PCNN Forecasting Model Based on Wavelet Transform and Its Application

Qiang Fu[1] Yan Feng[1] Dengchao Feng[2]

[1] College of Water Conservancy & Architecture, Northeast Agricultural University, Harbin 150030, P. R. China
[2] College of Electronics & Information Engineering, Tianjin University, Tianjin 300072, P. R. China

Abstract

Pulse Coupled Neural Network (PCNN) is widely used in image processing for its basic characteristics of coupling mechanism and achieved some results. PCNN model has been improved as follows: Firstly, correlation coefficient is used to control the bonding strength. Secondly, the threshold setting is adjusted by the least error. Thirdly, A Trous transform is combined with PCNN model to form the combination forecasting model. The improved combination model was implemented in annual rainfall forecasting to check its feasibility.

Keywords: Pulse coupled neural network, A Trous wavelet transform, Forecasting, Correlation analysis

1. Introduction

Pulse coupled neural network (PCNN), called the third generation of artificial neural network, and is a new neural network with the development over the past ten years, which is different from the traditional neural network. In 1990,Mammalian neuron model was obtained by Eckhorn et al. through analyzing the oscillation phenomenon of synchronous pulse bursts in the cat visual cortex[1]-[3]. In 2000, a new neuron model was proposed by Kilter and Leo to analyse the correlation of a couple of adjacent neuronal ignition quality. In the same year, Bressloff and Coombes[4] made an intensive study of the dynamic behavior of PCNN with strong coupling and pointed out how the phase locking state enters into unstable state with the enhancement of the coupling strength in the weak-coupling limit. Finally, PCNN model was formed. PCNN model with some of the characteristics, such as strong adaptive capturing ignition, internal coupling, dynamic alignment threshold to control impulse-firing, has been widely applied to image denoising[5], image smoothing[6], image processing [7], image segmentation[8], image fusion[9]. It's also partly used in the shortest path optimization[10], Structural layout optimization[11], etc.

Little research has been conducted in applying PCNN to other fields, even no related references. Therefore, the traditional PCNN was improved in the paper, combined with wavelet transform to form combination forecasting model. It's applied to forecasting rainfall and got good results, then verified the feasibility of the model.

2. Improved PCNN forecasting model

2.1. Original PCNN model

A single PCNN neuron model [12] is shown in Figure 1. It's simplified and approximated for real neurons, consisting of acceptance domain, modulation and pulse excitation.

Acceptance domain: receiving other neurons and external input. The input value will be transmitted via two channels (*L* channel and *F* channel). The corresponding equations are as follows:

$$L_j = \sum_k L_{kj} = \sum_k \left[W_{kj} e\left(-\alpha_{kj}^L t\right)\right] \otimes Y_k(t) + J_j \qquad (1)$$

$$F_j = \sum_k F_{kj} = \sum_k \left[M_{kj} e\left(-\alpha_{kj}^F t\right)\right] \otimes Y_k(t) + I_j \qquad (2)$$

where W_{kj} , M_{kj} W_{kj} are synaptic connection weights， α_{kj}^L , α_{kj}^F are time constant； J_j, I_j is input constant.

Modulation: Signal L_i from L channel adds a positive offset, and then multiply modulation is operated with signal F_i from F channel. The formula is as follows:

$$U_j = F_j\left(1 + \beta_j L_j\right) \qquad (3)$$

where offset is 1, β_j is the connecting strength. Because of the variety of the signal F_i slower than signal L_i , signal U_j caused by multiply modulation is approximately considered as the constant signal superimposed a fast change signal.

