The mathematic model of evaluation of enterprise activity

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Abstract—It is a general concerned by whole society that how long the firn can last. Investigating on enterprise's survival state, it research on the life table of enterprise based on the concept of life table in demographic. So as to gain the period that high death rate take place and the average life expectancy. According to the information above, the enterprise can make corresponding strategy and government can conduct the enterprise in healthy developing ways.But le this model has certain problems from personal views. Hence it also brings out the revised commercial age model on the basis of Japanese research.

Keywords-enterprise; activity; natural age; commercial age; mathematic model

I. INTRODUCTION

As an organic community in the social ecological environment, the enterprise, like other individual organisms, follows the circulation law of nature— birth, growth, decline and death. While in different life cycle of enterprises have different dynamic performance.The purpose of this paper is to throuth the study of natural age,business enterprise age and enterprise survival situation on enterprise mortality ,so as to gain the period that high death rate take place and the average life expectancy According to the information above, the enterprise can make corresponding strategy and government can conduct the enterprise in healthy developing ways.

II. ENTERPRISE AGE- SPECIFIC MORTALITY RATE AND AGE-SPECIFIC MORTALITY RATE

Age indicates the situation of life entity and the measurement of its survival condition in its entire life circle. The natural age of the enterprise refers to the time span the enterprise undergoes since its establishment, showing the length of the enterprise's survival time.

The analysis of the enterprise's lifetime includes dynamic analysis and static analysis. The dynamic analysis studies the whole process of a group from the emergence of the first enterprise to the extinction of the last enterprise. While the static analysis analyzes from one cross section the experience of survival and death of a group during a certain period. In Demography, the age-specific mortality rate and death rate are the most basic indicators used in analyzing the death of population. Here, enterprise age-specific mortality rate means the possibility of an enterprise dies in a certain age range. There is the formula:

$$q_i = \frac{\text{Death formed within the age range}(x_i, x_{i+1})}{\text{When } x_i \text{ is still surviving companies}} \quad (1)$$

Besides this, the enterprise age- specific mortality rate can also be defined as the ratio between the numbers of the dead companies and those of the survival companies within a certain age range. Here is the formula:

$$M_i = \frac{\text{In the age range } (x_i, x_{i+1}) \text{ for death companies}}{\text{In the age range } (x_i, x_{i+1}) \text{ for alive companies}} \quad (2)$$

In figure1, X-axis indicates the enterprise ages, while the axis of bank L(x) indicates the survived enterprise numbers. From this curve we can know: dx_i indicates the number of the dead enterprises in the interval $(x_i, x_i + 1)$, and the formula is : $dx_i = L_i - L_{I+1}$ (3)

while the age-specific mortality rate can be indicated like this: $q_i = dx_i / L$ (4)

The estimated formula of the age-specific mortality rate is :

$$M_i = \frac{dx_i}{\int_{x_i}^{x_{i+1}} L(x)dx} \qquad x_i < x \le x_{i+1} \quad (5)$$

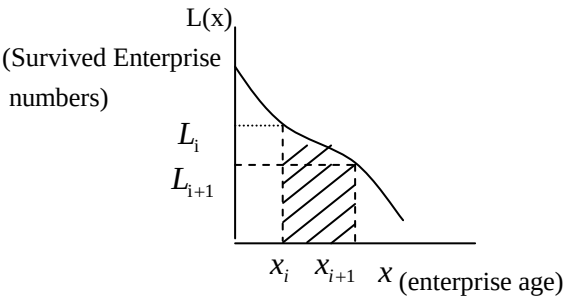


Figure 1. The Relation Between Enterprise Numbers and Ages

As the area of the shaded part can be viewed as the sum of rectangle and triangle, the integral formula can be replaced by ni(Li－dxi)+0.5nidxi , ni is the length of the

age interval, so that the formula above can be converted into the following: $$M_i = \frac{dx_i}{n_i[L_i - dx_i] + 0.5 n_i dx_i}$$

III. THE COMPILATION METHOD OF THE ENTERPRISE LIFE TABLE

A. *The Compilation Theory of Life Table*

Life table is the chart that can most clearly and most directly reveal the process in which how the biotic population stays alive and dies. It's also a useful tool when describing the process. Life table can be divided into two types: dynamic life table and static life table.

The dynamic life table compiles a life table based on observing the data obtained from process in which a group of enterprises, established at the same time, die and survive. It records the practical experience of death from the appearance of the first enterprise to the disappearance of the last one in one enterprise group.

