

Ruin Problems Under the Renewal Risk Model with Interest Force

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Abstract—this paper discusses ruin problems in the renewal risk model with interest force. By using inductive method, the recursive expressions of the distribution of the ruin probability, the distribution of maximum surplus before the ruin and distribution of minimum surplus before the ruin are obtained, and the corresponding integral equation for the distributions are obtained.

Keywords—risk model; Interest force; renewal risk model; ruin probability; maximum surplus; minimum surplus

I. INTRODUCTION

In recent years, the classic risk model has received a remarkable amount of attention and there have been many generalizations. Sundt and Teugels (1995,1997) considered a compound Poisson model with a constant interest force, by using renewal techniques, upper and lower bounds for the ruin probability and the integral equation satisfied by the ruin probability were obtained. Yang and Zhang (2001a, 2001b, 2001c) used the techniques of Sundt and Teugels (1995), some related problems were obtained. Yang (1998) considered a discrete time risk model with a constant interest force, both Lundberg-type inequality and non-exponential upper bounds for ruin probabilities were obtained by using martingale inequalities. Renewal risk model with interest force as a generalization of the classic risk model was considered in Wu and Du (2002), the problem was translated discrete, then used Markov property and transition probability to derive the explicit expression for the ruin probability. Lin and Wang (2005) adopted a different discrete techniques, derived the distribution of surplus immediately before ruin and that of deficit at ruin, further the integral equations of these distributions were obtained.

In this paper, we consider renewal risk model with interest force, by using the techniques of Lin and Wang (2005), the ruin probability, the minimum surplus before the ruin, the maximum surplus before the ruin and its corresponding integral equations for those distributions are obtained.

The paper is organized as follows: we definition of the model in section II; then, in section III, IV, V, we discuss the distribution of the ruin probability, the minimum surplus before the ruin, the maximum surplus before the ruin, the recursive expressions and integral

equation for the distributions are obtained; finally conclude is in section VI.

II. DEFINITION OF THE MODEL

Let (Ω, F, P) be a complete probability space. We consider the renewal risk model with interest force. Suppose $S(t)$ denote the amount of claim in the time

interval $(0, t]$, i.e. $S(t) = \sum_{i=1}^{N(t)} X_i$, where $\{X_i, i \geq 1\}$ is

independent and identically distributed (i.i.d.) random variables with common distribution function $F(x)$, denotes the amount of the i th claim. The counting process $\{N(t), t \geq 0\}$ denotes the number of claims up to time t and is defined as $N(t) = \max\{k : W_1 + W_2 + \dots + W_k \leq t\}$, where the inter-claim times $\{W_i, i \geq 1\}$ are assumed to be i.i.d. random variables with common distribution function $K(w)$. Further, we assume the sequences $\{W_i, i \geq 1\}$ and $\{X_i, i \geq 1\}$ are independent, and that $cE(W_1) > E(X_1)$, providing a positive safety loading factor.

Let $U_\delta(t)$ denotes the insurance company's surplus at time t . From the above assumption, it follows that

$$dU_\delta(t) = cdt + U_\delta(t)\delta dt - dS(t) \quad (1)$$

From (1), we know that

$$U_\delta(t) = ue^{\delta t} + c\bar{S}_{t|} - \int_0^t e^{\delta(t-v)} dS(v) \quad (2)$$

where

$$\bar{S}_{t|} = \int_0^t e^{\delta v} dv = \begin{cases} t & \text{if } \delta = 0 \\ \frac{e^{\delta t} - 1}{\delta} & \text{if } \delta > 0 \end{cases}$$

$u > 0$ is initial surplus of insurance company, $c > 0$ is the premium income of unit time, δ is constant interest force.

Definition 1

If $T = \inf\{t > 0 : U_\delta(t) < 0\}$ ($T = \infty$ if the set is empty), T is the ruin time. Obviously, it's a stopping time.

