Application of Markov prediction method in the decision of insurance company

Yuzhen Lu
Dalian Maritime University
Dalian, China
e-mail: dlyuzhen@dlmu.edu.cn

Ting Yu
Dalian Maritime University
Dalian, China
e-mail: mengxiere@163.com

Min Zhang*
Dalian Maritime University
Dalian, China
e-mail: zhangmin071@sina.com

Minghui Qu
Dalian Maritime University
Dalian, China
e-mail: 984114859@163.com

Abstract—This paper studies the theory of Markov knowledge, what's more, it surveys and analysis the market share. Then for three specific companies, the paper builds a model using Markov theory based on companies of insurance market. Through the model, it analysis several possible circumstances and puts forward the way to deal with them. Furthermore, it predicts and analyzes the distribution of market shares in the later period, which provides references for companies.

Keywords-Markov; prediction; distribution; market; share

I. INTRODUCTION

With the improvement of people's living standards, more and more people choose to insure all kinds of things. Therefore, it's necessary for company to attract more customers to increase the market share, and it's sensible for company to accurately know about the market conditions and access its competitiveness. Markov prediction method is an effective way to analyze the current situation and make predictions the future. Since Markov, the Russian mathematician put forward the Markov Chain, Markov Chain Theory and Methods are widely used in various fields. Especially with rapid economic development, Markov prediction method is more widespread applied in economic analysis of the market. In 2002, LI Zhenfeng and JI Ling[1]study the application of System Situation Transition Probability Matrix in Transportation Market, and analyze the market share in the future. In 2004, CHEN Jike[2]et al. describe Markov Chain in economic management and applied it in the prediction of market share. In 2011, JIN Shengfu[3]use State Transfer Probability Matrix of Markov Model to predict the market share based on the "Black Box Theory". In the same year, XU Lingling[4][5][6][7][8][9][10] et al. also research into the theory of Markov and using it to give advice to actual problems. This paper analyzes the current situation based on the Markov theory, and predicts the company's market share in the future, thus providing reference and guidance for the company to open market.

II. BASIC THEORY OF MARKOV

A. Markov Chain

Markov process refers to the state of system transferring from one to another. It will be called a Markov Chain if its time and state is discrete. Markov chain should satisfy the following conditions: the state of system is only related to the current situation, it has nothing to do with the state before. As time goes on, the system is gradually getting close to the stable state, and has nothing to do with the initial state.

B. Transferring probability matrix

Assuming that the current state of the system is i, the probability of transferring to the state j by one step is P_{ij}, then the one-step transition probability matrix can be expressed as

$$p = \begin{pmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{pmatrix} \quad (1)$$

And it should be satisfied

$$p_{ij} \geq 0, \sum_{j=1}^{n} p_{ij} = 1(i, j = 1, 2, \cdots n)$$

If the system transfers from state i to state j by n steps, then the n-steps transition probability matrix can be expressed as

$$p^{(n)} = \begin{pmatrix} p^{(n)}{}_{11} & \cdots & p^{(n)}{}_{1n} \\ \vdots & \ddots & \vdots \\ p^{(n)}{}_{n1} & \cdots & p^{(n)}{}_{nn} \end{pmatrix} \quad (2)$$

It can be proved that n-steps state transition matrix can be expressed as one-step transition probability matrix's power of n, namely $p^{(n)} = p^n$.

C. Markov forecasting model

Set the initial probability distribution is q_0 ,one-step transition probability matrix is p , the next period distribution could be predicted as $s = q_0 p$,the distribution after k period should be expressed as

$$s^{(k)} = s^{(k-1)} p = q_0 p^{(k-1)} \quad (3)$$

From the above formula we can know that the market share of any period in the later can be predicted if one step transition probability matrix p and the initial share q_0 is known, thus it can provide references for the company if taking steps or not.

