L_2-L_∞ Fuzzy Filtering for Nonlinear Time-delay Jump Systems with Uncertain Parameters

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Abstract

The L_2-L_∞ fuzzy filtering problem for a class of nonlinear time-delay jump systems with uncertain parameters is considered. By using the constructed Lyapunov-Krasovskii function, a sufficient condition that the solution of the L_2-L_∞ fuzzy filter existed is given and proved. The results are derived by the form of linear matrix inequalities (LMIs) and the design problem of L_2-L_∞ fuzzy filter is described as an optimization one. The presented fuzzy filter makes the error systems have exponentially stochastically stability and satisfy the given L_2-L_∞ performance index. Simulation results demonstrate the validity of the proposed approaches.

Keywords: Time-delay jump systems, Uncertain, Fuzzy filtering, L_2-L_∞ performance, Linear matrix inequalities(LMIs).

1. Introduction

Since the Kalman filtering theory [1] has been introduced in the early 1960s, the filtering problem has been extensively investigated, whose objective is to estimate the unavailable state variables (or a linear combination of the states) of a given system. But in Kalman filtering scheme, it has some limitations in practical applications because it assumes the system model be a certain well-posed one and unknown inputs be Gaussian noise disturbances. During the past decades, the robust H_∞ filtering technique [2]-[5] regains increasing interest when the systems are described by uncertain models or have stochastic noise disturbances with unknown statistics. Compared with the conventional Kalman filtering, the noise sources in the H_∞ filtering setting are arbitrary signals with bounded energy or average power, and the H_∞ filter has been shown to be much more robust to parameter uncertainty in a control system. H_∞ filtering guarantees a prescribed bounded for the induced L_2 norm of the operator from the unknown noise inputs to the filtering error, but in practical engineering applications, the peak values of filtering error should be considered. This kind of performance criterion firstly discussed in [6], but the L_2-L_∞ filtering problem [7] [8] has received less attention. Compared with H_∞ filtering, the stochastic noise disturbances are both assumed energy bounded in these two filtering techniques, but L_2-L_∞ filtering setting requires the L_2-L_∞ performance prescribed bounded from unknown noise disturbances to filtering error.

On the other hand, a lot of dynamical systems have variable structures subject to random abrupt changes, which may result from abrupt phenomena such as sudden environment changes, subsystem switching, system noises, failures occurred in components or interconnections and executor faults, etc. Markov jump systems (MJS) are special class of hybrid systems with two components which are the mode and the state in state vectors, may be employed to model the above problems. In MJS, the dynamics of the jump modes and continuous states are respectively modeled by a finite state Markov chain and differential equations. Since the pioneering work of Krasovskii and Lidskii on quadratic control [9] of MJS in the mid 1960s, MJS regain increasing interest owing to the application of them are more comprehensive, for instance, economic systems [10], solar thermal receiver systems [11] and communication systems [12], etc. The existing results about MJS cover a large variety of problems such as stochastic stability [13] [14], stochastic controllability [15] [16] and references therein. In recent years, the filtering problem [17] [18] has gained a great deal of attention, but L_2-L_∞ filtering problem for MJS has received less consideration.

It has been recognized that the time-delays, parameter uncertainties and nonlinearities are inherent features of many physical process and often encountered in engineering systems, their presence must be considered in realistic stability analysis. Therefore, for the nonlinear systems with time-delays and parameter uncertainties, it is necessary to study the robust L_2-L_∞ filtering problem. For a class of linear time-invariant systems, a robust guarantee L_2-L_∞ filter was designed in [7]. The linear L_2-L_∞ design filtering problem was tackled in [8] for the time-delay stochastic system. In this paper, we discuss the L_2-L_∞ fuzzy filtering problem for a class of nonlinear time-delay jump systems with uncertain parameters. By means of Takagi-Sugeno (T-S) fuzzy models [19], the fuzzy filter is constructed, and the dynamics of filtering error generator is also obtained. By using the constructed Lyapunov-Krasovskii function and linear matrix inequalities (LMIs), a sufficient condition that the solution of the L_2-L_∞ fuzzy filter existed is given and proved, and the design problem of L_2-L_∞ fuzzy filter is described as an optimization one. The presented fuzzy filter makes the error systems have exponentially stochastically stability and satisfy the given L_2-L_∞ fuzzy filtering norm index. Simulation results demonstrate the validity of the proposed approaches.

In the sequel, the following notion will be used: R^n denotes n-dimensional Euclidean space, A^{T} and A^{-1} denote the transpose and the inverse of any matrix or vector, $diag\{A,B\}$ denotes the block-diagonal matrix of A and B, $\|\cdot\|$ denotes the Euclidean norm of vectors, $\mathrm{E}\{\cdot\}$ denotes the mathematics statistical expectation of the stochastic process or vector, $L_2^n[0,\infty)$ is the space of n-dimensional square integrable function vector over $[0,\infty)$, I is the unit matrix with appropriate dimensions, 0 is the zero matrix with appropriate dimensions, $*$ means the symmetric terms in a symmetric matrix.