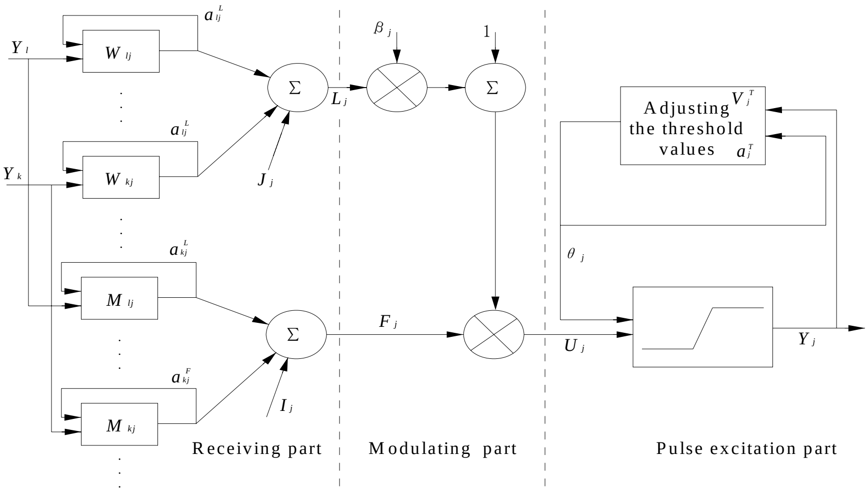

Fig.1: PCNN signal neuron model.

Pulse Excitation: consisting of comparator with variable threshold and pulse generator. When neuronal threshold θ_j is larger than U_j , pulse generator is turned off and the impulse-firing is stopped as well. Then, the threshold began to fall according to exponent decent. When threshold value is less than U_j , pulse generator is turned on and neuron is ignited, namely activated state. An impulse or impulse sequence is input accordingly. Therefore, the maximum frequency of neuronal output impulse should not be larger than that caused by impulse generator. In Equ.(4), V_j^T is the amplitude coefficient and α_j^T is the time constant respectively. The comparator of pulse excitation and impulse generator can be replaced by step function if only one pulse is output when neuron is ignited [13]. The corresponding equation is as follows:

$$\frac{d\theta}{dt} = -\alpha_j^T \theta_j + V_j^T Y_j(t) \tag{4}$$

$$Y_j = Step(U_j - \theta_j) \tag{5}$$

2.2. Improved PCNN Model

After a great deal of application research, the basic PCNN model was simplified by G.Kuntimad and H.S.Ranganath. The figure is as follows.

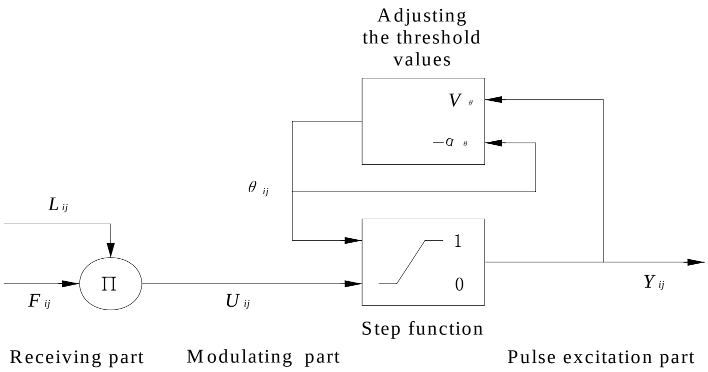

Fig. 2: Neural cell model of simplification PCNN.

Jointing Section: $W_{ij,kl}$ is the weight between neuron ij and kl , V_L is the jointing amplitude coefficient, $Y_{kl}(n-1)$ is the tag of neuron ignition.

Feedback input is as follows.

$$F_{ij}(n) = x_{ij} \tag{6}$$

Jointing input is as follows.

$$L_{ij}(n) = V_L \sum_{k,l} W_{ij,kl} Y_{kl}(n-1) \tag{7}$$

Modulation: The internal action of Neuron ij is produced by modulation between feedback input and jointing input. The Equ.(8) is as follows.

$$U_{ij}(n) = F_{ij}(n)(1 + \beta L_{ij}(n)) \tag{8}$$

where, β is bonding strength. Binary output is produced by neuronal impulse generator according to a step function of internal action $U_{ij}(n)$ and the threshold θ_{ij} is adjusted adaptively according to the ignition state of neuron ij .If neuron ij is ignited, θ_{ij} is adjusted as follows:

$$\theta_{ij}(n)=e^{-\alpha\theta}\theta_{ij}(n-1)+V_\theta Y_{ij}(n-1) \tag{9}$$

where α_θ is time attenuation constant, V_θ is the threshold constant.

Pulse Excitation is as follows.

$$Y_{ij}(n)=\begin{cases}1 & U_{ij}\geq\theta_{ij}\\ 0 & U_{ij}<\theta_{ij}\end{cases} \tag{10}$$

The operating principle of improved model in Fig.2 is the same as the basic model in Fig.1.The purpose of improved PCNN is to reduce model parameter and improve the model precision. Therefore, PCNN model will be improved in different fields.