And if one-step transition probability matrix is known, the distribution of the market after a certain period can be known through the formula

$$\begin{cases} (p_1^{'}, p_2^{'}, \cdots p_n^{'}) p = (p_1^{'}, p_2^{'}, \cdots p_n^{'}) \\ p_1^{'} + p_2^{'} + \cdots p_n^{'} = 1 \end{cases} \quad (4)$$

III. Application of Markov model

Supposing there are three insurance companies on the market, namely A、B、C, and the total number of customers is S（$S = S_A + S_B + S_B$）,of whom S_A is the number of company A , S_B is the number of company B ,and S_C is the number of company C .After one period, the number of each company's customers is changed, that is, each company's market share has changed mainly for the following situation.

A. There aren't new customers getting into or old customers exiting the insurance market

It's namely the total number of customers has not changed. The customers have a certain flow among the three companies. By calculating gains and losses rate and the retention rate, one-step transition probability matrix can be obtained and expressed by

$$p = \begin{pmatrix} \frac{s_{AA}}{s_A} & \frac{s_{AB}}{s_A} & \frac{s_{AC}}{s_A} \\ \frac{s_{BA}}{s_B} & \frac{s_{BB}}{s_B} & \frac{s_{BC}}{s_B} \\ \frac{s_{CA}}{s_C} & \frac{s_{CB}}{s_C} & \frac{s_{CC}}{s_C} \end{pmatrix}$$

Where the rows of the matrix indicate the probability of a company losing customers to other companies, the columns of the matrix can be expressed as a probability of a company getting customers from other companies. For example, $\frac{S_{AA}}{S_A}$ represent the retention rate of company A, $\frac{S_{AB}}{S_A}$ represent the probability of customers transferring from company A to B, $\frac{S_{BA}}{S_B}$ represent the probability of company B getting customers form company A.After one step transition, each share of company can be expressed as

$$q_1 = \left(\frac{s_A + S_{BA} + S_{CA} - S_{AB} - S_{AC}}{S}, \frac{S_B + S_{AB} + S_{CB} - S_{BA} - S_{BC}}{S}, \frac{S_C + S_{AC} + S_{BC} - S_{CA} - S_{CB}}{S}\right).$$

If no company takes steps to change the current situation, namely one-step transition probability matrix is unchanged, then the market share after k period can be predicted through the initial probability matrix and one-step transition probability matrix namely $p^{(k)} = q_1 p^{k-1}$ .Assuming that after a certain period, the distribution of the market reached a stable state, $p_A^{'}, p_B^{'}, p_c^{'}$ is respectively the market share of each company, and $p_A^{'}, p_B^{'}, p_c^{'}$ can be obtained by the formula

$$\begin{cases} (p_A^{'}, p_B^{'}, p_c^{'}) p = (p_A^{'}, p_B^{'}, p_c^{'}) \\ p_A^{'} + p_B^{'} + p_c^{'} = 1 \end{cases} \quad (5)$$

B. There are new customers getting into the insurance market (no old customers withdraw the market)

Assuming 3m customers have access to insurance markets, and they are shared by the three companies equally, namely, each company increase m customers, then the one-step transition probability is

$$p=\begin{pmatrix}\frac{s_{AA}+m}{s_A+m} & \frac{s_{AB}}{s_A+m} & \frac{s_{AC}}{s_A+m}\\ \frac{s_{BA}}{s_B+m} & \frac{s_{BB}+m}{s_B+m} & \frac{s_{BC}}{s_B+m}\\ \frac{s_{CA}}{s_C+m} & \frac{s_{CB}}{s_C+m} & \frac{s_{CC}+m}{s_C+m}\end{pmatrix} \quad (6)$$

Each company's market share is

$$q_1=(\frac{S_A+S_{BA}+S_{CA}-S_{AB}-S_{AC}+m}{S+3m},$$

$$\frac{S_B+S_{AB}+S_{CB}-S_{BA}-S_{BC}+m}{S+3m},$$

$$\frac{S_C+S_{AC}+S_{BC}-S_{CA}-S_{CB}+m}{S+3m})$$

If the situation remains the same, then the market share after k period can be predicted by the equation

$$p^{(k)}=q_1p^{k-1}, \quad (7)$$

similarly, after some period the market is in a stable state, the market share at that time also can be predicted by the formula

$$\begin{cases}(p_A^{'},p_B^{'},p_c^{'})p=(p_A^{'},p_B^{'},p_c^{'})\\ p_A^{'}+p_B^{'}+p_c^{'}=1\end{cases}$$