2. Problem Formulation

Given a probability space (Ω, F, P) where Ω is the sample space, F is the algebra of events and P is the probability measure defined on F. Let the random form process $\{r_t, t \geq 0\}$ be the Markov stochastic process taking values on a finite set $M=\{1,2,\ldots,N\}$ with transition rate matrix $\Pi=\{\pi_{rk}, r,k \in M\}$, and define the following transition probability from mode r at time t to mode k at time $t+\triangle t$ as:

$$P_{rk} = P(r_{t+\Delta t} = k | r_t = r) = \begin{cases} \pi_{rk}\Delta t + o(\Delta t), & if \quad r \neq k \\ 1 + \pi_{rr}\Delta t + o(\Delta t), & if \quad r = k \end{cases} \tag{1}$$

with transition probability rates $\pi_{rk} \geq 0$ for $r,k \in M$, $r \neq k$ and $\sum\limits_{k=1,k\neq r}^{N} \pi_{rk} = -\pi_{rr}$, where $\triangle t > 0$ and $\lim\limits_{\triangle t \to 0} \frac{o(\triangle t)}{\triangle t} \to 0$.

Consider a class of nonlinear stochastic MJS described by T-S fuzzy model with time-delay and uncertainties over the space (Ω, F, P):

Subsystem i:

If $\mu_1(t)$ is F_1^i , $\mu_2(t)$ is F_2^i, and $\cdots$, $\mu_g(t)$ is F_g^i, then

$$\begin{aligned} \dot{x}(t) &= [A_i(r_t) + \Delta A_i(r_t)]x(t) \\ &\quad +[A_{hi}(r_t) + \Delta A_{hi}(r_t)]x(t-h) + B_i(r_t)d(t) \\ y(t) &= C_i(r_t)x(t) + D_i(r_t)d(t) \\ z(t) &= L_i(r_t)x(t) \\ x(\xi) &= \eta(\xi), r(\xi) = r_0, \xi \in [-h,0], i = 1,2,\cdots,S \end{aligned} \tag{2}$$

where $x(t) \in R^n$ is the state, $y(t) \in R^l$ is the measured output, $d(t) \in L_2^q[0,\infty)$ is the unknown input, $z(t) \in R^p$ is the controlled output, $h > 0$ is the unknown delay constant, $\eta(\xi) \in L_2[-h,0]$ is a continuous vector-valued initial function, r_0 is the initial mode. $\mu_1(t)$, $\mu_2(t)$, $\cdots$, $\mu_g(t)$ are the premise variables that depend on the states in many cases. F_j^i, $i = 1,2,\cdots,S$, $j = 1,2,\cdots,g$ are the fuzzy sets, S is the numbers of Subsystem i. $A_i(r_t)$, $A_{hi}(r_t)$, $B_i(r_t)$ $C_i(r_t)$, $D_i(r_t)$, $L_i(r_t)$ are known mode-dependent constant matrices with appropriate dimensions.

In the above, $\Delta A_i(r_t)$ and $\Delta A_{hi}(r_t)$ are uncertain matrices with appropriate dimensions defined as follows:

$$\begin{aligned} &[\Delta A_i(r_t), \Delta B_i(r_t)] \\ &= M_i(r_t)F_i(r_t,t)\,[N_i(r_t), N_{hi}(r_t)] \end{aligned} \tag{3}$$

where $M_i(r_t)$, $N_i(r_t)$, $N_{hi}(r_t)$ are known mode-dependent matrices with appropriate dimensions and $F_i(r_t,t)$ is the time-varying unknown matrix function with Lebesgue norm measurable elements satisfying $F_i^{\mathrm{T}}(r_t,t)F_i(r_t,t) \leq I$. By using a singleton fuzzifier, product inference and a center-average defuzzifer [20], the following dynamic global model can be obtained:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{S} h_i(\mu(t))[(A_i(r_t) + \Delta A_i(r_t))x(t) \\ &\quad +[A_{hi}(r_t) + \Delta A_{hi}(r_t)]x(t-h) + B_i(r_t)d(t)] \\ y(t) &= \sum_{i=1}^{S} h_i(\mu(t))[C_i(r_t)x(t) + D_i(r_t)d(t)] \\ z(t) &= \sum_{i=1}^{S} h_i(\mu(t))L_i(r_t)x(t) \\ x(\xi) &= \eta(\xi), r(\xi) = r_0, \xi \in [-h,0], i = 1,2,\cdots,S \end{aligned} \tag{4}$$

where $\mu(t) = [\mu_1(t), \mu_2(t), \cdots, \mu_g(t)]$. And for $\forall i = 1, 2, \cdots, S$,

$$\begin{aligned} h_i(\mu(t)) &= u_i(\mu(t))/\sum_{i=1}^{S} u_i(\mu(t)) \\ u_i(\mu(t)) &= \prod_{j=1}^{g} F_j^i(\mu_j(t)) \end{aligned} \tag{5}$$

in which $F_j^i(\mu(t))$ is the grade of membership of $\mu_j(t)$ in the fuzzy set F_j^i . In this paper, we assume $u_i(\mu(t)) \geq 0$ and $\sum_{i=1}^{S} u_i(\mu(t)) \geq 0$. Then we can obtain the following conditions:

$$\begin{aligned} &\sum_{i=1}^{S} h_i(\mu(t)) = 1 \\ &1 \leq h_i(\mu(t)) \leq 1, i = 1, 2, \cdots, S \end{aligned} \tag{6}$$

