An Improvement of Extend TOPSIS Method for Armament System of Systems Selection with Interval Data

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Abstract—**An Improvement of Extend TOPSIS (IE-TOPSIS) method is presented for Armament System of Systems (ASoS) selection. In this paper, ASoS selection problem is a multiple criteria decision-making problem. Because determining the specific value of the attribute for each ASoS is difficult, the criteria score is considered as interval vague value. The IE-TOPSIS method can deal with the interval data value score to determine the most preferable decision among all the choice. The weight is calculate with analysis of the implicit function behind the data, which is different of the classic TOPSIS method. Finally an illustrative example with interval data is shown for the Air-Defense ASoS selection problem.**

Keywords- extend topsis; multiple criteria decision-making; Armament System of Systems; air-defence ASoS

I. INTRODUCTION

Armament System of Systems (ASoS) is generally regarded as a kind of SoS composed by multiple armament systems, which are usually geographic distribution, operational and managerial independent. Just as SoS, ASoS has many complex features, such as nonlinearity, emergent behavior, multiple dependency, and that makes it really difficult to be evaluated and selected.[1] So ASoS Selection problem is often a multiple criteria decision-making problem of finding the best ASoS portfolio from all the feasible alternatives. And there are plenty of selection methods, such as classic ADC method, exponent method, Lanchester method[2], expert evaluation method, ANP method[3], VFT method[4][5], fuzzy analysis method[6]. These methods are based on a specific data set about the criteria of each candidates from the expert score or simulation value. However, due to the complexity and uncertainty involved in the real-world decision problems and incomplete information or knowledge, it is very difficult to provide a precise numerical value[7], usually only an interval data can be gave out. For example, human judgements including preferences are often vague and cannot estimate his preference with an exact numerical data, there for these data may be have some structures such as bounded data, ordinal data, interval data, and fuzzy data.

Recently researchers proposed some method handling the multiple criteria decision-making problem with the interval data set. Researcher Kress set up a method to calculate the approximate articulation of preference and priority deviation for inconsistent interval comparison matrices.[8] Merono-Jimenez analysis the distribution of possible ranking of alternatives in a small interval reciprocal comparison matrix.[9] And Haines proposed a statistic method for interval reciprocal comparison matrices.[10] And researcher G.R. Jahanshahloo proposed an algorithmic method to extend TOPSIS for decision-making problems with interval data.[11] The extend TOPSIS method proved to be an efficient method for this multiple criteria decision-making problem, however, the weight of the criteria was gave by experts, which is still a complex work even for experts. This paper give an improvement of extend TOPSIS method for ASoS Selection with interval data by calculate the weight from the interval comparison matrices. At present, the weights obtained methods from the interval data such as a fuzzy preference programming (FPP) method[12][13], a goal programming (GP) method[14]. Researcher Fang Liu gave an acceptable consistency analysis of interval reciprocal comparison matrix by judging the importance of different criteria by an extend method of classic AHP.[7]

In this paper, we proposed an improvement of extend TOPSIS (IE-TOPSIS) method to analysis the ASoS selection problem under a condition of decision makers provide an interval data matrix of each ASoS over different criteria. The rest of the paper is organized as follows: section 2 describes the main problem of ASoS selection in mathematics, section 3 proposed the improvement of extend TOPSIS method, section 4 validated with a general case, and conclusions are drawn in section 5. The IE-TOPSIS method will be discussed in chapter 3.2.

II. PROBLEM DESCRIPTION

ASoS selection problem can be described as a multiple criteria decision-making problem. And the symbols are defined as follows, let P_i be the ith portfolio (ASoS), C_i be the ith criteria of the index, W_i be the weight of the ith criteria, A be the interval data score matrix and the R_i be the rank of the ith portfolio.