Definition 2

Let $\Psi_\delta(u)$ denote the ultimate ruin probability with initial reserve u . That is

$$\Psi_\delta(u) = P\{\cup_{t \geq 0} (U_\delta(t) < 0) \mid U_\delta(0) = u\} \quad (3)$$

III. THE RUIN PROBABILITY

Let T_n denote the time of the n th claim happening,

i.e. $T_n = \sum_{i=1}^n W_i$. By (2), we have

$$U_\delta(t) = ue^{\delta t} + c \frac{e^{\delta t} - 1}{\delta} - \sum_{i=1}^n X_i e^{\delta(t-T_i)}$$

when $t = T_n$, we have

$$\begin{aligned} U_\delta(T_n) &= ue^{\delta T_n} + c \frac{e^{\delta T_n} - 1}{\delta} - \sum_{i=1}^n X_i e^{\delta(T_n-T_i)} \\ &= ue^{\delta T_n} - \sum_{i=1}^n [X_i - c \frac{e^{\delta W_i} - 1}{\delta}] e^{\delta(T_n-T_i)} \\ &= ue^{\delta \sum_{i=1}^n W_i} - \sum_{i=1}^n Y_i e^{\delta(T_n-T_i)} \\ &= ue^{\delta \sum_{i=1}^n W_i} - \sum_{i=1}^n Y_i e^{\delta \sum_{j=i+1}^n W_j} \end{aligned}$$

where $Y_i = X_i - c \frac{e^{\delta W_i} - 1}{\delta}$, $i \geq 1$.

Obviously $\{(Y_i, W_i), i \geq 1\}$ are independent and have the same distribution $G(y, w)$,

$$\begin{aligned} G(y, w) &= P\{Y_1 = X_1 - c \frac{e^{\delta W_1} - 1}{\delta} \leq y, W_1 \leq w\} \\ &= \int_0^w P\{X_1 - c \bar{S}_{\bar{t}_1} \leq y\} dK(t) \\ &= \int_0^w F(y + c \bar{S}_{\bar{t}_1}) dK(t) \end{aligned}$$

Theorem 1 Let $\Psi_\delta(u)$ be defined as (3), then we have

$$\Psi_\delta(u) = P\{T < \infty\} = \sum_{n=1}^{\infty} f_n(u)$$

where

$$\begin{aligned} f_1(u) &= \int_0^w \bar{F}(ue^{\delta t} + c \bar{S}_{\bar{t}_1}) dK(t) \\ f_n(u) &= \int_0^{\infty} \int_0^{ue^{\delta t} + c \bar{S}_{\bar{t}_1}} f_{n-1}(ue^{\delta t} + c \bar{S}_{\bar{t}_1} - y) dF(y) dK(t) \end{aligned}$$

Proof

$$\Psi_\delta(u) = P\{T < \infty\} = \sum_{n=1}^{\infty} P\{T = T_n\}$$

$$= \sum_{n=1}^{\infty} P\{U_\delta(T_1) \geq 0, U_\delta(T_2) \geq 0,$$

$$\dots, U_\delta(T_{n-1}) \geq 0, U_\delta(T_n) < 0\}$$

$$= \sum_{n=1}^{\infty} f_n(u)$$

According to definition (3), when $n = 1$

$$\begin{aligned} f_1(u) &= P\{U_\delta(T_1) < 0\} \\ &= P\{ue^{\delta T_1} - Y_1 < 0\} \\ &= \int_0^{\infty} \int_0^{\infty} P\{ue^{\delta W_1} - Y_1 < 0 \mid Y_1 = y, W_1 = t\} dG(y, t) \\ &= \int_0^{\infty} \int_{ue^{\delta t}}^{\infty} dF(y + c \bar{S}_{\bar{t}_1}) dK(t) \\ &= \int_0^{\infty} \int_{ue^{\delta t} + c \bar{S}_{\bar{t}_1}}^{\infty} dF(y) dK(t) \\ &= \int_0^{\infty} \bar{F}(ue^{\delta t} + c \bar{S}_{\bar{t}_1}) dK(t) \\ f_2(u) &= P\{U_\delta(T_1) \geq 0, U_\delta(T_2) < 0\} \\ &= P\{ue^{\delta T_1} - Y_1 \geq 0, ue^{\delta T_2} - e^{\delta(T_2-T_1)} Y_1 - Y_2 < 0\} \\ &= \int_0^{\infty} \int_0^{\infty} P\{ue^{\delta W_1} - Y_1 \geq 0, ue^{\delta(W_1+W_2)} \\ &\quad - e^{\delta W_2} Y_1 - Y_2 < 0 \mid Y_1 = y, W_1 = t\} dG(y, t) \\ &= \int_0^{\infty} \int_{-c \bar{S}_{\bar{t}_1}}^{ue^{\delta t}} P\{ue^{\delta(t+W_2)} - e^{\delta W_2} y - Y_2 < 0\} dF(y + c \bar{S}_{\bar{t}_1}) dK(t) \\ &= \int_0^{\infty} \int_0^{ue^{\delta t} + c \bar{S}_{\bar{t}_1}} P\{(ue^{\delta t} + c \bar{S}_{\bar{t}_1} - y) e^{\delta W_2} - Y_2 < 0\} dF(y) dK(t) \\ &= \int_0^{\infty} \int_0^{ue^{\delta t} + c \bar{S}_{\bar{t}_1}} f_1(ue^{\delta t} + c \bar{S}_{\bar{t}_1} - y) dF(y) dK(t) \end{aligned}$$