Similar to the fuzzy observer design [21], we now consider the following fuzzy filter for the estimate of $z(t)$:
Filter i:
If $\mu_1(t)$ is F_1^i , $\mu_2(t)$ is F_2^i , and $\cdots$, $\mu_g(t)$ is F_g^i, then

$$\begin{aligned} \dot{\hat{x}}(t) &= A_{fi}(r_t)\hat{x}(t) + B_{fi}(r_t)y(t) \\ \hat{z}(t) &= L_{fi}(r_t)\hat{x}(t) \\ \dot{\hat{x}}(0) &= \hat{x}(\varphi), i = 1, 2, \cdots, S \end{aligned} \tag{7}$$

and the dynamic global filter model can be constructed as

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{S} h_i(\mu(t))[A_{fi}(r_t)\hat{x}(t) + B_{fi}(r_t)y(t)] \\ \hat{z}(t) &= \sum_{i=1}^{S} h_i(\mu(t))L_{fi}(r_t)\hat{x}(t) \\ \dot{\hat{x}}(0) &= \hat{x}(\varphi), t = 0,\ i = 1, 2, \cdots, S \end{aligned} \tag{8}$$

where $\hat{x}(t) \in R^n$ and $\hat{z}(t) \in R^p$ respectively represent the filter state and output, $A_{fi}(r_t)$, $B_{fi}(r_t)$ and $L_{fi}(r_t)$ are matrices to be determined. Here, we first introduce the state estimate error $e(t) = x(t) - \hat{x}(t)$ and output error $\bar{z}(t) = z(t) - \hat{z}(t)$, and define $\bar{x}(t) = [x^{\mathrm{T}}(t), e^{\mathrm{T}}(t)]^{\mathrm{T}}$. For notational simplicity, $h_i(\mu(t))$, $A_i(r_t)$, $\bar{x}(t)$ and $x(t-h)$ are respectively denoted as h_i, $A_i(r)$, $\bar{x}$ and x_h, and the other vectors and expressions are defined similarly. Then we can get the following filtering error dynamics from systems (4)-(8):

$$\begin{aligned} \dot{\bar{x}} &= \sum_{i=1}^{S} h_i \sum_{j=1}^{S} h_j[\bar{A}_{ij}(r)\bar{x} + \bar{A}_{hi}(r)E\bar{x}_h + \bar{B}_{ij}(r)d] \\ \bar{z} &= \sum_{i=1}^{S} h_i \sum_{j=1}^{S} h_j \bar{L}_{ij}(r)\bar{x} \\ \bar{x}(\xi) &= [\eta^{\mathrm{T}}(\xi), \hat{x}^{\mathrm{T}}(\varphi)]^{\mathrm{T}},\ \xi \in [-h, 0] \end{aligned} \tag{9}$$

where

$$\bar{A}_{ij}(r) = \begin{bmatrix} A_i(r) + \Delta A_i(r) & 0 \\ A_i(r) + \Delta A_i(r) - A_{fj}(r) - B_{fj}(r)C_j(r) & A_{fj}(r) \end{bmatrix},$$

$$\bar{A}_{hi}(r) = \begin{bmatrix} A_{hi}(r) + \Delta A_{hi}(r) \\ A_{hi}(r) + \Delta A_{hi}(r) \end{bmatrix}, E = \begin{bmatrix} I & 0 \end{bmatrix},$$

$$\bar{B}_{ij}(r) = \begin{bmatrix} B_i(r) \\ B_i(r) - B_{fj}(r)D_j(r) \end{bmatrix},$$

$$\bar{L}_{ij}(r) = \begin{bmatrix} L_i(r) - L_{fj}(r) & L_{fj}(r) \end{bmatrix}.$$

Definition 1. The nonlinear jump system (4) is said to be exponentially stochastically stable if, for every system mode and every $\bar{x}(\xi) \in L_2[-h, 0]$, there exist scalars $a > 0$ and $k > 0$, such that

$$\mathrm{E}\,\|\bar{x}(t)\| \leq a e^{-kt} \sup_{-h \leq \xi \leq 0} \mathrm{E}\|\bar{x}(\xi)\| \tag{10}$$

where $k > 0$ is called the degree of exponential stability.

