

# Evaluation of commercial trainer effectiveness based on DEA cross-efficiency model

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**Abstract-**In order to evaluate the trainer training effectiveness precisely, DEA cross-efficiency model is used to modify the Masefield - Burdak model by introducing cross efficiency .The trainer training effectiveness model is established again. On calculating evaluation matrixes of the aircraft's Performance indexes, it is normalized by being distributed into benefit-type, and cost-type. Finally, the correctness of this model was verified through the courses of pilot school commercial flight license training with C172S, P2010, DA40 and P2006T .The result is accorded with reality.

**Key word-**Air transportation; The trainer; Flight training; Cross-efficiency model; Training effectiveness evaluation .

## I. INTRODUCTION

The earliest research on trainer training effectiveness evaluation is about the second generation of jet military trainer cost effectiveness analysis model [1], proposed by Dott Bazzocchi in 1978. In 1990's, Swiss scholars O. L.P.Masefield and E.A.P.Burdak made revision to it and put forward a method for trainer training effectiveness evaluation, noted Masefield-Burdak model [2]. Few studies have been done on training effectiveness by Domestic scholars, they mainly use this model for military trainer's training effectiveness evaluation [3-6]. This method is based on multi-index comprehensive evaluation theory, includes three steps: selecting evaluation indexes, determining the weight of each index, and establishing function to calculate the efficiency value of the each trainer based on evaluation indexes and weight coefficient. Because of lacking comprehensive consideration on the relation between input and output indexes of decision making units (evaluation units), using multi-index comprehensive evaluation theory merely would reduce the accuracy of evaluation results [7].

Data envelopment analysis (DEA) is a quantity analysis method to evaluate unit of production efficiency, put forward by the famous American operational research expert Charnes. It has some classic models such as  $C^2R$  and  $BC^2$  [8]. Academia considers there are some disadvantages in existing DEA modes. The self-

assessment multiplier completely relying on self-assessment could not reflect the evaluation results objectively. In response to this problem, Sexton et al put forward cross-efficiency evaluation model based on  $C^2R$  [9]. The method used "self-evaluation" and "peer-evaluation" strategy, making up for the shortage of completely relying on self-evaluation. In this way, the evaluation result becomes more objective and credible.

In order to evaluate the trainer training effectiveness precisely, this article uses cross-efficiency evaluation model to modify Masefield-Burdak model by introducing cross-efficiency, and establishes DEA cross-efficiency trainer effectiveness evaluation model. the correctness of this model is verified through a pilot school's commercial single training course. Result shows that it is consistent with Masefield - Burdak model.

## II. MASEFIELD-BURDAK TRAINER TRAINING EFFECTIVENESS EVALUATION MODEL

Scholars O. L.P.Masefield and E.A.P.Burdak think that training effectiveness of trainer has some relationship with performance indexes, flight training courses and training time of the courses. The mathematical model [2] is:

$$T_E = S' \times W \times F \quad (1)$$

where:

$T_E$  - value of trainer training effectiveness ;  $S$  - evaluating indexes normalization matrix (associated with the performance index of the plane);  $W$  - weight of evaluation indexes to training courses;  $F$  - proportion of training courses time.

## III. TRAINING EFFECTIVENESS EVALUATION MODEL BASED ON DEA CROSS- EFFICIENCY MODEL

### A. The DEA cross-efficiency model of evaluation unit

When evaluating trainer training effectiveness, evaluation indexes could be divided into benefit-type and cost-type. Cost-type indexes should be the smaller the better, benefit-type on the contrary [10]. To DEA, the

input index should be as small as possible, the greater the output indexes should be better, so we can use cost-type index as input index of evaluation unit, benefit-type index as output index, and then use DEA method to evaluate [11].

Suppose the number of trainer which will be evaluated is  $k$  and treat every trainer as a evaluation unit  $DMU_d$  ( $d=1,2,\dots,k$ ). To  $DMU_d$ , its Input/output vector are  $X_d > 0$  and  $Y_d > 0$ .  $X_d$  represents the vector of the  $d_{th}$  trainer's cost-type evaluation indexes and  $Y_d$  represents the vector of the  $d_{th}$  trainer's benefit-type

evaluation indexes. We defines  $h_d = \frac{U_d'Y_d}{V_d'X_d}$  as the  $d_{th}$  trainer's efficiency exponent. So using the  $h_d$  as a target and other trainer's efficiency exponent as constraints, fractional Programming model is as follow:

$$\begin{aligned} \max \quad & h_d = \frac{U_d'Y_d}{V_d'X_d} \\ \text{s.t.} \quad & \frac{U_d'Y_j}{V_d'X_j} \leq 1; \\ & j = 1, 2, \dots, k; \\ & V_d \geq 0; \\ & U_d \geq 0. \end{aligned} \quad (2)$$

$U_d$ 、 $V_d$  is as input and output index weight vectors respectively.