3. PCNN forecasting model based on wavelet transform

PCNN model in the paper is made further improvement based on the improved model in section1.2 to form PCNN forecasting model based on wavelet transform. The improved algorithm mainly focuses on how to change the formula, select the parameters, improve the operational mechanism in order to get the rational PCNN forecasting model.

3.1. Wavelet transform-A Trous

A Trous is a simple algorithm with small computational complexity. Defining $\phi(t)$ as the scaling function and $\varphi(t)$ as the wavelet function, the scaling function is as follows.

$$\frac{1}{2}\phi\left(\frac{t}{2}\right)=\sum_n h(n)\phi(t-n) \tag{11}$$

where $h(n)$ is the coefficient of lowpass filter.

Defining $C^0(t)$ as the approximate sequence by inner product between signal $f(t)$ and scaling function $\phi(t)$ in 0 scale, the corresponding equation is as follows.

$$C^0(t)=\langle f(t),\phi(t)\rangle \tag{12}$$

Correspondingly, the approximate sequence $C^j(t)$ in scale j is the inner product between approximate sequence $C^{j-1}(t)$ and $\phi(t)$ in scale $j-1$.

$$C^j(t)=\langle C^{j-1}(t),\phi(t)\rangle \tag{13}$$

With Equ.(11) and Equ.(13), $C^j(t)$ can be expressed as follows.

$$C^j(t)=\sum_n h(n)C^{j-1}(t+2^j n) \tag{14}$$

The formula manipulation between wavelet function $\varphi(t)$ and scaling function $\phi(t)$ is as follows.

$$\frac{1}{2}\varphi\left(\frac{t}{2}\right)=\phi(t)-\frac{1}{2}\phi\left(\frac{t}{2}\right) \tag{15}$$

then wavelet coefficient $W^j(t)$ in scale j is the difference between scale j and $j-1$. The equation is as follows.

$$W^j(t)=C^{j-1}(t)-C^j(t) \tag{16}$$

where $\{W^1(t),W^2(t),\cdots,W^j(t),C^J(t)\}$ is the wavelet transform sequence in scale J , J is the maximum decomposing scale. Usually, the maximum scale is $\lg N$ （ N is the length of sequence）. Equ.(16) is A Trous decomposing algorithm.

The reconstruction formula of A Trous is as follows.

$$C^0(t)=C^J(t)+\sum_{j=1}^{J}W^j(t) \tag{17}$$

3.2. Combining forecasting model principle

In order to make the combining forecasting model principle simple and luminous, it'll be illustrated by the example of annual rainfall forecasting. Firstly, A Trous transform is applied to decompose the annual rainfall runoff data to get the needed data, including random entry, trend term and periodic term. Secondly, the three kind of decomposed data are input to PCNN to forecast them. Finally, these three parts of forecasting data are reconstructed by A Trous. According to the above process, the forecasting data of annual rainfall data is realized.

Supposing the data of annual rainfall runoff in M years, namely M samples; each sample has n indexes, namely n kinds of decomposing component by wavelet transform. $(n=3, m=1,2,3,\cdots,M, i=1,2,3,...,n)$

Input sample is as follows.

$$F_i(m)=x_i(m) \tag{18}$$

where $x_i(m)$ is the i^{th} kind of annual rainfall runoff in the m^{th} year, m=1,2,3,…,M.

Linearity link is as follows.

$$L_i(m)=\sum_{i=1}^{n}w_i Y_i(m-1) \tag{19}$$

Pulse Output $Y_i(m-1)$ is the result of weighted summation.

Internal action is as follows.

$$U_i(m)=F_i(m)[1+\beta L_i(m)] \tag{20}$$

where β is the bonding strength constant.

Equ.(2) fully embodies the non-linear characteristic. The correlation analysis of Excel software is adopted in the paper to get the average correlation analysis $\overline{\rho}$ among these samples, then

$$\beta = 1 - \overline{\rho} \quad (21)$$

Error control is as follows.

$$E_i(m) = U_i(m) - x_i(m+1) \quad (22)$$

$E_i(m)$ is absolute error, namely the difference between the m^{th} sample and the $m+1^{th}$ sample.