By inductive assumption, when $n \geq 3$, we have

$$\begin{aligned} f_n(u) &= P\{U_\delta(T_1) \geq 0, U_\delta(T_2) \geq 0, \\ &\quad \dots, U_\delta(T_{n-1}) \geq 0, U_\delta(T_n) < 0\} \\ &= P\{ue^{\delta W_1} - Y_1 \geq 0, ue^{\delta(W_1+W_2)} - e^{\delta W_2} Y_1 - Y_2 \geq 0, \\ &\quad \dots, ue^{\delta \sum_{j=1}^n W_j} - \sum_{i=1}^n Y_i e^{\delta \sum_{j=i+1}^n W_j} < 0\} \\ &= \int_0^{\infty} \int_{-c \bar{S}_{\bar{t}_1}}^{ue^{\delta t}} P\{(ue^{\delta t} - y) e^{\delta W_2} - Y_2 \geq 0, \\ &\quad \dots, (ue^{\delta t} - y) e^{\delta \sum_{j=2}^n W_j} - \sum_{i=2}^n Y_i e^{\delta \sum_{j=i+1}^n W_j} < 0\} dG(y, t) \end{aligned}$$

$$\begin{aligned}
&= \int_0^\infty \int_{-c\bar{S}_{\bar{t}_l}}^{ue^{\delta t}} f_{n-1}(ue^{\delta t} - y) dF(y + c\bar{S}_{\bar{t}_l}) dK(t) \\
&= \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} f_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y) dF(y) dK(t)
\end{aligned}$$

Obviously $\sum_{n=1}^\infty f_n(u)$ is convergence, that $\Psi_\delta(u)$ has the following theorem 2

Theorem 2 Let $\Psi_\delta(u)$ be defined as (3), then

$\Psi_\delta(u)$ satisfies the following integral equation

$$\begin{aligned}
\Psi_\delta(u) &= \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}_l}) dK(t) \\
&\quad + \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} \Psi_\delta(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y) dF(y) dK(t)
\end{aligned}$$

Proof

$$\begin{aligned}
\Psi_\delta(u) &= \sum_{n=1}^\infty f_n(u) = f_1(u) + \sum_{n=2}^\infty f_n(u) \\
&= f_1(u) + \sum_{n=2}^\infty \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} f_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y) dF(y) dK(t) \\
&= f_1(u) + \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} \sum_{n=2}^\infty f_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y) dF(y) dK(t) \\
&= f_1(u) + \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} \Psi_\delta(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y) dF(y) dK(t) \\
&= \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}_l}) dK(t) \\
&\quad + \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} \Psi_\delta(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y) dF(y) dK(t)
\end{aligned}$$

IV. THE DISTRIBUTION OF MAXIMUM SURPLUS BEFORE THE RUIN

Denote the distribution of maximum before the ruin with the initial reserve u by

$$H(u, x) = P\{\sup_{0 \leq t < T} U_\delta(t) \leq x, T < \infty \mid U_\delta(0) = u\} \quad (4)$$

Theorem3 Let $H(u, x)$ be defined as (4), then we can get

(1) When $x < u$, we have

$$H(u, x) = 0;$$

(2) When $x \geq u$, we have

$$H(u, x) = \sum_{n=1}^\infty h_n(u, x)$$

where $n = 1$, we have

$$h_1(u, x) = \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}_l}) dK(t)$$

and $n \geq 2$, we have

$$h_n(u, x) = \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}_l} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} h_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y, x) dF(y) dK(t)$$