We mark the optimal solution of model (2) as  $(h_d^*, U_d^*, V_d^*)$ , and the cross-efficiency of  $DMU_d$  is

$$E_d = \frac{1}{k} \sum_{j=1}^k \frac{U_j^* Y_d}{V_j^* X_d} \quad (3)$$

#### B. Introducing cross-efficiency to establish trainer training effectiveness evaluation model

Using the cross efficiency calculated from formula (3) to modify Masfield - Burdak model, a revised model is:

$$T_E = E \times (S' \times W \times F) \quad (4)$$

where  $E = \text{diag}(E_1, E_2, \dots, E_k)$ .

The difference between and model (1) is that model (4) has considered the cross efficiency of evaluation unit, making up for the lack of considering the deficiency of the input and output indexes comprehensive relations of the model (1). Using this model to evaluate trainer training effectiveness could make the evaluation results more precisely.

### V. CASE ANALYSIS

We take a pilot school' commercial single-engine training at stage 1 as an example, and select commonly

### IV. COMMERCIAL TRAINER TRAINING EFFECTIVENESS EVALUATION

#### A. The normalization of evaluation index

Considering trainer aircraft flight training characteristics, determined 9 evaluation indexes are: Take-off distance(ground roll), Take-off distance(50 foot obstacle), landing distance(ground roll), landing distance(50-foot obstacle), Maximum rate of climb, Ceiling, Cruise speed of design, Maximum range and Maximum take-off weight. Evaluation indexes should be normalized.

To maximum take-off weight, maximum rate of climb, ceiling, Cruise speed of design and maximum range, the values can be the greater the better. its greater value could give students more operating margin, so it can be classified as benefit-type index. While restricted by pilot school actual situation, the value of Take-off distance(ground roll), Take-off distance(50 foot obstacle), landing distance(ground roll), landing distance(50-foot obstacle) would be smaller the better, it could be classified as cost-type.

Index normalized method is given below. Suppose the number of trainer which will be evaluated is  $k$  and evaluation indexes is  $\beta_j$  ( $j=1,\dots,n$ ). The value of the  $i_{th}$  ( $i=1,\dots,k$ ) trainer's evaluation index  $\beta_j$  is  $x_{ij}$ . The maximum and minimum of Evaluation index  $\beta_j$  is  $r_{\max}^j = \max_{1 \leq i \leq k} \{x_{ij}\}$  and  $r_{\min}^j = \min_{1 \leq i \leq k} \{x_{ij}\}$  respectively. The normalized value uses  $s_{ij}$  represent it.

If evaluation index  $\beta_j$  is benefit-type, then

$$s_{ij} = \frac{x_{ij}}{r_{\max}^j} \quad (5)$$

If Evaluation index  $\beta_j$  is cost-type, then

$$s_{ij} = 1 - \frac{x_{ij} - r_{\min}^j}{r_{\max}^j - r_{\min}^j} \quad (6)$$

For this, the normalized matrix  $S = (s_{ij})_{n \times k}$  [11] could be obtained.

#### B. The determination of weight matrix

Flight training course set by training program. We get the weighting matrix by expert scoring through questionnaire survey. In the questionnaire, The importance of evaluation indexes in each training course sets "0-3" classes. If the relationship between them is small scoring 0, has certain relation scoring 1, has a close relation scoring 2, has a very important relationship scoring 3. Then normalized the data of questionnaire investigation according to flight course. The handled matrix names sample matrix. And then, put all the sample matrix weighted average, get the weight matrix.

trainer C172S, P2010, DA40 and SR20 for efficiency calculation. The time of each training course shows in

table 1. Table 2 and table 3 show the value of aircraft scoring, evaluation indexes and comprehensive weight by expert

TABLE I. THE TRAINING TIME OF EACH COURSE

	Training course						Tot-al
	Course1	Course 2	Course 3	Course 4	Course 5	Course 6	
Time	4 hours	6 hours	18 hours	6 hours	6 hours	3 hours	43 hours