Definition 2. In the Euclidean space $\{R^n \times M \times R_+\}$, we introduce the stochastic Lyapunov function of system (4) as $V(x(t), r_t, t > 0) {=} V(x, r)$, the weak infinitesimal operator of which satisfies

$$\begin{aligned} \Gamma V(x, t) = \lim_{\Delta t \to 0} \tfrac{1}{\Delta t}[&\mathrm{E}V(x(t + \Delta t), r_{t+\Delta t}, t + \Delta t) \\ &- V(x(t), r, t)] \end{aligned} \tag{11}$$

Definition 3. Consider system (4) and (8), if there exist the parameters $A_{fi}(r)$, $B_{fi}(r)$ and $L_{fi}(r)$, and a positive scalar γ , such that the filtering error dynamic system (9) is exponentially stochastically stable and the output error satisfies the following cost function inequality for all admissible $d \in L_2^q[0, \infty)$,

$$\mathrm{E}\|\bar{z}\|_\infty^2 \leq \gamma^2 \mathrm{E}\|d\|_2^2 \tag{12}$$

where

$$\mathrm{E}\|\bar{z}\|_\infty^2 = \mathrm{E}\left\{\sup_{t>0}[\bar{z}^{\mathrm{T}}\bar{z}]\right\}, \mathrm{E}\|d\|_2^2 = \mathrm{E}\left\{\int_0^\infty d^{\mathrm{T}} d dt\right\},$$

then the filter (8) is said to be the robust L_2-L_∞ fuzzy filter for system (4).

Remark 1. In robust L_2-L_∞ fuzzy filtering, the unknown noise d are assumed to be arbitrary deterministic signals of bounded energy, and the problem of this paper is to design an fuzzy filter that guarantees a prescribed bounded for the induced L_2-L_∞ norm of the operator from the unknown noise inputs to the output error , i.e. the designed robust fuzzy filter is supposed to satisfy inequality (12) with attenuation γ.

3. Design of Robust Jump L_2-L_∞ Fuzzy Filter

Before proceeding with the study on filter design, the following lemmas will be useful in designing the expected L_2-L_∞ fuzzy filter for the nonlinear MJS (4).

Lemma 1 [13]. Stochastically stable is equivalent to nearly asymptotically stable.

Lemma 2 [22]. The stochastic nonlinear system $\dot{x}(t)=f(x(t),t)$ with $f(0,t)=0$ is globally exponentially stochastically stable if, there exist positive-definite function $V(x,t) \in \{R^n \times R_+\}$, and positive scalars $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, such that

$$\lambda_1\|x\|^2 \le V(x,t) \le \lambda_2\|x\|^2 \tag{13}$$

$$\Gamma V(x,t) \le -\lambda_3\|x\|^2 \tag{14}$$

Theorem 1. For the given scalar $\gamma>0$, if there exist a set of mode-dependent symmetric positive-definite matrix $P(r)$ and symmetric positive-definite matrix Q, satisfying the following matrix inequalities for all $r \in M$ and $1 \le i \le j \le S$,

$$\Pi_{ij}(r) = \begin{bmatrix} \Pi_1(r) & \Pi_2(r) & \Pi_3(r) \\ * & -2Q & 0 \\ * & * & -2I \end{bmatrix} < 0 \tag{15}$$

$$[\bar{L}_{ij} + \bar{L}_{ji}]^{\mathrm{T}}[\bar{L}_{ij} + \bar{L}_{ji}] < 4\gamma^2 P(r) \tag{16}$$

where

$\Pi_1(r) = P(r)[\bar{A}_{ij}(r) + \bar{A}_{ji}(r)] + [\bar{A}_{ij}(r) + \bar{A}_{ji}(r)]^{\mathrm{T}} \times P(r) + 2E^{\mathrm{T}}QE + \sum\limits_{r=1}^{N} \pi_{rk}P(k)$,

$\Pi_2(r) = P(r)[\bar{A}_{hi}(r) + \bar{A}_{hj}(r)]$,

$\Pi_3(r) = P(r)[\bar{B}_{ij}(r) + \bar{B}_{ji}(r)]$.

Then the filtering error dynamic system (9) is exponentially stochastically stable and the fuzzy filter (8) is said to be the robust L_2-L_∞ fuzzy filter for system (4) with attenuation γ.

Proof: Under the conditions of the theorem, we first study the exponentially stochastically stability of system (9). For the given symmetric positive-definite matricx $P(r)$, $r \in k$, define the following Lyapunov-Krasovskii function

$$V(\bar{x},r,t) = \bar{x}^{\mathrm{T}}P(r)\bar{x} + \int_{t-h}^{t} \bar{x}^{\mathrm{T}}(\tau)E^{\mathrm{T}}QE\bar{x}(\tau)d\tau$$

Then along the solution of (9) with $d=0$, applying definition 2, the weak infinitesimal operator $V(\bar{x},r,t)$ of (9) is given by

$$\begin{aligned}&\Gamma V(\bar{x},r,t) = \tfrac{1}{2}\sum_{i=1}^{S} h_i \sum_{j=1}^{S} h_j\{\bar{x}^{\mathrm{T}}P(r)[(\bar{A}_{ij}(r) \\ &+\bar{A}_{ji}(r))\bar{x} + (\bar{A}_{hi}(r) + \bar{A}_{hj}(r))x_h] + [(\bar{A}_{ij}(r) \\ &+\bar{A}_{ji}(r))\bar{x} + (\bar{A}_{hi}(r) + \bar{A}_{hj}(r))x_h]^{\mathrm{T}}P(r)\bar{x} \\ &+\bar{x}^{\mathrm{T}}\sum_{r=1}^{N}\pi_{rk}P(k)\bar{x}\} + \bar{x}^{\mathrm{T}}E^{\mathrm{T}}QE\bar{x} - x_h^{\mathrm{T}}Qx_h = \\ &\tfrac{1}{2}\sum_{i=1}^{S} h_i \sum_{j=1}^{S} h_j \begin{bmatrix} \bar{x} \\ x_h \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} \Pi_1(r) & \Pi_2(r) \\ * & -2Q \end{bmatrix} \begin{bmatrix} \bar{x} \\ x_h \end{bmatrix}\end{aligned}$$