Proof

(1) when $x < u$, according to definition(4), we easily get

$$H(u, x) = 0$$

(2) when $x \geq u$, we have

$$H(u, x) = P\{\sup_{0 \leq t < T} U_\delta(t) \leq x, T < \infty\}$$

$$\begin{aligned}
&= \sum_{n=1}^\infty P\{\sup_{0 \leq t < T} U_\delta(t) \leq x, T = T_n\} \\
&= \sum_{n=1}^\infty P\{0 \leq U_\delta(T_1) \leq x, 0 \leq U_\delta(T_2) \leq x, \dots, 0 \leq U_\delta(T_{n-1}) \leq x, U_\delta(T_n) > x\} \\
&= \sum_{n=1}^\infty h_n(u, x)
\end{aligned}$$

According to definition (4), when $n = 1$

$$\begin{aligned}
h_1(u) &= P\{0 \leq u \leq x, U_\delta(T_1) < 0\} \\
&= P\{ue^{\delta T_1} - Y_1 < 0\}
\end{aligned}$$

$$= \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}_l}) dK(t)$$

$$h_2(u) = P\{0 \leq U_\delta(T_1) \leq x, U_\delta(T_2) < 0\}$$

$$= P\{0 \leq ue^{\delta T_1} - Y_1 \leq x, ue^{\delta T_2} - e^{\delta(T_2 - T_1)} Y_1 - Y_2 < 0\}$$

$$= \int_0^\infty \int_0^\infty P\{0 \leq ue^{\delta W_1} - Y_1 \leq x, ue^{\delta(W_1 + W_2)} - e^{\delta W_2} Y_1 - Y_2 < 0 \mid Y_1 = y, W_1 = t\} dG(y, t)$$

$$= \int_0^\infty \int_{ue^{\delta t} - x}^{ue^{\delta t}} P\{(ue^{\delta t} - y)e^{\delta W_2} - Y_2 < 0\} dF(y + c\bar{S}_{\bar{t}_l}) dK(t)$$

$$= \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}_l} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} P\{(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y)e^{\delta W_2} - Y_2 < 0\} dF(y) dK(t)$$

$$= \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}_l} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}_l}} h_1(ue^{\delta t} + c\bar{S}_{\bar{t}_l} - y, x) dF(y) dK(t)$$

By inductive assumption, when $n \geq 3$, we have

$$h_n(u) = P\{0 \leq U_\delta(T_1) \leq x, 0 \leq U_\delta(T_2) \leq x,$$

$$\dots, 0 \leq U_\delta(T_{n-1}) \leq x, U_\delta(T_n) < 0\}$$

$$= P\{0 \leq ue^{\delta W_1} - Y_1 \leq x, 0 \leq ue^{\delta(W_1 + W_2)} - e^{\delta W_2} Y_1 - Y_2 \leq x,$$

$$\dots, ue^{\delta \sum_{j=1}^n W_j} - \sum_{i=1}^n Y_i e^{\delta \sum_{j=i+1}^n W_j} < 0\}$$

$$= \int_0^\infty \int_{ue^{\delta t} - x}^{ue^{\delta t}} P\{0 \leq (ue^{\delta t} - y)e^{\delta W_2} - Y_2 \leq x,$$

$$\cdots, (ue^{\delta t} - y)e^{\delta \sum_{j=2}^n W_j} - \sum_{i=2}^n Y_i e^{\delta \sum_{j=i+1}^n W_j} < 0\} dG(y, t)$$

$$= \int_0^\infty \int_{ue^{\delta t} - x}^{ue^{\delta t}} h_{n-1}(ue^{\delta t} - y, x) dF(y + c\bar{S}_{\bar{t}|}) dK(t)$$

$$= \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}|}} h_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

We know that $\sum_{n=1}^\infty h_n(u, x)$ is convergence. Then, we can get the follow theorem

Theorem4 Let $H(u, x)$ be defined as (4), then $H(u, x)$ satisfies the following integral equation

$$H(u, x) = \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}|}) dK(t)$$

$$+ \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}|}} H(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

