Study on the Timing of Investment Decision of Real Estate Projects Based on the Real Option Theory

Yang Yang
School of Economics & Management
Chongqing Normal University
Chongqing, China
784140182@qq.com

Huashan Tan
College of Computer and Information Science
Chongqing Normal University
Chongqing, China
6510388@qq.com

Abstract—Real estate investment project has the real option characteristics such as long time development, high capital and high investment risk. Contrasting with the traditional evaluation method, the real option theory is more scientific one that considers the value of the management flexibility and strategic investment. Considering the management flexibility, the timing of investment can improve the value of the project. This paper builds the optimal investment timing model to explain why the developers will delay the investment. The factors affecting the optimal investment time are discussed through the sensitivity analysis.

Keywords-real options; timing of investment decisions; real estate projects; volatility; Sensitivity analysis

I. INTRODUCTION

Real estate investment project has the characteristics such as high investment, high transaction costs, long time development cycle, flexible management and the tendency by outside influence. It makes the real estate industry more favorable to delay the investment and wait for the best time when the future could look brighter. This management flexibility is very important, but the traditional evaluation method such as DCF (Discounted Cash Flow) can't reflect the value. SO Myers and Ross pointed out that the potential investment opportunities of the risk project can be regarded as another form of option [1-2]. It is called as the real option that has been widely used in project evaluation. The real option has three features. The fist one is irreversibility of investment. Once the real estate project starts, the investment cost is reduced to a sunk cost. If the building on the land has been used for a particular purpose, it is very difficult to be also used for other purpose. The second one is uncertainty of investment which come from the gain on investment and the external environment. The project's coming earnings will be affected by many internal and external factors. For example, the change of the national related policy will stop the project. If the adverse external environment is appear, the project can shrink or even terminate at last. The third one is that the timing investment can be selected. After obtaining land, Real estate developers will wait for the best time to investment by hoarding the land. Because the project has trait of long development cycle and it is a multi-stage process. It is also a dynamic process in continuous decision making. According to the implementation of the project and market conditions, manages can decide whether to go on the follow-up investment. In the real estate industry, idle land or the first phase development will create an option to go on the project when market conditions become conductive later. Waiting to delivery option at some point in the future is a better choice than execution at once. But, what time of delivery option or how to choose the optimal timing of investment is very important and difficulty. Study on the field is not sufficient. In the existing literature, Dixit and Pindyck discussed the value of investment opportunity and the optimal timing under conditions of uncertainty [3]. Bernaroch and Schwartz researched the timing of investment of IT industry [4-5].

This paper uses the real option theory to research whether a real estate project is worthy to invest or not and when to invest. By building models under certain and uncertain conditions, it presents different solutions of the model in different situation. It also analyses that how different parameters influence the optimal timing of investment. This can help managers to make the optimal investment decisions.

II. THE TIMING MODEL OF REAL ESTATE PROJECT BASED ON THE REAL OPTION THEORY

A. *The model assumptions and parameters*

According to Black-Schloes model and contingent liabilities model by Merton, this article supposes as follows:

- The capital market is perfect in which there are not transaction cost and tax, and all the traders can be free to obtain complete information without cost.
- Risk-free interest rate is known and constant.
- Asset price volatility is known and constant.
- Asset can be concessive trading and divided limitlessly. The price changes are continuous.
- Within the period of validity of the option, there is no cash flow and no dividends should be paid.
- All investors are risk neutral.

Asset pricing by option theory plays an important role in investment valuation. There is some internal connection in the value between options of real estate investment and financial options. Holding an investment opportunity likes holding a financial option. Comparison between them is shown below.

TABLE I. COMPARISON OF THE RELATED PARAMETERS OF FINANCIAL OPTION AND REAL OPTION

Financial option	Real option of real estate investment
The current market price of stock	The total discount of the cash flow in the future (V)
Strike price	Capitalized cost (I)
The due date	The time from initial outlay to lost investment opportunities (T)
The volatility of stock price	The uncertainty of project value (σ)
Risk-free rate	Risk-free rate (r)

B. The Timing Model of real estate project under certain conditions

With obvious characteristics of real option in real estate investment, the opportunity can be regarded as an American call option which provides a right to wait till it is favorable. The factors impacted on the value of real estate investment project are often uncertain, such as competition, market environment, economy policy. We assume that the value in the investment project follows a Wiener process[6-8].