Obviously, by the Schur complement formula, $\Gamma V(\bar{x},r,t)<0$ can be guaranteed by matrix inequality (15). This implies that there always exists a scalar $\lambda_3 > 0$, such that

$$\Gamma V(\bar{x},r,t) \le -\lambda_3\|x\|^2$$

Meanwhile, considering the Lyapunov-Krasovskii function $V(\bar{x},r,t)$, there will exist scalars $\lambda_1 > 0$, $\lambda_2 > 0$, such that

$$\lambda_1\|x\|^2 \le V(\bar{x},r,t) \le \lambda_2\|x\|^2$$

Therefore, by Lemma 2 and Definition 1, we have that system (9) is exponentially stochastically stable. So dynamic error system (9) is exponentially asymptotically stable according to Lemma 1.

Furthermore, we consider the L_2-L_∞ filtering performance for dynamic error system (9) while $d \ne 0$. In zero initial condition, for $T > 0$, we introduce the following cost function for system (9) by Definition 3,

$$J = \mathrm{E}\{V(\bar{x},r,t)\} - \mathrm{E}\left\{\int_0^T d^{\mathrm{T}}d dt\right\}$$

While define $Z = [\bar{x}^{\mathrm{T}}, x_h^{\mathrm{T}}, d^{\mathrm{T}}]^{\mathrm{T}}$, we have

$$\begin{aligned}&J = \mathrm{E}\left\{\Gamma V(\bar{x},r,t) - \textstyle\int_0^T d^{\mathrm{T}}d dt\right\} \\ &< \textstyle\int_0^{\mathrm{T}} \left[\tfrac{1}{2}\sum\limits_{i=1}^{S} h_i^2 Z^{\mathrm{T}}\Pi_{ii}(r)Z + \sum\limits_{i=1}^{S} h_i h_j Z^{\mathrm{T}}\Pi_{ij}(r)Z\right] dt\end{aligned}$$

By the Schur complement formula, $J<0$ holds if $\Pi_{ij}(r)$. Therefore, it can be verified that

$$\mathrm{E}\{\bar{x}^{\mathrm{T}}P(r)\bar{x}\} \le \mathrm{E}\{V(\bar{x},r,t)\} < \mathrm{E}\left\{\int_0^T d^{\mathrm{T}}d dt\right\}$$

On the other hand, we get

$$\begin{aligned}&\bar{z}^{\mathrm{T}}z = \tfrac{1}{4}\sum_{i=1}^{S} h_i \sum_{j=1}^{S} h_j \sum_{a=1}^{S} h_a \sum_{b=1}^{S} h_b \\ &\times \bar{x}^{\mathrm{T}}[\bar{L}_{ij}(r) + \bar{L}_{ji}(r)]^{\mathrm{T}}[\bar{L}_{ab}(r) + \bar{L}_{ba}(r)]\bar{x} \le \\ &\tfrac{1}{4}\sum_{i=1}^{S} h_i \sum_{j=1}^{S} h_j \bar{x}^{\mathrm{T}}[\bar{L}_{ij}(r) + \bar{L}_{ji}(r)]^{\mathrm{T}}[\bar{L}_{ij}(r) + \bar{L}_{ji}(r)]\bar{x}\end{aligned}$$

Recall matrix inequality (15), for $T > 0$, we can see that

$$\mathrm{E}\{\bar{z}^{\mathrm{T}}\bar{z}\} \le \gamma^2\mathrm{E}\{\bar{x}^{\mathrm{T}}P(r)\bar{x})\} < \gamma^2\mathrm{E}\left\{\int_0^T d^{\mathrm{T}}ddt\right\}$$

Obviously, $\mathrm{E}\{\bar{z}^{\mathrm{T}}\bar{z}\} < \gamma^2\mathrm{E}\left\{\int_0^\infty d^{\mathrm{T}}ddt\right\}$ holds while $T\to\infty$ for any nonzero $d \in L_2^q[0,+\infty)$. This completes the proof.

Remark 2. From Theorem 1, we can see clearly that the matrix inequalities (14) (15) are unlikely to resolve in practice for the uncertainties and nonlinearities in the inequalities. In order to obtain the robust L_2-L_∞ fuzzy filter of nonlinear jump system (4), it is necessary to get the LMIs optimized condition which without comprising the uncertainties and nonlinearities by relative Lemmas. On the previous assumption, the following Theorem 2 can be considered.

