \dots, e_n)$.

The definition of entropy of the j index:

$$e_j = -k \sum_{i=1}^m p_{ij} \ln p_{ij} \quad (2)$$

$$\text{And } p_{ij} = r'_{ij} / \sum_{i=1}^m r'_{ij}, k = \frac{1}{\ln m} \quad (3)$$

(Among $i=1, 2, 3, \dots, m, j=1, 2, 3, \dots, n$)

Step3: According to the entropy e_j and calculate entropy weight of every index W_2

$W_2 = (w'_1, w'_2, w'_3, \dots, w'_n)$, Type in $j=1, 2, 3, \dots, n$,

$$\text{And } w'_j = (1 - e_j) / \sum_{j=1}^n (1 - e_j) \quad (4)$$

2.3 Evidence theory

The Dempster-Shafer evidence reasoning theory (D-S theory) was proposed by Dempster in 1967, Shafer develop it, then form D-S evidence theory [14]. The paper adopt the D S theory of synthetic rules are combined with the result of two groups of independent index weight

Definition 1: Set Θ as a recognition framework, If the function $m: 2^\Theta \rightarrow [0,1]$ (2^Θ as Θ Power set, Represent Θ a collection of all subsets) content $m(\Phi) = 0, \sum_{A \subseteq \Theta} m(A) = 1, m$ call $mass$ function, basic probability assignment of frame Θ

Definition2: Set $\forall A \subseteq \Theta$, by $Bel(A) = \sum_{B \subseteq A} m(B)$ defined function $Bel: 2^\Theta \rightarrow [0,1]$ as reliability function of Θ

Definition3: The $Pl(A) = 1 - Bel(\bar{A})$ function defined $Pl: 2^\Theta \rightarrow [0,1]$ as plausible function. $Pl(A)$ called the plausibility of A. It is said that A is reliable or plausibility degree, or is not suspected degree.

Definition4: Set Bel_1 and Bel_2 as two reliability function with a frame Θ of discernment (A as the focal element), $\sum_{A \cap B = \Phi} m_1(A) m_2(B) < 1$, The orthogonal and expression:

$$m(A) = m_1(A) \oplus m_2(A) = \frac{1}{K} \sum_{B \cap C = A} m_1(B) m_2(C) \quad (5)$$

among K for the normalization constant, and

$$K = \sum_{B \cap C \neq \Phi} m_1(B) m_2(C) = 1 - \sum_{B \cap C = \Phi} m_1(B) m_2(C) \quad (6)$$

For multiple evidence combination problem, can use each comprehensive approach to evidence Dempster combination.

2.4 Establish the process of coupling model

Step1: Get the weight of each index W1 based on the subjective weighting method of AHP method to solve the index weight, and then constitute a mass1 function

Step2: By using the method of information entropy theory as the representative method to solve the index weight, and get the weight of each index W2, and then constitute a function of mass2

Step3: Combination rules of evidence theory, which synthesize the two set of mass function, which is the combination of the weights of the indexes, obtains each target combined weight W3

The coupled model:

$$\begin{aligned} W_3(x_j) &= mass3(x_j) = mass1(x_j) \oplus mass2(x_j) \\ mass1(x_j) &= W_1(x_j) \in W_1(x_1, x_2, \dots, x_j, \dots, x_n) \\ mass2(x_j) &= W_2(x_j) \in W_2(x_1, x_2, \dots, x_j, \dots, x_n) \end{aligned} \quad (7)$$

3. The Empirical Analysis

In a repair tender [15], A total of A, B, C, D, E five companies participate in the bid, the owners reference standard mainly includes the evaluation scheme, repair quality system, as well as understanding level of different enterprise. There are 5 assessment indexes in the tender, respectively repair quotation X1, repair schedule X2, quality assurance system X3, business situation x4, and in recent year management performance X5 (including the number of bearing similar repair project and related services). The qualitative indexes as the expert evaluation score, quantitative indexes as the original statistical data. All data of each enterprise are in Table 1.