$$\frac{dV_t}{V_t} = \alpha dt + \sigma dz_t \quad (1)$$

In this equation, the parameter says the total discount of the cash flow in the future at time . The parameter says the growth rate of project value. The parameter says the volatility which can reflect uncertainty. The parameter says an increment of standard Wiener process. Then the value of this call option is as follows:

$$R(V) = E\max\left[\left(V_T - I\right)e^{-rT}, 0\right] \quad (2)$$

In this equation, the parameter says the initial investment capital. The parameter says the value of real estate project in the final time . The parameter is the discount rate of the project. The optimal timing of investment is the time which could maximum the value of option. We must seek the parameter to satisfy the equation as follows:

$$R^*(V) = \max_T E\max\left[\left(V_T - I\right)e^{-rT}, 0\right] \quad (3)$$

In order to simplify the model, we first discuss the condition under which the volatility is zero. It's mean to seek the optimal timing of project under certain conditions without considering the uncertainty. Now the value of investment in real estate is increasing with the growth rate . We can assume that inequality is true. If not, the value of the project will be infinite. For manager to delay investment is the best decision. Over time , the total discount of the cash flow in the future will become variable . It meets the equations as follows:

$$V_T = V_0 e^{\alpha t} \quad (4)$$

We can get the decision model as follows:

$$R^*(V) = \max_T E\max\left[\left(V_0 e^{\alpha T} - I\right)e^{-rT}, 0\right] \quad (5)$$

By the optimization fist-order condition, the model solution is as follows:

$$\frac{dR}{dt} = \left(\alpha - r\right)V_0 e^{(\alpha - r)T} + rIe^{-rT} = 0 \quad (6)$$

We can get solution as follows:

$$T = \frac{1}{\alpha}\ln\left[\frac{rI}{\left(r-\alpha\right)V_0}\right] \quad (7)$$

III. THE TIMING MODEL OF REAL ESTATE PROJECT UNDER UNCERTAIN CONDITIONS

More generally, we discuss the optimal investment timing model of real estate projects under uncertain conditions. For all variable-value of the expected cash flow, there must be a critical value . If the project value is greater than the critical value, the managers decided to invest in. Otherwise, they will delay the investment. When the project value is V, the value of investment opportunity is exactly equal to net income of investment. It can be stated as follows:

$$R(V) = V^* - I \quad (8)$$

We next determined function value of the real estate project investment opportunities. Under conditions of uncertainty, changes of the project value would submit to Wiener process. We use the pricing theory on convertible bond to eliminate effect from Brownian motion. It needs to build up a risk-free portfolio that is made up with an investment project associated with value of opportunity and a few short-shorts positions related to the expected cash flow at the time . Then the value of the portfolio is as follows:

$$\gamma = R(V) - nV \quad (9)$$

The Stochastic return on investment is as follows:

$$d\gamma = dR(V) - ndV \quad (10)$$

By the Ito lemma, we can get the decision model as follows:

$$dR(V) - ndV$$
$$= \left[\alpha V \frac{\partial R(V)}{\partial V} + \frac{1}{2}(\sigma V)^2 \frac{\partial^2 R(V)}{\partial V^2} \right] dt$$
$$+ \sigma V \frac{\partial R(V)}{\partial V} dz_t - n(\alpha V dt + \sigma V dz_t) \tag{11}$$
$$= \left[\alpha V \frac{\partial R(V)}{\partial V} + \frac{1}{2}(\sigma V)^2 \frac{\partial^2 R(V)}{\partial V^2} - n\alpha V \right] dt$$
$$+ \sigma V \left[\frac{\partial R(V)}{\partial V} - n \right] dz_t$$

In order to make the construction portfolio risk free, thereby eliminating uncertainties, we can choose the right amount of short positions. We assume that the equality $n = \frac{\partial R(V)}{\partial V}$ holds. In the risk neutral condition, the parameter is replaced by the parameter $\hat{\alpha} = r - \delta$. The parameter δ is the difference which is below the equilibrium return. Due to the holders of each unit short-term position must pay the corresponding holders of long-term position a certain amount of return, the parameter δ is in the fixed income paid by holders of short-term position. The return of the portfolio in the risk neutral environment is the return calculated by risk-free rate [9-10]. So we can get the overall return of the portfolio as follows:

$$d\gamma - n\delta V dt = r\gamma dt \tag{12}$$

If we put equation (12) into the equation (11), we can get partial differential equation as follows:

$$\frac{1}{2}(\sigma V)^2 \frac{\partial^2 R(V)}{\partial V^2} + (r-\delta)V \frac{\partial R(V)}{\partial V} + rR(V) = 0 \tag{13}$$