In this paper, the decision-making problem is to choose the optimistic P_i from the feasible alternatives. The index of evaluation can be described as the vector $C = (C_1, C_2, ..., C_n)$. Here we transform all the criteria to benefit criteria to make it easier understanding . And the weight of the criteria is $W = (W_1, W_2, ..., W_n)$. The score value is presented in Interval data score matrix A.

$$A = \begin{bmatrix} [x_{11}^L, x_{11}^U] & [x_{12}^L, x_{12}^U] & \cdots & [x_{1m}^L, x_{1m}^U] \\ \vdots & \vdots & & \vdots \\ [x_{n1}^L, x_{n1}^U] & [x_{n2}^L, x_{n2}^U] & \cdots & [x_{nm}^L, x_{nm}^U] \end{bmatrix} ,$$

Where the lower value can be equal with the upper value.

Then calculate the rank of each portfolio and make your decision to choose the best portfolio.

III. METHODOLOGY

The IE-TOPSIS method is here to solve this ASoS selection problem. Before illustrating the IE-TOPSIS method, TOPSIS and Extend TOPSIS method will be discussed.

A. TOPSIS Method & Extend TOPSIS method

TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang[15], with reference to Hwang and Yoon[16]. TOPSIS is a multiple criteria method to identify solutions from a fi-nite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. And the procedure of TOPSIS can be referenced at the related articles.

Extend TOPSIS method[11] is proposed to handle the internal data. It gets the negative ideal solution from the lower data matrix of Interval data score matrix A. And the upper data to be the positive ideal solution. Then the specific weights gave by experts in the classic TOPSIS method. The detail of the method can be found at [11].

B. IE-TOPSIS method

IE-TOPSIS is a new method to develop TOPSIS method in multiple criteria decision-making problem with interval data sore matrix as

	C_1	C_2	$\dots$	C_m
P_1	$[x_{11}^L, x_{11}^U]$	$[x_{12}^L, x_{12}^U]$	$\dots$	$[x_{1m}^L, x_{1m}^U]$
P_2	$[x_{21}^L, x_{21}^U]$	$[x_{22}^L, x_{22}^U]$	$\dots$	$[x_{2m}^L, x_{2m}^U]$
P_n	$[x_{n1}^L, x_{n1}^U]$	$[x_{n2}^L, x_{n2}^U]$	$\dots$	$[x_{nm}^L, x_{nm}^U]$

The IE-TOPSIS method is executed in the following steps.

Step 1. Data Normalization Matrix

For the value of different criteria usually have different measure scale, the data matrix should be normalized[11] at first. Then we noted the normalized matrix N.

$$N = \begin{bmatrix} [n_{11}^L, n_{11}^U] & [n_{12}^L, n_{12}^U] & \cdots & [n_{1m}^L, n_{1m}^U] \\ \vdots & \vdots & & \vdots \\ [n_{n1}^L, n_{n1}^U] & [n_{n2}^L, n_{n2}^U] & \cdots & [n_{nm}^L, n_{nm}^U] \end{bmatrix}$$

$$n_{ij}^L = \frac{x_{ij}^L}{\sqrt{\sum_{j=1}^n [(x_{ij}^L)^2 + (x_{ij}^U)^2]}}, \ i = 1,2,...,m; \ j = 1,2,...n$$

$$n_{ij}^U = \frac{x_{ij}^U}{\sqrt{\sum_{j=1}^n [(x_{ij}^L)^2 + (x_{ij}^U)^2]}}, \ i = 1,2,...,m; \ j = 1,2,...n$$

(1)

Step 2. Criteria Comparison Matrix

In order to find a weight to distinguish the portfolios influentially from the implicit function behind the data, we can get the comparison matrices of the criteria importance with the normalized matrix N as follows.

	C_1	C_2	$\dots$	C_m
C_1	1	$P(C_1 \ge C_2)$	$\dots$	$P(C_1 \ge C_m)$
C_2	$P(C_1 \le C_2)$	1	$\dots$	$P(C_2 \ge C_m)$
C_m	$P(C_1 \le C_m)$	$P(C_2 \le C_m)$	$\dots$	1

Assuming the normalized criteria has the same measure scale and the same distribution. Because we have little information of the data, the distribution can be regarded as uniform distribution.[17][18] Consider only one portfolio, if we make it score value as the importance value of this criteria, then the weight of each criteria will be positive correlation, and the aggregation result will be the optimistic maximum for this specific portfolio. And if get the different weight of each portfolio, then every weight combined the value to optimistic maximum of it's portfolio, so the mean weight of different portfolio can show the total information of all the portfolios. This kind of set weight method is similarly as rotate all the dimensions to find a most distinguishable dimension combination. The specific procedure is as follows.