TABLE II. THE VALUE OF PERFORMANCE INDEX

		Trainer			
		C172S	P2010	DA40	SR20
Performance index	Take-off distance(ground roll)(m)	293	245	352	409
	Take-off distance(50 foot obstacle)(m)	204	139	158	188
	landing distance(ground roll)(m)	175	200	241	313
	landing distance(50-foot obstacle)(m)	232	113	281	309
	Maximum rate of climb(m/s)	3. 71	5. 10	5. 75	4. 20
	Ceiling (km)	4. 27	4. 57	5. 00	5. 33
	Cruise speed of design(km/h)	230. 0	260. 0	227. 8	287. 0
	Maximum range(km)	1185	1324	1341	1454
	Maximum take-off weight(kg)	1157	1199	1198	1361

TABLE III. THE VALUE OF COMPREHENSIVE WEIGHT BY EXPERT

		Training course					
		Course 1	Course 2	Course 3	Course 4	Course 5	Course 6
Weight Of Performance index	Take-off distance, ground roll(m)	0.0963	0.1033	0.0997	0.1053	0.0969	0. 1191
	Take-off distance,50 foot obstacle(m)	0.1036	0.1033	0.2665	0.1053	0.0969	0. 1082
	landing distance, ground roll(m)	0.1036	0.1033	0.0997	0.1053	0.0969	0. 1040
	landing distance,50-foot obstacle(m)	0.0720	0.0533	0.0411	0.0553	0.0508	0. 0952
	Maximum rate of climb(m/s)	0.1013	0.1029	0.1084	0.0962	0.1046	0. 0997
	Ceiling (km)	0.1013	0.0951	0.1036	0.0962	0.0969	0. 1125
	Cruise speed of design(km/h)	0.1408	0.1386	0.1036	0.1493	0.1551	0. 1179
	Maximum range(km)	0.1384	0.1468	0.1384	0.1405	0.1551	0. 1330
	Maximum take-off weight(kg)	0.1036	0.1033	0.0997	0.0966	0.0969	0. 0973

From table 1, we know

$$F = [0.0930 \quad 0.1395 \quad 0.4186 \quad 0.1395 \quad 0.1395 \quad 0.0689]^T$$

The normalized matrixes  $S$  is calculated from table 2 using formula (5) and (6). Its value is

$$S = \begin{bmatrix} 0.8826 & 1.0000 & 0.7384 & 0.5590 \\ 0.8802 & 0.4115 & 1.0000 & 0.9219 \\ 1.0000 & 0.9201 & 0.7891 & 0.5591 \\ 1.0000 & 0.7412 & 0.8435 & 0.7540 \\ 0.6452 & 0.8870 & 1.0000 & 0.7304 \\ 0.8011 & 0.9574 & 0.9381 & 1.0000 \\ 0.7986 & 0.9028 & 0.7910 & 1.0000 \\ 0.8150 & 0.9106 & 0.9223 & 1.0000 \\ 0.8501 & 0.8523 & 0.8802 & 1.0000 \end{bmatrix}$$

According to formula(2) and (3) ,the cross-efficiency vector could be calculated, its value is

$$E = \text{diag}(0.8827 \quad 0.6279 \quad 0.8047 \quad 0.6783)$$

At last, put  $S$ 、 $E$ 、 $W$  and  $F$  to formula (1) and (4).The result is shown in table 4.

TABLE IV. THE RESULTS OF TWO KINDS OF MODEL

	Masefield-Burdak model	Based on DEA cross-efficiency model
C172S	0.871	0.770
DA40	0.867	0.696
SR20	0.837	0.570
P2010	0.837	0.526

Table 4 shows that for this pilot school' commercial single engine training ,the C172S training effectiveness is the highest, followed by the DA40, SR20 and P2010.Results show that the trainer training effectiveness evaluation model based on DEA cross-efficiency is consistent with Masefield - Burdak model. Correctness of the model is verified. The results also shows more precision than Masefield - Burdak model.

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#### VI. CONCLUSION

This article discusses the Masefield-Burdak trainer training effectiveness evaluation model, and uses DEA cross-efficiency model to revise it, making up the shortage of it. When using revised model for evaluation, the benefit-type indexes of evaluation unit is used as cross efficiency model output indexes, cost-type indexes as input indexes. The Correctness of the model is

verified, consistency with Masefield -Burdak model evaluation results, shows that the model is feasible.

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