Pulse output is as follows.

$$Y_i(m) = \begin{cases} 1 & E_i(m) \le \theta \\ 0 & E_i(m) > \theta \end{cases} \quad (23)$$

where θ is the constrained the least error. When $E_i(m) \le \theta$, it illuminates the correlation between these two samples, namely $Y_i(m) = 1$; when $E_i(m) > \theta$,it means the noncorrelation or little correlation between the two samples, namely $Y_i(m) = 0$.

Network Operational Mechanism: When M sample can meet the error requirement, $U_i(m+1)$ can be obtained by $M+1$ times calculation, namely the forecasting data.

Forecasting Algorithm: Due to the correlation and regularity among annual rainfall runoff data to some extend, the forecasting results can be obtained by recursion.

4. Example of application (PCNN model based on wavelet for the rainfall forecasting)

Wavelet transform has been widely used in hydrology and water resources with good results. PCNN, called the third artificial neural network, is just a start to research. Therefore, it's no application in hydrology and water resources .In this paper, the improved model was used in annual rainfall forecasting, and made a satisfactory seat.

4.1. Basic data

Precipitation not only affects the utilization and exploitation in water resources, but also closely related to the occurrence, development and intensity of nature disasters, such as drought, flood, waterlogging, etc. Eventually, it will destroy ecological equilibrium. Therefore, forecasting precipitation accurately will provide scientific basis for the exploitation and utilization of water resources, effectively direct to fight floods and provide disaster relief. To some extend, it'll make a great contribution to maintain ecosystem.

Bie Lahong River locates the hinterland of Sanjiang plain, traversing the whole Fubian delta from west to east. Bie Lahong station was built in 1956, mainly observes water level, runoff, rainfall and ice regime. Detecting rain instrument is the self-record ombrometer with 20 cm .In this paper, the measured data of annual precipitation time sequence in the station was used to analyze the example. Figure.3 shows these data in the station from 1956 to 2004.

4.2. Algorithm principle and the corresponding steps

Annual rainfall runoff data is decomposed by A Trous transform to obtain periodic term, trend term and random entry. Then, the decomposed data are input to PCNN to forecast the corresponding data. Finally, the above three parts will be reconstructed by wavelet transform to forecast annual rainfall.

In this example, it includes the rainfall runoff data in 49 years, namely 49 samples; Each sample has 3 indexes, namely periodic term, trend term and trend term from A Trous transform.. $x_i(m)$ is the sample sequence, $x'_i(m)$ is the normalization sample. $(m = 1,2,3,\cdots,49)$, $(i = 1,2,3)$ .The calculation procedure is as follows:

(1)Normalize the rainfall runoff data in 49 years. The corresponding formula is as follows:

$$x'_i(m) = \frac{0.9 - 0.1}{x_{\max} - x_{\min}} x_i(m) + \left(0.9 - \frac{0.9 - 0.1}{x_{\max} - x_{\min}} x_{\max} \right) \quad (24)$$

(2)Based upon the length of the sample in the paper, the maximum decomposing scale is $J = 3$ and the lowpass filter is db4 wavelet. Program is designed in MATLAB to obtain the component in time domain. High frequency component is obtained in the second decomposition, almost including all the random entry, namely $s(m) = high2(m)$;After the third filtering, all the high frequency are strained away and the low frequency is component is the trend term, namely $q(m) = low3(m)$;Periodic term is the result of subtraction by annual rainfall runoff sequence and random entry, namely $z(m) = x'_i(m) - (s(m) + q(m))$.

(3)Correlation Analysis to ensure bonding strength β and random weight w_i .Based upon Step (2), $s(m)$ 、 $q(m)$ 、 $z(m)$ are obtained to input PCNN model. According to formula (18)-(23), the simulating forecasting is gained by recursive calculation.

(4)According to the operation of step (3), PCNN has the forecasting function. The last group of output data $y_i(49)$ is as the forecasting sample in the next year to forecast the rainfall runoff component $y_i(50)$ in 2005.Finally; the rainfall runoff component will be obtained in some years.

(5)All the components obtained by the above operation process are reconstructed by wavelet transform to get the forecasting rainfall runoff data.