The comparison of C_i and C_j from one portfolio normalized interval data ($P(C_i \ge C_j)$) can be calculated as areas of s_1 and s_2 in figure 1.[19]
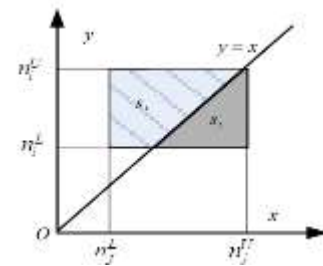

Figure 1. Criteria comparison from a portfolio normalized interval data

So the possibility of $C_i \geq C_j$ is

$$P(C_i \geq C_j) = \begin{cases} 1, & n_i^L \geq n_j^U \\ \dfrac{Area(S_1)}{Area(S_1)+Area(S_2)}, & others \\ 0, & n_i^U \leq n_j^L \end{cases} \quad (2)$$

That is

$$P(C_i \geq C_j) = \begin{cases} 1, & n_i^L \geq n_j^U \\ \dfrac{(2n_i^U - n_j^U - n_j^L)}{2(n_i^U - n_i^L)}, & n_i^L \leq n_j^L \leq n_j^U \leq n_i^U \\ \dfrac{(n_i^U - n_j^L)^2}{2(n_i^U - n_i^L)(n_j^U - n_j^L)}, & n_i^L \leq n_j^L \leq n_i^U \leq n_j^U \\ 1 - \dfrac{(n_j^U - n_i^L)^2}{2(n_j^U - n_i^L)(n_j^U - n_j^L)}, & n_j^L \leq n_i^L \leq n_j^U \leq n_i^U \\ \dfrac{(n_i^L + n_i^U - 2n_j^L)}{2(n_j^U - n_j^L)}, & n_j^L \leq n_i^L \leq n_i^U \leq n_j^U \\ 0, & n_i^U \leq n_j^L \end{cases}$$

$$(3)$$

And,

$$P(C_i \leq C_j) = 1 - P(C_i \geq C_j) \quad (4)$$

Then the criteria comparison matrix noted as CM_p of one portfolio P can be calculated from $P(C_i \geq C_j)$. And the change mapping ($P(C_i \geq C_j)$ to CM_{ij}) is shown in the following the picture.
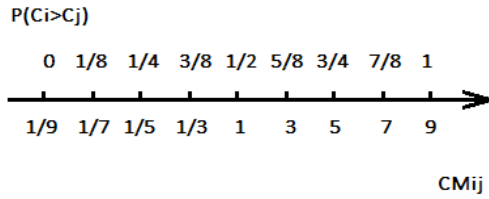

Figure 2.   Mapping criterion

Then we can get the CM_p matrix.

$$CM_p = \begin{bmatrix} 1 & cm_{12}^P & \cdots & cm_{1m}^P \\ cm_{21}^P & 1 & \cdots & cm_{2m}^P \\ & & & \\ cm_{m1}^P & cm_{m2}^P & \cdots & 1 \end{bmatrix} \quad (4)$$

Step 3. Consistent Analysis & Compute the Weight

After getting the CM matrix of all the portfolios, it is necessary to confer whether the comparison judgment matrix is consistent. Like classic AHP method, the comparison matrix can be seen as the relative importance of each criteria, so the consistent analysis can be processed as follows, which is initially set up by Saaty [20][21].

When the matrix CM is m by m dimension, the level of consistency can be measured by a CI and CR method[20]. That is,

$$CI = \frac{\lambda_{max} - m}{m-1}, \quad CR = CI/RI \quad (5)$$

Where λ_{max} is the largest eigenvalue of matrix CM, and m is dimension of CM. RI is a random index, which is the average CI of a large number of randomly generated simulations, and the value are gave out in Table 2.