Proof

$$H(u, x) = \sum_{n=1}^\infty h_n(u) = h_1(u) + \sum_{n=2}^\infty h_n(u)$$

$$= h_1(u) + \sum_{n=2}^\infty \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}|}} h_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

$$= h_1(u) + \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}|}} \sum_{n=2}^\infty h_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

$$= h_1(u) + \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}|}} H(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

$$= \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}|}) dK(t)$$

$$+ \int_0^\infty \int_{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x}^{ue^{\delta t} + c\bar{S}_{\bar{t}|}} H(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

V. THE DISTRIBUTION OF MINIMUM SURPLUS BEFORE THE RUIN

Denote the distribution of minimum before the ruin with the initial reserve u by

$$K(u, x) = P\{\inf_{0 \leq t < T} U_\delta(t) \geq x, T < \infty | U_\delta(0) = u\} \quad (5)$$

Theorem3 Let $K(u, x)$ be defined as (5), then we can get

(3) When $x < u$, we have

$$K(u, x) = 0;$$

(4) When $x \geq u$, we have

$$K(u, x) = \sum_{n=1}^\infty k_n(u, x)$$

where $n = 1$, we have

$$k_1(u, x) = \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}|}) dK(t)$$

and $n \geq 2$, we have

$$k_n(u, x) = \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x} k_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

Proof

(1) when $x < u$, according to definition (5), we easily get

$$K(u, x) = 0$$

(2) when $x \geq u$, we have

$$K(u, x) = P\{\inf_{0 \leq t < T} U_\delta(t) \leq x, T < \infty\}$$

$$= \sum_{n=1}^\infty P\{\inf_{0 \leq t < T} U_\delta(t) \leq x, T = T_n\}$$

$$= \sum_{n=1}^\infty P\{U_\delta(T_1) \geq x, U_\delta(T_2) \geq x,$$

$$\cdots, U_\delta(T_{n-1}) \geq x, U_\delta(T_n) < 0\}$$

$$= \sum_{n=1}^\infty k_n(u, x)$$

According to definition (5), when $n = 1$

$$k_1(u, x) = P\{u \geq x, U_\delta(T_1) < 0\}$$

$$= P\{ue^{\delta T_1} - Y_1 < 0\}$$

$$= \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}|}) dK(t)$$

$$k_2(u, x) = P\{U_\delta(T_1) \geq x, U_\delta(T_2) < 0\}$$

$$= P\{ue^{\delta T_1} - Y_1 \geq x, ue^{\delta T_2} - e^{\delta(T_2 - T_1)} Y_1 - Y_2 < 0\}$$

$$= \int_0^\infty \int_0^\infty P\{ue^{\delta W_1} - Y_1 \geq x, ue^{\delta(W_1 + W_2)}$$

$$- e^{\delta W_2} Y_1 - Y_2 < 0 | Y_1 = y, W_1 = t\} dG(y, t)$$

$$= \int_0^\infty \int_{-c\bar{S}_{\bar{t}|}}^{ue^{\delta t} - x} P\{(ue^{\delta t} - y)e^{\delta W_2} - Y_2 < 0\} dF(y + c\bar{S}_{\bar{t}|}) dK(t)$$

$$= \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x} P\{(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y)e^{\delta W_2} - Y_2 < 0\} dF(y) dK(t)$$

$$= \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}|} - x} k_1(ue^{\delta t} + c\bar{S}_{\bar{t}|} - y, x) dF(y) dK(t)$$

By inductive assumption, when $n \geq 3$, we have

$$k_n(u, x) = P\{U_\delta(T_1) \geq x, U_\delta(T_2) \geq x, \cdots, U_\delta(T_{n-1}) \geq x, U_\delta(T_n) < 0\}$$

$$= P\{0 \leq ue^{\delta W_1} - Y_1 \leq x, 0 \leq ue^{\delta(W_1 + W_2)} - e^{\delta W_2} Y_1 - Y_2 \leq x,$$

$$\cdots, ue^{\delta \sum_{j=1}^n W_j} - \sum_{i=1}^n Y_i e^{\delta \sum_{j=i+1}^n W_j} < 0\}$$

$$= \int_0^\infty \int_{-c\bar{S}_{\bar{t}|}}^{ue^{\delta t} - x} P\{(ue^{\delta t} - y)e^{\delta W_2} - Y_2 \leq x,$$

$$\begin{aligned}
& \dots, (ue^{\delta t} - y)e^{\delta \sum_{j=2}^n W_j} - \sum_{i=2}^n Y_i e^{\delta \sum_{j=i+1}^n W_j} < 0 \} dG(y, t) \\
& = \int_0^\infty \int_{-c\bar{S}_{\bar{t}_i}}^{ue^{\delta t} - x} k_{n-1}(ue^{\delta t} - y, x) dF(y + c\bar{S}_{\bar{t}_i}) dK(t) \\
& = \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_i} - x} k_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}_i} - y, x) dF(y) dK(t)
\end{aligned}$$