Theorem 2. For the given scalar $\gamma > 0$, if there exist a set of mode-dependent symmetric positive-definite matrix $P(r) = diag\{P_1(r), P_2(r)\}$, $r \in k$, symmetric positive-definite matrix Q, and a set of mode-dependent matrices $\bar{A}_{fi}(r)$, $\bar{B}_{fi}(r)$ and $\bar{L}_{fi}(r)$, $r{\in}M$, $i = 1, 2, \cdots, S$, satisfying the following LMIs

$$\Lambda_{ij}(r) = \begin{bmatrix} \Lambda_{1ij}(r) & \Lambda_{2ij}(r) & \Lambda_{3ij}(r) & \Lambda_{4ij}(r) \\ * & \Lambda_{5ij}(r) & 0 & 0 \\ * & * & -2I & 0 \\ * & * & * & -[\alpha_{ij}(r)+\alpha_{ji}(r)]I \end{bmatrix} < 0,$$

$$1 \le i \le j \le S \tag{17}$$

$$\begin{bmatrix} P(r) & \Lambda_{6ij}(r) \\ * & 4\gamma^2 I \end{bmatrix} > 0 \tag{18}$$

where

$$\Lambda_{1ij}(r) = \begin{bmatrix} \Lambda_{11ij}(r)+\Lambda_{11ji}(r) & * \\ \Lambda_{12ij}(r)+\Lambda_{12ji}(r) & \Lambda_{13ij}(r)+\Lambda_{13ji}(r) \end{bmatrix},$$

$$\Lambda_{11ij}(r) = P_1(r)A_i(r) + A_i^{\mathrm{T}}(r)P_1(r) + Q + \tfrac{1}{2}\sum_{r=1}^{N}\pi_{rk}P_1(r) + \alpha_{ij}(r)N_i^{\mathrm{T}}(r)N_i(r),$$

$$\Lambda_{12ij}(r) = P_2(r)A_i(r) - \bar{A}_{fj}(r) - \bar{B}_{fj}(r)C_j(r),$$

$$\Lambda_{13ij}(r) = \bar{A}_{fj}(r) + \bar{A}_{fj}^{\mathrm{T}}(r) + \tfrac{1}{2}\sum_{r=1}^{N}\pi_{rk}P_2(k),$$

$$\Lambda_{2ij}(r) = \begin{bmatrix} [P_1(r)[A_{hi}(r)+A_{hj}(r)] + \alpha_{ij}(r)N_i^{\mathrm{T}}(r)N_{hi}(r) \\ +\alpha_{ji}(r)N_j^{\mathrm{T}}(r)N_{hj}(r)] \\ P_2(r)[A_{hi}(r)+A_{hj}(r)] \end{bmatrix},$$

$$\Lambda_{3ij}(r) = \begin{bmatrix} P_1(r)[B_i(r)+B_j(r)] \\ P_2(r)[B_i(r)+B_j(r)] - \bar{B}_{fi}(r)D_i(r) - \bar{B}_{fj}(r)D_j(r) \end{bmatrix},$$

$$\Lambda_{4ij}(r) = \begin{bmatrix} P_1(r)[M_i(r)+M_j(r)] \\ P_2(r)[M_i(r)+M_j(r)] \end{bmatrix},$$

$$\Lambda_{5ij}(r) = -2Q + \alpha_{ij}(r)N_{hi}^{\mathrm{T}}(r)N_{hi}(r) + \alpha_{ji}(r)N_{hj}^{\mathrm{T}}(r)N_{hj}(r),$$

$$\Lambda_{6ij}(r) = \begin{bmatrix} [L_i(r)+L_j(r)]^{\mathrm{T}} - [\bar{L}_{fi}(r)+\bar{L}_{fj}(r)]^{\mathrm{T}} \\ \bar{L}_{fi}^{\mathrm{T}}(r) + \bar{L}_{fj}^{\mathrm{T}}(r) \end{bmatrix}.$$

Then the filtering error dynamic system (9) is exponentially stochastically stable and the fuzzy filter (8) is said to be the robust L_2-L_∞ fuzzy filter for system (4) with attenuation γ. The corresponding L_2-L_∞ fuzzy filter of the form (8) will be constructed by:

$$A_{fi}(r) = P_2^{-1}(r)\bar{A}_{fi}(r),\ B_{fi}(r) = P_2^{-1}(r)\bar{B}_{fi}(r),\ L_{fi}(r) = \bar{L}_{fi}(r) \tag{19}$$

Proof: For the further convenient analysis, we set

$$P(r) = diag\{P_1(r), P_2(r)\}, \bar{A}_{fi}(r) = P_2A_{fi}(r),$$
$$\bar{B}_{fi}(r) = P_2B_{fi}(r), \bar{L}_{fi}(r) = L_{fi}(r)$$

where $P_1(r)$, $P_2(r)$, $r \in M$ are mode-dependent symmetric positive-definite matrices. Substitute the above matrices into matrix inequalities (14) and (15), use some changes of variables, and recall following matrix inequality [23],

$$HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} \le \varepsilon^{-1}HH^{\mathrm{T}} + \varepsilon E^{\mathrm{T}}E$$

where $\varepsilon > 0$ is a real scalar and H, F, E are real matrices of appropriate dimensions with $\|F\| \le I$. Matrix inequalities (14) and (15) can be easily changed into the LMIs (17) and (18). Therefore, the L_2-L_∞ filtering problem can be solved by using MATLAB LMI Control Toolbox. This completes the proof.