This period the number of customers doesn't increase in the same proportions, it's a promotion of the first case. Assuming the number of increasing customers is respectively m_1, m_2, m_3 ,the one-step transition probability matrix can be expresses by

$$p=\begin{pmatrix}\frac{s_{AA}+m_1}{s_A+m_1} & \frac{s_{AB}}{s_A+m_1} & \frac{s_{AC}}{s_A+m_1}\\ \frac{s_{BA}}{s_B+m_2} & \frac{s_{BB}+m_2}{s_B+m_2} & \frac{s_{BC}}{s_B+m_2}\\ \frac{s_{CA}}{s_B+m_3} & \frac{s_{CB}}{s_B+m_3} & \frac{s_{CC}+m_3}{s_B+m_3}\end{pmatrix}$$

The market share is

$$q_1=(\frac{S_A+S_{BA}+S_{CA}-S_{AB}-S_{AC}+m_1}{S+m_1+m_2+m_3},$$

$$\frac{S_B+S_{AB}+S_{CB}-S_{BA}-S_{BC}+m_2}{S+m_1+m_2+m_3},$$

$$\frac{S_C+S_{AC}+S_{BC}-S_{CA}-S_{CB}+m_3}{S+m_1+m_2+m_3})$$

Similarly, the market share after k period and the stable period can be predicted.

The previous two cases basically sum up the general situation, they take three companies for examples, while, the method doesn't confine to three examples, it can be extended to more than one instance.

IV. APPLICATION IN EXAMPLES

Three companies flow from Jan.2011 to Jan.2012 is shown in the following TABLE I.

Then we can obtain the reserve rate and share of each company in TABLE II.

Then the one-step transition probability can be expressed by

$$p=\begin{pmatrix}160/200 & 20/200 & 20/200\\ 35/500 & 450/500 & 15/500\\ 25/300 & 20/300 & 255/300\end{pmatrix}$$

And through the formula

$$p^{(k)}=q_1p^{k-1}=(0.22,0.49,0.29)\begin{pmatrix}160/200 & 20/200 & 20/200\\ 35/500 & 450/500 & 15/500\\ 25/300 & 20/300 & 255/300\end{pmatrix}^{k-1}$$

The share after k period can be calculated. Assuming the share of each company is separately $p_A^{'},p_B^{'},p_c^{'}$ when the market reaches a steady state, through the formula

$$\begin{cases}(p_A^{'},p_B^{'},p_c^{'})p=(p_A^{'},p_B^{'},p_c^{'})\\ p_A^{'}+p_B^{'}+p_c^{'}=1\end{cases}$$

We can obtain the values of $p_A^{'},p_B^{'},p_c^{'}$:

$$p_1^{'}=\frac{250}{981},\ p_2^{'}=\frac{793}{1667},\ p_3^{'}=\frac{260}{981}$$

If there is company who isn't satisfied with the predicted results and wanting to improve the market share, then it may face the following conditions.

TABLE I. NUMBER OF CUSTOMERS(UNIT: 10 THOUSAND)

company		A	B	C
Jan.2011		200	500	300
From other companies	From A	0	20	20
	From B	35	0	15
	From C	25	20	0
To other companies	To A	0	35	25
	To B	20	0	20
	To C	20	15	0
Jan.2012		220	490	290

TABLE II. RATE AND SHARE

company	A	B	C
Jan.2011	200	500	300
lost customers	40	50	45
reserved customers	160	450	255
reserved rate of 2012	0.8	0.9	0.85
share	0.22	0.49	0.29

A. The total number of customers remains the same.

The company who wants to improve its share should keep its customers and getting customers from other companies at the same time, take company A for example. Assuming the reserving rate of company A increase x scale, getting y proportion of customers of company B, z of company C, $0 \le x, y, z \le 1$. The one-step transition probability matrix is

$$p = \begin{pmatrix} 0.8+x & 0.1+y & 0.1+z \\ 0.07 & 0.09 & 0.03 \\ 0.083 & 0.067 & 0.85 \end{pmatrix}$$

Market share is

$$q_1 = \left(\frac{220+m}{1000+3m}, \frac{490+m}{1000+3m}, \frac{290+m}{1000+3m}\right)$$

When the market reaches a stable situation, the share of company A can be calculated and the value is

$$p_1 = \frac{250}{3610y+3700z+1231}$$

$$\left(p_2 = \frac{9000y+5000z+1400}{10830y+11100z+3693}, p_3 = \frac{600y+2000z+260}{3610y+3700z+1231}\right)$$

By knowing the value of y and z, A could know whether taking steps or not to change its situation in the future.