TABLE I.   THE MEAN CONSISTENCY INDEX (CI)

m	1	2	3	4	5	6	7	8	9
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46

For CM is usually a positive matrix, λ_{max} the largest eigenvalue can be calculated by formula (6).[22]-[24]

$$\lambda_{max} = \min_{v \in V} \max_i \sum_{j=1}^{m} cm_{ij} \frac{v_j}{v_i}, i = 1,2,...,m,$$

$$where\, V = \{(v_1, v_2,...,v_m)/v_i > 0, i = 1,2,...,m\}$$

$$(6)$$

When CR<0.10, the comparison matrix is considered to be acceptably consistent. While CR>0.10, the CM is said to be of unacceptable consistency, which should be adjusted to that with acceptable consistency to ensure the rationality of decisions.

The weight of can be computed in root method.

$$w_i^P = \frac{\left(\prod_{j=1}^{n} cm_{ij}^P\right)^{\frac{1}{n}}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} cm_{ij}^P\right)^{\frac{1}{n}}} \quad (7)$$

Then the weight of each criteria is

$$w_i = \sum_{i=1}^{n} w_i^P / n = \frac{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} cm_{ij}^P\right)^{\frac{1}{n}}}{n\sum_{i=1}^{n}\left(\prod_{j=1}^{n} cm_{ij}^P\right)^{\frac{1}{n}}} \quad (8)$$

Step 4. IE-TOPSIS Calculation

In the same procedure of classic TOPSIS, IE-TOPSIS calculation is list as follows.

Considering the difference importance of each criteria. Calculate the weighted normalized interval value matrix V.

$$V = \begin{bmatrix} [v_{11}^L, v_{11}^U] & [v_{12}^L, v_{12}^U] & \cdots & [v_{1m}^L, v_{1m}^U] \\ \vdots & \vdots & & \vdots \\ [v_{n1}^L, v_{n1}^U] & [v_{n2}^L, v_{n2}^U] & \cdots & [v_{nm}^L, v_{nm}^U] \end{bmatrix}.$$

And,

$$v_{ij}^L = w_i n_{ij}^L, \quad i = 1,2,...,m; \ j = 1,2,...n$$

$$v_{ij}^U = w_i n_{ij}^U, \quad i = 1,2,...,m; \ j = 1,2,...n \quad (9)$$

Then calculate the positive ideal solution and negative ideal solution as, because we assumed all the criteria have been transformed into benefit criteria.

$$I^{positive} = \{v_1^{positive}, v_2^{positive}, ..., v_m^{positive}\} = \{\max_i(v_{ij}^U), j = 1,2,...,m\}$$

$$I^{negative} = \{v_1^{negative}, v_2^{negative}, ..., v_m^{negative}\} = \{\min_i(v_{ij}^L), j = 1,2,...,m\}$$

$$(10)$$

Then the separation of each alternative from the positive and negative ideal solution can be scaled by n-dimension Euclidean distance.

$$D_j^{positive} = \sum_{j=1}^{n}(I^{positive} - v_{ij}^U)^2$$

$$D_j^{negative} = \sum_{j=1}^{n}(I^{negative} - v_{ij}^L)^2 \quad (11)$$

Then

$$D_j^* = \frac{D_j^{negative}}{D_j^{positive} + D_j^{negative}} \quad (12)$$

Finally, sort these D_j^* to get the rank R_i of each portfolio. Select the top rank portfolio as the best ASoS selection.

IV. CASE STUDY

The method proposed in this paper can make decision of the ASoS selection problem. Here is a numerical sample to illustrate it.