We know that $\sum_{n=1}^\infty k_n(u, x)$ is convergence. Then, we can get the follow theorem

Theorem4 Let $K(u, x)$ be defined as (5), then $K(u, x)$ satisfies the following integral equation

$$\begin{aligned}
K(u, x) &= \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}_i}) dK(t) \\
&+ \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_i} - x} K(ue^{\delta t} + c\bar{S}_{\bar{t}_i} - y, x) dF(y) dK(t)
\end{aligned}$$

Proof

$$\begin{aligned}
K(u, x) &= \sum_{n=1}^\infty k_n(u) = k_1(u) + \sum_{n=2}^\infty k_n(u) \\
&= k_1(u) + \sum_{n=2}^\infty \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_i} - x} k_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}_i} - y, x) dF(y) dK(t) \\
&= k_1(u) + \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_i} - x} \sum_{n=2}^\infty k_{n-1}(ue^{\delta t} + c\bar{S}_{\bar{t}_i} - y, x) dF(y) dK(t) \\
&= k_1(u) + \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_i} - x} K(ue^{\delta t} + c\bar{S}_{\bar{t}_i} - y, x) dF(y) dK(t) \\
&= \int_0^\infty \bar{F}(ue^{\delta t} + c\bar{S}_{\bar{t}_i}) dK(t) \\
&+ \int_0^\infty \int_0^{ue^{\delta t} + c\bar{S}_{\bar{t}_i} - x} K(ue^{\delta t} + c\bar{S}_{\bar{t}_i} - y, x) dF(y) dK(t)
\end{aligned}$$

VI. CONCLUSION

In this paper, we have studied the renewal risk model with interest force. Via inductive method technique, some important distributions are obtained. Main results are:

1. The recursive expression of the distribution of the ruin probability is obtained, and its corresponding integral equation for the distribution is obtained.

2. The distribution of maximum surplus before the ruin is obtained, and its corresponding integral equation for the distribution is obtained.

3. The distribution of minimum surplus before the ruin is obtained, and its corresponding integral equation for the distribution is obtained.

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REFERENCES

- [1] B. Sundt, J. L. Tveit, "Ruin estimates under interest force", Insurance: Mathematics and Economics, 1995, pp: 7-22.
- [2] B. Sundt, J. L. Tveit, "The adjustment function in ruin estimates under interest force", Insurance: Mathematics and Economics, 1997, pp: 85-94.
- [3] H. L. Yang, L. H. Zhang, "On the distribution of surplus immediately after ruin under interest force", Insurance: Mathematics and Economics, 2001a, pp: 247-255.
- [4] H. L. Yang, L. H. Zhang, "On the distribution of surplus immediately after ruin under interest force[J]. Insurance: Statistics and probability letters, 2001b, pp: 329-338.
- [5] H. L. Yang, L. H. Zhang, "The joint distribution of surplus immediately before ruin and the deficit at ruin under interest force", North American Actuarial Journal, 2001c, pp: 92-103.
- [6] R. Wu, Y. H. Du, "The renewal risk model with constant interest force", Journal of engineering mathematics, 2002, pp: 46-54.
- [7] Q. M. Lin, R. M. Wang, "Calculation of ruin probabilities under a renewal risk model with interest force", Journal of east china normal university, 2005, pp: 46-52.
- [8] H. B. Hao, J. X. Hu, C. P. Li, "A joint Distribution for the Discrete Time Insurance risk Model With Dependent Rates", Journal of Xiaogan University, 2007, pp: 34-36.
- [9] J. X. Su, X. J. Zhao, X. H. Li, "Ruin probability under interest rates with autoregressive structure order 2", Journal of Lanzhou University, 2004, pp: 1-4.
- [10] C. P. Li, H. B. Hao, "A extreme value distribution on a renewal risk model with interest force", mathematics in economics, 2007, pp: 121-124.