Remark 3. For the further convenient analysis, mode-dependent matrix $P(r)$ is selected as a diagonal one, so it will bring some conservative performances. To overcome it, $P(r)$ can be selected as full order matrix, and the solution of fuzzy filter will be quite difficult and require the further research. On the other hand, Theorem 2 guarantees the positive-definite performance of $P(r)$, thus the L_2-L_∞ fuzzy filter parameters can be easily presented by solving LMIs (17) and (18).

Remark 4. To obtain an optimized L_2-L_∞ fuzzy filtering performance against unknown inputs and system uncertainties, the attenuation lever γ can be reduced to the minimum possible value such that LMIs (17) and (18) are satisfied. The LMIs optimization problem can be described as follows:

$$\begin{aligned} &\min_{\bar{A}_{fi}(r),\bar{B}_{fi}(r),\bar{L}_{fi}(r),P_1(r),P_2(r),Q,\rho} \rho \\ &s.t. \text{ LMIs (17) } and \text{ (18) } with\, \rho=\gamma^2 \end{aligned} \tag{20}$$

4. Numerical Example

Considering a class of nonlinear stochastic MJS described by T-S fuzzy model with time-delay and uncertainties parameters given by:

Subsystem 1: If $x_2(t)$ is F_1, then

$$\begin{aligned} \dot{x} &= [A_1(r)+M_1(r)F_1(r,t)N_1(r)]x+ \\ &[A_{h1}(r)+M_1(r)F_1(r,t)N_{h1}(r)]x_h+B_1(r)d \\ y &= C_1(r_t)x+D_1(r)d \\ z &= L_1(r)x \end{aligned}$$

where

$$A_1(1)=\begin{bmatrix} 2 & 4 \\ -4.5 & -5 \end{bmatrix}, A_1(2)=\begin{bmatrix} -5 & -1.6 \\ 2.1 & -4 \end{bmatrix},$$

$$A_{h1}(1)=\begin{bmatrix} -0.4 & 0.2 \\ -0.1 & -0.3 \end{bmatrix}, B_1(1)=\begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix},$$

$$A_{h1}(2)=\begin{bmatrix} -0.1 & 0.5 \\ 0.2 & -0.3 \end{bmatrix}, B_1(2)=\begin{bmatrix} 0.3 \\ -0.2 \end{bmatrix},$$

$$M_1(1)=M_1(2)=\begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$

$N_1(1)=N_1(2)=\begin{bmatrix} 0.2 & 0.1 \end{bmatrix}$,

$L_1(1)=L_1(2)=\begin{bmatrix} 0.1 & -0.3 \end{bmatrix}$,

$N_{h1}(1)=N_{h1}(2)=\begin{bmatrix} 0.1 & 0.2 \end{bmatrix}$,

$C_1(1)=C_1(2)=\begin{bmatrix} 0.2 & 0.3 \end{bmatrix}$,

$D_1(1)=D_1(2)=0.1$;

Subsystem 2: If $x_2(t)$ is F_2, then

$$\begin{aligned} \dot{x} &= [A_2(r)+M_2(r)F_2(r,t)N_2(r)]x+ \\ &[A_{h2}(r)+M_2(r)F_2(r,t)N_{h2}(r)]x_h+B_2(r)d \\ y &= C_2(r_t)x+D_2(r)d \\ z &= L_2(r)x \end{aligned}$$

where

$$A_2(1)=\begin{bmatrix} -2.5 & -0.8 \\ 0.5 & -3.2 \end{bmatrix}, A_2(2)=\begin{bmatrix} -5 & 1 \\ 2 & -7 \end{bmatrix},$$

$$A_{h2}(1)=\begin{bmatrix} -0.3 & 0.5 \\ -0.2 & -0.3 \end{bmatrix}, B_2(1)=\begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix},$$

$$A_{h2}(2)=\begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, B_2(2)=\begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix},$$

$$M_2(1)=M_2(2)=\begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},$$

$N_2(1)=N_2(2)=\begin{bmatrix} 0.1 & 0.2 \end{bmatrix}$,

$L_2(1)=L_2(2)=\begin{bmatrix} -0.1 & 0.2 \end{bmatrix}$,

$N_{h2}(1)=N_{h2}(2)=\begin{bmatrix} 0.1 & -0.2 \end{bmatrix}$,

$C_2(1)=C_2(2)=\begin{bmatrix} -0.2 & 0.2 \end{bmatrix}$,

$D_2(1)=D_2(2)=0.2$.