The boundary conditions of the equation are as follows:

$$R(0) = 0 \qquad R(V^*) = V^* - I \tag{14}$$

Because equation (13) is a second-order linear equation, the general solution can be regarded as the linear combination of any two special solutions. We can assume that equation $R(V) = AV^\beta$ is a special solution and put it into equation (13). Then we get equation (15) as follows:

$$\frac{1}{2}\sigma^2(\beta-1)\beta + (r-\delta)\beta - r = 0 \tag{15}$$

From equation (15), it is easy to see that a solution of equation is greater than 0, while another solution is less than 0. We assume that the general solution is as follows:

$$R(V) = A_1 V^{\beta_1} + A_2 V^{\beta_2} \tag{16}$$

In the equation, variable β_1 and variable β_2 are two solutions. Here variable β_1 is greater than 0 and variable β_2 is less than 0. The parameter A_1 and parameter A_2 are the two undetermined coefficients. Considering the boundary conditions (14) and the inequality $\beta_2 < 0$, we can conclude that the equation $A_2 = 0$ is true. Then the value of expected cash flow when we get maximized Investment opportunity is as follows:

$$V^* = \frac{\beta_1}{\beta_1 - 1} I$$

$$\beta_1 = \left(\frac{1}{2} - \frac{\hat{\alpha}}{\sigma^2} \right) + \sqrt{\left(\frac{\hat{\alpha}}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} \tag{17}$$

$$\hat{\alpha} = r - \delta$$

Finally we get the investment timing of real estate project is as follows:

$$T = \frac{1}{\hat{\alpha}} \ln \left[\frac{\beta_1 I}{(\beta_1 - 1)V_0} \right] \tag{18}$$

IV. Sensitivity Analysis of the Optimal Investment Time of Real Estate Project

Based on the timing model of real estate project, sensitivity analysis of the optimal investment time under certain and uncertain conditions is made.

A. *Sensitivity analysis under certain conditions*

We can draw the relationship between growth rate and the optimal investment time when we assign these values as: $I = 100, V_0 = 100, r = 0.1, 0.15, 0.2$ (see Fig 1). From the Fig 1, we can see that the optimal investment timing depends on two main factors, the growth rate and discount rate. When other conditions are same, it starts to move in inverse relation to the growth rate and discount rate. If the growth rate is unchanged, the bigger are the discount rate, the smaller will be the threshold for investment and the earlier will be the investment time. This is because the loss of the time value of money and income will exceed the cost in project investment in the long term. Contrarily if the discount rate is unchanged, the bigger are the growth rate, the later will be the investment time. Waiting for investment is more favorable than doing it at once.

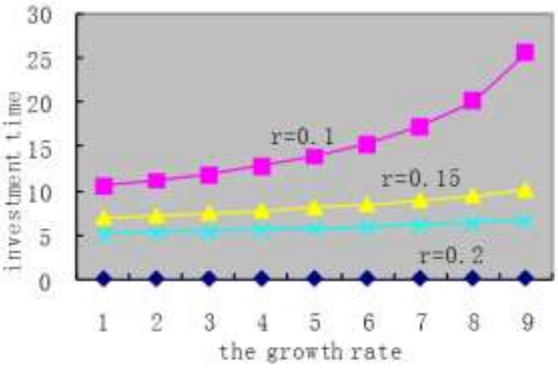

Figure 1. Relationship between growth rate and the optimal investment time under certain conditions

B. Sensitivity analysis under uncertain conditions

1) The relationship between growth rate and the optimal investment time

We can draw the relationship between growth rate and the optimal investment time when we assign these values as: $I = 100, V_0 = 100, r = 0.1, \sigma = 0, 0.1, 0.2$ (see Fig 2). Here we consider the uncertain conditions of real estate project, and we selected three values of changes in volatility σ.

From the Fig 2, we can see that uncertain conditions increased the investment threshold. The optimal timing of investment project extends compared with certain conditions. When project growth rate is small, the optimal investment time is greatly affected by the volatility. But as the growth rate grows, the volatility effects on it reduce gradually.