Table 1. The initial data of Five enterprises in different indicators

Enterprise numbers	Repair quotation (million)	Repair schedule (month)	Quality assurance (points)	Management level(points)	Performance (points)
A	750	10	50	65	86.7
B	930	7	64	74	85.6
C	980	8	50	99	70.2
D	890	7	40	76	65.3
E	790	10	70	80	94.5

3.1 Using the coupling model to certain the comprehensive index weight

(1) To solve the index weight W1 following the content of 2.1

Given in this paper, the result of the model is synthesis of AHP method and information entropy theory results, and compared with decision results of the reference sample, as a result, the same method of the same data processing results should be identical [15]. According to step1 ~ step3 in 2.1, can solve satisfactory feature vector W1. As a result of the in the direct introduction reference literature

$$W_1 = (0.5978, 0.1324, 0.1521, 0.0313, 0.0864)$$

(2) To solve the index weight W2 following the content of 2.2

$$B = \begin{pmatrix} 1.0000 & 0.7000 & 0.7143 & 0.6566 & 0.9175 \\ 0.8065 & 1.0000 & 0.9143 & 0.7475 & 0.9058 \\ 0.7653 & 0.8750 & 0.7143 & 1.0000 & 0.7429 \\ 0.8427 & 1.0000 & 0.5714 & 0.7677 & 0.6910 \\ 0.9494 & 0.7000 & 1.0000 & 0.8081 & 1.0000 \end{pmatrix}$$

According to the result of step2 ask out in 2.2

$$E = (0.9968, 0.9922, 0.9880, 0.9939, 0.9942)$$

And, $k=0.1263$

According to the result of step3 ask out in 2.2

$$W_2 = (0.0903, 0.2227, 0.3448, 0.1756, 0.1666)$$

(3) To solve the index weight W3 following the content of 2.3

According to the relevant definition of evidence theory, In this case the recognition framework $\Theta = (x_1, x_2, x_3, x_4, x_5)$, Two groups of mass function on recognition framework Θ distribution as shown in Table 2, Among Function relation of index and weight which was obtained by the AHP method is mass1, Function relation of index and weight which was obtained by The information entropy theory is mass2. By definition 3 and 4, If each element in recognition framework does not exist intersection, the probability distribution function, trust function value and plausibility function values of an element are equal.

Table 2. Two sets of mass distribution

Mass function	W(x1)	W(x2)	W(x3)	W(x4)	W(x5)
Mass1(AHP method)	0.5978	0.1323	0.1521	0.0313	0.0864
Mass2(Information entropy theory method)	0.0903	0.2227	0.3448	0.1755	0.1666

According to the 2.3 definition 4, the synthesis results:

$$W_3 = (0.3464, 0.1892, 0.3367, 0.0353, 0.0924)$$

Among $1/k = 6.4189$

(4) The fuzzy evaluation results

According to the membership degree principle, the formula $V = B * W$ of fuzzy evaluation select the optimal project from the 5 enterprises, and the results were compared with the literature [15]. And

$$V = B * W^T = \begin{pmatrix} 1.0000 & 0.7000 & 0.7143 & 0.6566 & 0.9175 \\ 0.8065 & 1.0000 & 0.9143 & 0.7475 & 0.9058 \\ 0.7653 & 0.8750 & 0.7143 & 1.0000 & 0.7429 \\ 0.8427 & 1.0000 & 0.5714 & 0.7677 & 0.6910 \\ 0.9494 & 0.7000 & 1.0000 & 0.8081 & 1.0000 \end{pmatrix} * \begin{pmatrix} 0.3464 \\ 0.1892 \\ 0.3367 \\ 0.0353 \\ 0.0924 \end{pmatrix}$$

$$= (0.8273, 0.8865, 0.7751, 0.7645, 0.9189)$$

As $\max(0.8274, 0.8865, 0.7751, 0.7645, 0.9189) = 0.9189$, so E enterprise for the winning enterprise. The sequence for all the enterprises is that $V(E) > V(B) > V(A) > V(C) > V(D)$. In this paper, the evaluation results are fully consistent with the result of literature [15], as shown in the data in table 3.