The membership functions are selected as

$$h_1(x_2(t))=\frac{-x_2(t)+3}{6},\ h_2(x_2(t))=\frac{x_2(t)+3}{6}$$

and the transition rate matrix is defined by

$$\Pi=\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}.$$

By solving the LMIs optimization problem (20), the L_2-L_∞ optimized value $\gamma=0.0536$, and the fuzzy filter matrices are respectively as follows:

$$A_{f1}(1)=\begin{bmatrix} 1.5270 & 3.1885 \\ -5.8188 & -6.7746 \end{bmatrix},$$

$$A_{f1}(2)=\begin{bmatrix} -6.0080 & -3.0694 \\ 2.4240 & -3.4464 \end{bmatrix},$$

$$B_{f1}(1)=\begin{bmatrix} 2.4858 \\ 4.8549 \end{bmatrix}, B_{f1}(2)=\begin{bmatrix} 5.0145 \\ -1.8132 \end{bmatrix},$$

$L_{f1}(1)=\begin{bmatrix} 0.0901 & -0.2800 \end{bmatrix}$,

$L_{f1}(2)=\begin{bmatrix} 0.0883 & -0.2977 \end{bmatrix}$,

$$A_{f2}(1)=\begin{bmatrix} -1.9640 & -0.8875 \\ -0.0878 & -4.1463 \end{bmatrix},$$

$$A_{f2}(2)=\begin{bmatrix} -5.1533 & 1.3868 \\ 2.1576 & -7.3695 \end{bmatrix},$$

$$B_{f2}(1)=\begin{bmatrix} 1.5549 \\ 0.7841 \end{bmatrix}, B_{f2}(2)=\begin{bmatrix} -0.9597 \\ 1.1310 \end{bmatrix},$$

$L_{f2}(1)=\begin{bmatrix} -0.0894 & 0.1879 \end{bmatrix}$,

$L_{f2}(2)=\begin{bmatrix} -0.0886 & 0.1996 \end{bmatrix}$.

Assume the unknown inputs are random bounded noise (altitude is from -0.3 to 0.3), the initial condition of states about the original system and filtering system are respectively x_1=x_{f1}=1.0 and x_2=x_{f2}=0.8. Thus, the simiple simulation results about jump mode, system states (real state and estimate state), and controlled output signals are shown in Fig.1-Fig.4.

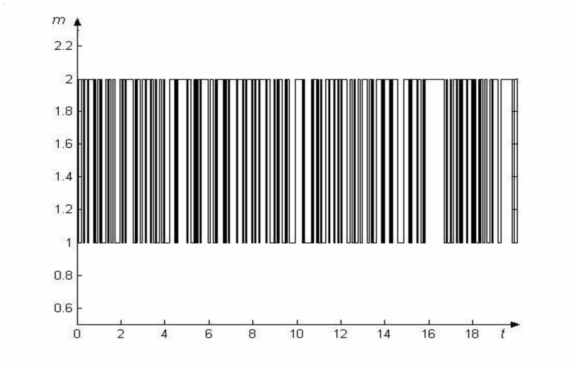

Fig. 1: Jump mode

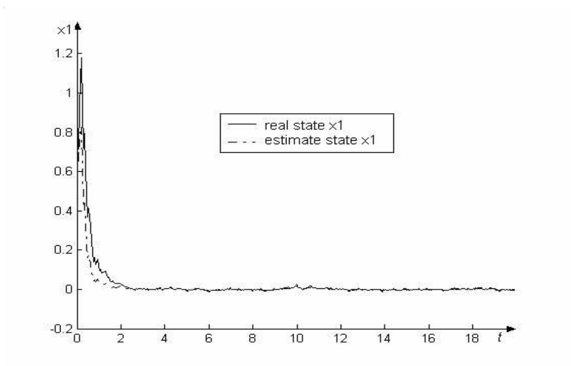

Fig. 2: System state x1

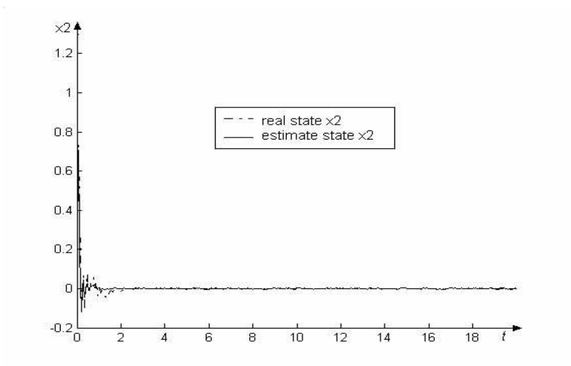

Fig. 3: System state x2

5. Conclusions

In the paper, we have addressed the design of L_2-L_∞ fuzzy filter for the nonlinear jump system with

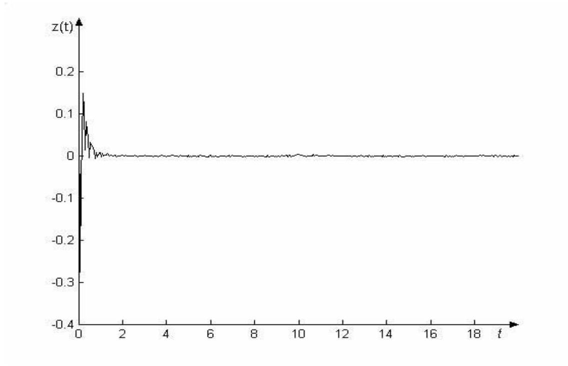

Fig. 4: Controlled output signals z(t)

time-delay and uncertainties. It ensures exponentially stochastically stable for the overall dynamic error system and a prescribed bound on the gain from the unknown noise to the estimation error. By selecting the appropriate Lyapunov-Krasovskii function and applying matrix transformation and variable substitution, the main results are provided in terms of LMIs form. Simulation example demonstrates the contribution of the main results.

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