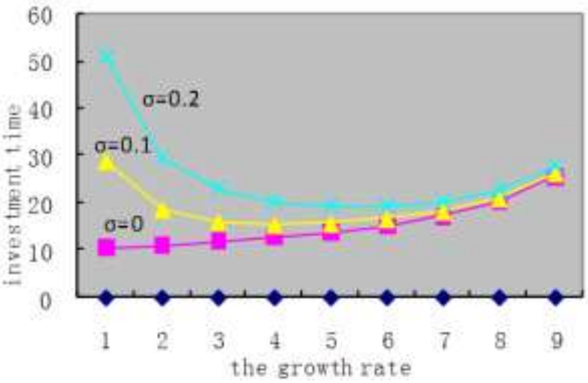

Figure 2. Relationship between growth rate and the optimal investment time under uncertain conditions

2) The relationship between volatility and the optimal investment time

In order to further discuss the effect of uncertainty on optimal timing, we can draw the relationship between volatility and the optimal investment time when we assign these values as: $I = 100, V_0 = 100, \hat{\alpha} = 0.05, r = 0.1, 0.2, 0.3$ (see Fig 3). Fig 3 shows the best investment opportunity depends on the volatility. When other conditions are same, the potential uncertainty in real estate investment is higher, the more favorable to postpone the project. Under the certain and uncertain conditions, If the growth rate is unchanged, the bigger are the discount rate, the earlier will be the investment time.

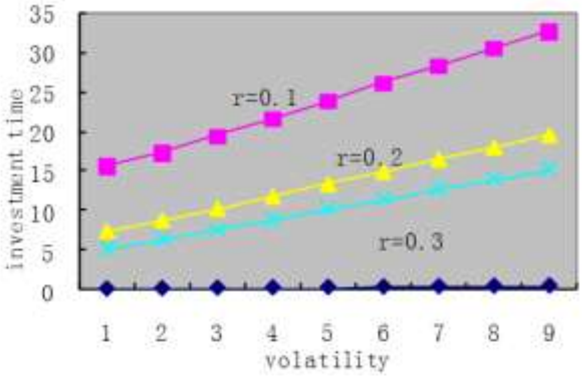

Figure 3. Relationship between volatility and the optimal investment time

V. CONCLUSIONS

In the real estate project investment decision-making, investment timing is difficult to manager. The investment is like buying an American call option. But the external uncertainty will reduce its value. This paper uses the real option theory to discuss the timing models of real estate project under certain and uncertain conditions. The sensitivity analysis of the parameters shows that the best investment time grows with expected return and is inversely with the investment cost. With the passage of time and the external uncertainty reduction, the optimal timing of investment is gradually reduced.

ACKNOWLEDGMENT

This work was supported by the Science Technology Researching Fund of Chongqing Education Committee (KJ100623, KJ110629).

REFERENCES

[1] Myers S C. Determinants of cooperate borrowing, Journal of Financial Economics, 1977, 5(2):47-176.

[2] Ross S A. A simple approach to the valuation of risky income streams, Journal of Buisness, 1978, 5(3):453-475.

[3] Dixit A.K, Pindyck R.S, The options approach to capital investment, Havard Business Revew, 1995, 73 (3):105-115.

[4] Bernaroch M, Kauffman R J, Justifying electronic banking network expansion using real options analysis, Mis Quarterly, 2000, 24(2): 197-225.

[5] Schwartz E S, Gorostiza C Z, Investment under uncertainty in information technology: acquisition and development projects, Management Science, 2003, 49(1):57 -70.

[6] Lander D M, Pinches G E, Challenges to the practical implementation of modeling and valuing real options, The Quarterly Review of Economics and Finance, 1998, 38(2):537-567.

[7] Berk J B, Green R C, Naik V, Valuation and return dynamics of new ventures, The Review of Financial Studies, 2004, 17(1):35.

[8] Hartman M, Hassan A, Application of real options analysis for pharmaceutical R&D project evaluation empirical results from a survey, Research Policy, 2006, 35(1):343 -354.

[9] Xu M, Xian Q, Research on application of real option in multiple phase investment, Manufacturing Industry Information , 2012, 41(5):54-57.

[10] Xiao G, Feng H, Chen C, R&D project multi-stage evaluation model based on real option under the circumstance of risks, Management Science, 2011, 55(6):72-75.