4.3. Analysis of the forecasting results

Based upon A Trous transform, the decomposed component includes Random entry, trend term and periodic term. It's shown in Figure.1 and the before and after normalized samples are listed in the figure as well. The high frequency in the second layer represents random entry, the low frequency in the third layer represents trend term and the periodic term is the subtraction between the original runoff sequence and the above two indexes, which is shown in figure 4~6 ;The reconstruction runoff sequence by A Trous transform is shown in figure 7.

Year	Sample	Normalized samples	Random entry	Trend term	Periodic term
1956	918.7	0.7401	0.1103	0.4652	0.1645
1957	786.3	0.5714	0.0421	0.4599	0.0695
1958	570.5	0.2966	-0.1029	0.4568	-0.0573
1959	773.4	0.5550	-0.0918	0.4541	0.1927
1960	683.3	0.4403	0.0216	0.4454	-0.0268
1961	713.0	0.4781	0.0407	0.4396	-0.0023
1962	802.8	0.5925	-0.0556	0.4329	0.2151
1963	702.9	0.4652	-0.0518	0.4183	0.0988
1964	663.7	0.4153	0.1363	0.4024	-0.1234
1965	627.3	0.369	0.1715	0.3893	-0.1918
1966	523.9	0.2373	-0.0276	0.3675	-0.1026
1967	640.2	0.3854	-0.1158	0.3442	0.1569
1968	619.0	0.3584	-0.0732	0.3318	0.0998
1969	742.6	0.5158	-0.0364	0.3202	0.232
1970	619.6	0.3592	0.0336	0.3172	0.0084
1971	758.5	0.536	0.0678	0.3289	0.1394
1972	619.9	0.3595	0.0247	0.3405	-0.0057
1973	618.2	0.3574	-0.0240	0.3488	0.0325
1974	519.7	0.2319	-0.0664	0.3655	-0.0672
1975	562.2	0.2861	-0.0567	0.3815	-0.0388
1976	488.5	0.1922	0.0194	0.3846	-0.2119
1977	592.1	0.3241	0.0597	0.3872	-0.1228
1978	568.3	0.2938	0.0418	0.385	-0.133
1979	597.3	0.3308	0.0315	0.376	-0.0767
1980	569.2	0.2950	0.0178	0.3738	-0.0967
1981	1044.3	0.9000	-0.0255	0.3779	0.5476
1982	539.7	0.2574	-0.0498	0.379	-0.0718
1983	696.3	0.4568	-0.0459	0.3839	0.1188
1984	667.7	0.4204	-0.0325	0.3975	0.0553
1985	612.9	0.3506	0.0274	0.4095	-0.0862
1986	443.1	0.1344	0.0529	0.4168	-0.3354
1987	710.2	0.4745	-0.0432	0.4152	0.1025
1988	622.1	0.3623	-0.0332	0.4017	-0.0061
1989	543.2	0.2619	0.1288	0.381	-0.248
1990	669.4	0.4226	0.0880	0.3549	-0.0203
1991	798.2	0.5866	-0.0667	0.3227	0.3306
1992	628.1	0.3700	-0.0356	0.2874	0.1182
1993	493.8	0.1989	-0.0329	0.2551	-0.0233
1994	664.4	0.4162	-0.1040	0.2352	0.2849
1995	633.3	0.3766	-0.0014	0.2358	0.1422
1996	646.0	0.3928	0.0868	0.2493	0.0567
1997	733.9	0.5047	-0.0150	0.2702	0.2496
1998	493.2	0.1982	-0.0250	0.302	-0.0788
1999	416.1	0.1000	0.1040	0.3386	-0.3426
2000	519.4	0.2316	0.0697	0.3766	-0.2148
2001	473.7	0.1734	-0.0805	0.4149	-0.161
2002	511.7	0.2217	-0.0945	0.4428	-0.1266
2003	460.7	0.1568	-0.0286	0.4557	-0.2702
2004	595.5	0.3285	0.0399	0.4625	-0.1739

Table 1: Decomposing results by A Trous transform.

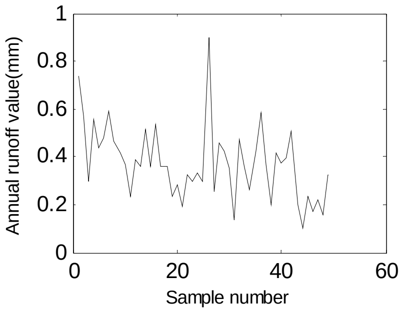

Fig.3:1956-2004 rainfall sample sequence.