B. There have new customers getting into the insurance market and no old customers exiting.

There are $3m$ customers into the market, each company increases m, then the one-step transition probability matrix and share is separately

$$p = \begin{pmatrix} \frac{160+m}{200+m} & \frac{20}{200+m} & \frac{20}{200+m} \\ \frac{35}{500+m} & \frac{450+m}{500+m} & \frac{15}{500+m} \\ \frac{25}{300+m} & \frac{20}{300+m} & \frac{255+m}{300+m} \end{pmatrix}, q_1 = \left(\frac{220+m}{s+3m}, \frac{490+m}{s+3m}, \frac{290+m}{s+3m}\right)$$

Equally, the distribution of insurance market after k period can be calculated by $p^{(k)} = q_0 p^k$, the share in the stable situation can be obtained by

$$\begin{cases} (p_1, p_2, p_3)p = (p_1, p_2, p_3) \\ p_1 + p_2 + p_3 = 1 \end{cases}$$

Assuming the quantities of increasing customers of three companies is separately m_1, m_2, m_3,
Then the one-step transition probability matrix is

$$p = \begin{pmatrix} \frac{160+m_1}{200+m_1} & \frac{20}{200+m_1} & \frac{20}{200+m_1} \\ \frac{35}{500+m_2} & \frac{450+m_1}{500+m_2} & \frac{15}{500+m_2} \\ \frac{25}{300+m_3} & \frac{20}{300+m_3} & \frac{255+m_3}{300+m_3} \end{pmatrix}$$

Then the distribution and share in the stable market can be predicted.

V. CONCLUSION

This paper built a model using Markov theory with companies of insurance market for example. Furthermore, it predicted and analyzed the distribution of market shares in the later period, which provided references for companies. Of course, the application of Markov doesn't confine to what's in this paper, it's greatly used in many other situations.

REFERENCES

[1] LI Zhenfeng, JI ling. Applications of System Situation Transition Probability Matrix in Transportation Market [J]. Journal of Tongji University.Aug.2002.the 8th issue of volume:955-958

[2] CHEN Jieke, XIA Hongsheng. Application of statistics in the economic management-Forecast and Analysis on Markov Chain market share[J].Commercial modernization.2004(19):23-24

[3] JIN Shengfu. State Transfer Probability Matrix of Markov Model to predict the market share based on the "Black Box Theory"[J]. Enterprise management: the productivity study.Aug.2008:175-177

[4] XU Lingling. Enterprise Market share prediction based on Markov decision method [J].Commercial age. The 9th period of 20：91-92

[5] GAO zhiwei. Application of Markov in the pharmaceutical market share analysis. China and Foreign Investment(Second Half)：2011（12）：142-142

[6] SHEN Jinhui. Application of Markov method in the prediction of market share. Journal of Chifeng University(ScienceEducation) 2011,11.Volume 3rd issue 11th:19-21

[7] LI Chunqing, YUAN Liang, MA Junping. Prediction of customer's behavior of bank based on Markov Chain probability matrix. Journal of Xi'an Technological University. Nov. 2011.Volume 31st issue 6th:554-558

[8] LI Yunxia. The forecast to the market share and development of Markov model, J. Statistics and Decision, Vol.12 (2009) pp.180-181.

[9] DONG Jianguo, HUI Shurong, Chen Zhongwei. Application of Markov Chain Model to Mathematics Education Evaluation, J. Journal of Shenyang Agricultural University (Social Sciences Edition)，Vol.12(2010).

[10] ZHANG Eryan, ZHU Xiaofeng. Markov Chain Model For the Structure of the Traditional Publishing Industry in China. International Conference on Education Technology and Management Science:1175-1178(2013).