A. Case Description

To illustrate the IE-TOPSIS method with multiple criteria decision-making problem, we presented a numerical experiment on Air-Defense weapon system of systems as shown in figure 2, 15 candidate ASoS portfolios, 4 criteria with lower and upper boundary interval data are considered.
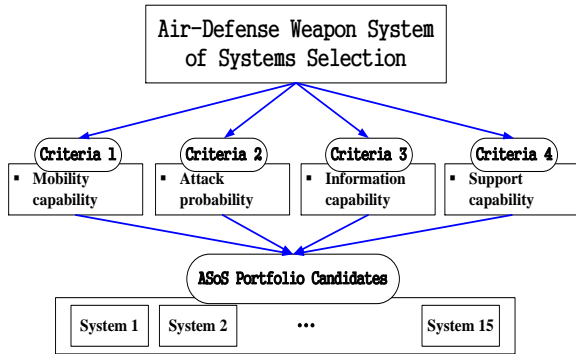

Figure 3. Air-Defense ASoS portfolio selection

The interval data in table 3 of this numerical experiment was referenced at al[11].

TABLE II.　　THE INTERVAL DATA

	C1	C2	C3	C4
	$[x_{i1}^L, x_{i1}^U]$	$[x_{i2}^L, x_{i2}^U]$	$[x_{i3}^L, x_{i3}^U]$	$[x_{i4}^L, x_{i4}^U]$
P1	[500.37,961.37]	[26969953,126798]	[2636438,254]	[965.97 6957.33]
P2	[873.70,1775.50]	[1027546,1061260]	[3791,50308]	[2285.03,3174.00]
P3	[95.93,196.39]	[1145235,1213541]	[22964,26846]	[207.98,510.93]
P4	[848.07,1752.60]	[390902,395241]	[492,1213]	[63.32,92.30]
P5	[58.69,120.47]	[144906,165818]	[18053,18061]	[176.58,370.81]
P6	[464.39,955.61]	[408163,416416]	[40539,48643]	[4654.71,5882.53]
P7	[155.29,342.89]	[335070,410427]	[33797,44933]	[560.26,2506.67]
P8	[1752.31,3629.50]	[700842,768593]	[1437,1519]	[58.89,86.86]
P9	[244.34,495.78]	[641680,696338]	[11418,24108]	[1070.81,2283.08]
P10	[730.27,1417.10]	[453170,481943]	[2719,2955]	[375.07,559.85]
P11	[454.75,931.24]	[309670,342598]	[2016,2617]	[936.62,1468.45]
P12	[303.58,630.01]	[286149,317186]	[14918,27070]	[1203.79,4335.24]
P13	[658.81,1345.50]	[321435,347848]	[6616,8045]	[200.36,399.80]
P14	[420.18,860.79]	[618105,835839]	[24425,40457]	[2781.24,4555.42]
P15	[144.68,292.15]	[119948,120208]	[1494,1749]	[282.73,471.22]

B. Result

Based on the EI-TOPSIS method described in chapter 3, the weight calculated result is bellow.

$$w = (\ 0.2819, 0.2487, 0.2414, 0.2280\ )$$

And the final result is shown in table 4.

TABLE III.　　THE EI-TOPSIS METHOD RESULT

	$D_j^{positive}$	$D_j^{negative}$	D_j^*	R_i
P1	0.0294	0.0107	0.2659	8
P2	0.0238	0.0137	0.3651	4
P3	0.0342	0.0075	0.1804	2
P4	0.0239	0.0149	0.3838	10
P5	0.0351	0.0072	0.1704	13
P6	0.0304	0.0070	0.1880	1
P7	0.0336	0.0059	0.1483	11
P8	0.0140	0.0282	0.6679	6
P9	0.0324	0.0061	0.1587	3
P10	0.0258	0.0125	0.3263	14
P11	0.0294	0.0087	0.2284	15
P12	0.0319	0.0059	0.1563	5
P13	0.0266	0.0121	0.3132	9
P14	0.0303	0.0066	0.1796	12
P15	0.0339	0.0072	0.1756	7

V. CONCLUSION

An IE-TOPSIS method to analysis the ASoS selection problem in this paper is more feasible to the real-world multiple criteria decision-making problems. Especially efficient to the interval data value score caused by the complexity and uncertainty in it.

The advantages of this method is that it can handle the interval data as well as the specific value, it needn't to specified the weight of each criteria, the weight calculation method is also feasible to small sample. And the procedural of the method is very clear and easy for computer programming. Besides the EI-TOPSIS method is also need to be feather study on getting the weight from the implicit function behind the data.

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