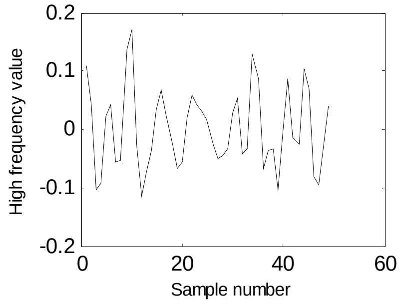

Fig.4: Random entry-high frequency component in the second layer.

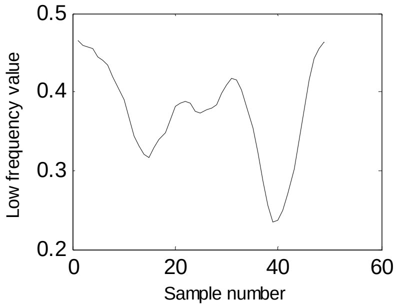

Fig.5: Trend term-Low frequency component.

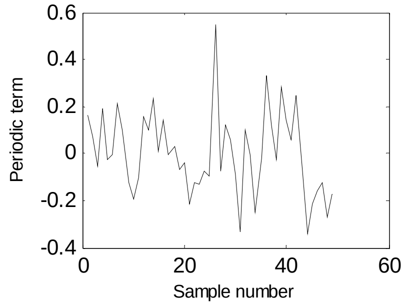

Fig.6: The trend of periodic sequence in the third layer.

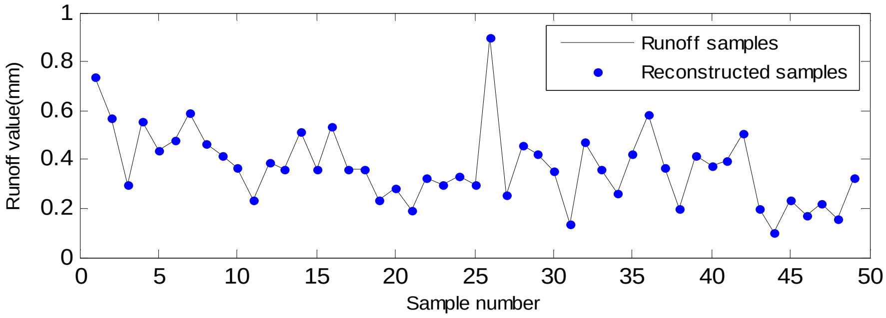

Fig. 7: The fitting chart between reconstructed samples and runoff samples.

The correlations of these three sets of values, random entry, trend term and periodic term in Table.1 were analyzed by Excel software to get the correlation coefficient, namely $\overline{\rho} = 0.82$. From the sample sequence and the corresponding chart, it's shown the total precipitation suddenly increase to $1044.3mm$ in 1981.However,it's $569.2mm$ in 1980 and it decreased to $539.7mm$ in 1982.Therefore,the correlation coefficient is -0.00428 from 1980 to 1981 and the correlation coefficient is 0.186824 from 1981 to 1982.Besides,the correlation coefficients are less than 0.5 due to the irregular change among 1990-1991,1993-1994, and 1996-1998 respectively; The rest correlation coefficient is larger than 0.9 among other years; Only a few coefficients are large than 0.6,0.7,0.8 respectively among some years .In Equ.(21), a very important point to note is the inverse relationship between bonding intensity β and the correlation coefficient $\overline{\rho}$,namely larger $\overline{\rho}$ with little difference among years. Only small β can overcome larger error E .Therefore, $\beta = 1 - \overline{\rho}$ is the reference value in the paper, where $\beta = 0.18$.

Hence it can also be seen that there are large correlation and periodicity of annual rainfall runoff sequence in Bie Lahong station in Sanjiang plain. Figure 5 shows the general trend of annual rainfall is gradually raising.

According as the improved PCNN forecasting principle in the paper, random entry, trend term and periodic term in Table.1 are input to PCNN. The three indexes in all 49 samples are trained and got good results, the corresponding forecasting value is shown in Table.2, where the original runoff value and reconstructed forecasting value in 2004 are listed in it to be the forecasting test, the data from 2005 to 2007 is the pure forecasting data without any sample to compare with.