Table 3. The form about the comparison of results from different methods of fuzzy evaluation

All method	V(A)	V(B)	V(C)	V(D)	V(E)
AHP method	0.8989	0.8552	0.7775	0.8068	0.9240
Information entropy theory method	0.7606	0.8929	0.8096	0.7457	0.8949
Model in literature [15]	0.7761	0.7996	0.7346	0.7316	0.8220
Coupling model	0.8273	0.8865	0.7751	0.7645	0.9189

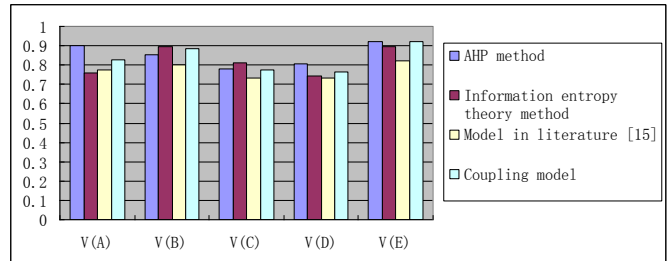


Figure 1. Comparison of Fuzzy evaluation between the results in different methods

3.2 Comparative interpretation of coupling results

The index weights calculation results are listed in this paper by using different methods obtained, shown in Table 4, figure 2

Table 4. Weights of indexes in different methods

All method	W(x1)	W(x2)	W(x3)	W(x4)	W(x5)
AHP method	0.5978	0.1323	0.1521	0.0313	0.0864
Information entropy theory method	0.0903	0.2227	0.3448	0.1755	0.1666
Model in literature [15]	0.3626	0.1744	0.2414	0.0979	0.1236
Coupling model	0.3464	0.1892	0.3367	0.0353	0.0924

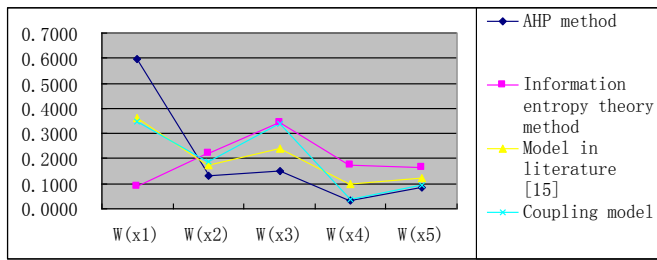


Fig.2 Comparison of index weights in different methods

First, the coupling model of synthetic index weight sort results accord with the results of literature [15]. By using the calculation principle of different methods analysis, AHP method is based on the importance of the project between the each indicators of subjective measure to calculate index weight. The literature [15] is based on information entropy and AHP linear combination weighting, combination results too mean, subjective intention of decisions without apparent consideration. According to the different methods for ranking of the index weight size, using AHP method solve the discrete degree of the index weight large, and the information entropy method solve index weight distribution relatively uniform. The correlation curve fitting by two different methods which can be found, they also has the very big difference in forming the trend, literature [15] is designed by both methods given a certain proportion, so that the weight value may extrude average. This paper is based on the coupling model of combination rules of evidence theory, in the calculation of the index weights. In addition to ensure the index weight close to ideal index decision value, also retained the data contained in different index information.

4. Conclusions

In the article, the synthesis rule of evidence theory has been used to solve the index weight synthetically based on the synthesis rule of evidence theory, which is applied to settle the multi-objective decision problem, and the conclusion accords with the practical situation. Compared with the existing determining weights method, the main advantages of the index weight coupling model are mainly reflected as follows:

First, when compared with the single way to solve the index weighting method, most methods only guarantee objectivity or subjectivity, which can't define qualitative and quantitative indicators about empowerment.

Second, the coupled model can compound the result of a variety of methods, rather than simply average or singly assign the multi-parameter method, but compound it to make the index features more obvious based on the coupling characteristics of synthesis instead, and avoid the index weighting multi-index too uniform in multi-index decision problem.

Third, the traditional linear combination is hard to be

applied to value in parameter selection about for multi-index coupling problem, which means coefficient selection is arbitrary. The more accurate method is based on TOPSIS with the idea of the smallest distance, whose the calculation process is tedious. Multi-index weight coupled model presented in this paper can function in compound result of the multi-factor, which makes the convergence rate of the results greatly improved, and provides a new way to determine the index weight process.

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