Forecasting year	Random entry	Trend term	Periodic term	Reconstruction runoff data	Measured runoff
2004	0.0399	0.4625	-0.1739	595.5296	595.5
2005	0.0462	0.5355	-0.2013	636.2590	
2006	0.0426	0.4940	-0.1857	613.0763	
2007	0.0507	0.5875	-0.2208	665.3247	

Table 2: The simulated forecasting data in PCNN model. Unit： *mm*

Table.2 shows the rising trend of annual rainfall sequence has the rising trend, was coincident with the decomposing results by A Trous algorithm in Figure 5.

5. Conclusions

In the paper, PCNN is improved based on the simplified PCNN: absolute error control formula is added to the model; correlation coefficient is used to control the bonding strength and the threshold setting is adjusted by the least error; the operation mechanism of network is different from the PCNN used in the past; A Trous transform is combined with PCNN model to form the combination forecasting model. The improved combining model is used in forecasting annual rainfall and the feasibility is testified by analyzing the examples. This study further shows that the model reduces the complication by adjusting and training the network parameters, overcome the local minimum caused by BP model and simplifies the operation process, which not only saves the operation time, but also gets good forecasting results.

PCNN combinatorial forecasting model based on wavelet transform is the further improvement by seeking for the bonding point between wavelet analysis and PCNN. This paper deals with the realization process of forecasting function by PCNN model, expanding the application field of PCNN and finding a new solution to the forecasting problem of hydrology and water resources. Let us hope that there will be more excellent experts and scholars to do the further theory study to associate with the practical application.

Acknowledgement

This work is partially supported by National Nature Science Foundation of China (Grant No. 30400275)

References

[1] Y. D. Ma, L. Li and Y. F. Wang, The principles of Pulse Coupled Neural Networks and its application. *Beijing, Science Press*, 2006.

[2] R. Eckhorn, H. J. Reitboeck and M. Arndt, Feature Linking via Synchronization among Distributed Assemblies:Simulation of Results from Cat Cortex”. *Neural Computer*, 3: 293-307, 1990.

[3] R. Eckhorn, A. Frien and R. Bauer, High Frequency Oscillations in Primary Visual Cortex of Awake Monkey. *NeuroRep*, 3: 243-246, 1993.

[4] C. M. Gray, W. Singer, Stimulus-specific neuronal oscillations in the orientation columns of cat visual cortex. *Proc, Nat. Academy Sci.*, 5: 1699-1702, 1989.

[5] X. D. Gu, S. D. Guo and D. H. Yu, A new approach for noise reducing of image based on PCNN. *Journal of Electronics and Information technology*, 10: 1304-1309, 2002.

[6] J. Y. Zhang, B. Y. Li, Feature extraction on image smoothness based on PCNNs. *Computer Simulation*, 9: 103-108, 2003.

[7] Y. Fang, F. H. Qi and B. Z. Pei, PCNN implementation and applications in image processing. *Journal Infrared Millimeter and Waves*, 4: 291-295, 2005.

[8] X. Q. Xu, B. Wang and J. C. Yu, Image segmentation based on an improved PCNN. *Journal of Projectiles, Rockets, Missiles and Guidance*, 1: 126-131, 2006.

[9] Q. G. Miao, B. S. Wang, A novel algorithm of multi-focus image fusion using adaptive PCNN”. *Journal of Electronics and Information Technology*, 3:466-470, 2006.

[10] Y. M. Song, D. L. Duan, An algorithm for finding the shortest path of labyrinth based on PCNN model. *Journal of Circuits and Systems*, 3: 72-75, 2005.

[11] H. Li, H. D. Cao and Y. C. Hu, Pulse transiently chaotic neural network for layout optimization. *Chinese Journal of Computational Mechanics*, 5: 623-628, 2005.

[12] X. D. Gu, D.H.Yu. PCNN's principles and applications. *Journal of Circuits and Systems*, 3: 45-50, 2001.

[13] J. L. Johnson , Pulse-coupled neural nets: translation, rotation, scale, distortion and intensity signal invariance for images. *Appl. Opt.*,26: 6239-6253, 1994