The static life table means to do an age-structural survey on death situation inside the enterprise population in a special period of time, on the basis of which to analyze, from a cross section, the death and living experience of an enterprise group within a certain period of time . it mainly depends on the age-specific mortality rate in the time interval when preparing the life table. The static life table is the most effective tool used to conglomerate the experience of living and death of enterprise group. The concrete enterprise life table is shown in table 1:

TABLE 1 ENTERPRISE LIFE TABLE

Age Interval	Remaining Companies (li)	Enterprise Deaths （di）	Mortality Rate (qi)	Enterprise average number of survival tears (*Yi*)	Total interprise age (*Ti*)	Average expectation of life (Ei)

The first rank indicates the Age Interval. If the length is 10, it can be divided into 6 intervals: 0-10, 10-20, 20-30, 30-40, 40-50 and above 50.

The second rank li indicates the number of the survival enterprises in age interval.

The third rank di indicates the number of the dead enterprises.

The fourth rank qi indicates the number of survival enterprises at the age of x, and the probability of the number of the dead enterprises at the age of x+10.

The fifth rank Yi indicates the survival years of all enterprises that are entering into age in the certain interval. The formula is that:

$$Y_i = \int_{x_i}^{x_{i+1}} L(x)dx \qquad x_i < x \le x_{i+1}$$

The function continuously changed in（xi,xi+1] can be approximately replaced by the weighted average of the number of enterprises at the beginning and the end of the period: $$Y_i = 0.5*(L_i + L_{i+1})$$

The sixth rank Ti indicates the summation of the survival years of Li enterprises after the age of xi, and the formula is : $$T_i = \sum_{n=i}^{\infty} Y_i$$

The seventh rank Ei indicates the estimate value, called life expectancy, of the average continuous living years of the enterprises enter into the age of x,. And the formula is:

$$E_i = T_i / Y_i$$

B. *The Compilation of the Specific Life Table*

The key for compiling a life table is to obtain the concrete data of the death of enterprise, and then synthesize the data, calculating the number di of dead enterprises at each step, such as the number of enterprises died at the age of 0-10, 10-20, 20-30and so on. After obtaining these data we can compile the life table. Table 2 is a concrete life table.

TABLE 2 ENTERPRISE LIFE TABLE IN AN AGE

Age $(x_i,x_{i+1}]$	Number of survival enterprises (l_i)	Number of dead enterprises （d_i）	Probability of death (q_i) %	Average enterprise survival years (Y_i)	Total enterprise ages (T_i)	Expectation of life(E_i)
0—10	412	155	37.62	3345	9420	22.86
11—20	257	47	18.29	2335	6075	23.63
21—30	210	65	30.95	1775	3740	17.81
31—40	145	32	22.07	1290	1965	13.55
41—50	113	102	90.27	620	675	5.97
50 以上	11	11	100.00	55	55	5

Data in the line has been given and the others are calculated based on formulas. The table above reflects that the two peaks of enterprise death occurs respectively at the age group of 40-50 and that of 0-10, when the probabilities of death are relatively higher, respectively 90.27% and 37.62%. From expectation of life, we can know that enterprise expectation of life reaches the highest point at the age group of 10-20, which showing strongest survival ability of enteprises.

IV. ENTERPRISE COMMERCIAL AGE

A. *The Conception Of Commercial Age*

Commercial age, which directly reflects the speed of development, competitive capacity, operating capability and economic benefit of an enterprise and other information, is the quantitative criteria of enterprise vitality.

It has nothing to do with enterprise age. Japanese research shows that enterprises, whose commercial age is usually younger than age, have a stronger competitive capability in each industry. Table 3 reflects the commercial age and age of some excellent Japanese enterprises.

Age just reflects the length of survival of enterprise and possesses irreversibility. While the commercial age reflects an enterprise's operating performance and growth, and it doesn't necessarily increase by the time going.

TEABLE 3 AGE AND COMMERCIAL AGE OF SOME TAPAN ENTERPRISE(1993)

Enterprise name	Established time（year/month）	Age（year）	Commercial age (year)
Toyota	1937.8	56	36.1
Honda Motor Co., Ltd.	1948.9	45	38.6
Sony	1946.5	47	33.6
Matsushita Electric Industrial Co	1918.3	75	36.5
Nippon Electronics Corp	1899.7	94	35.2
Fujitsu Corporation	1935.6	58	33.9
Canon	1937.8	56	33.1
Ricoh Corporation	1936.2	57	36.6
FUJIFILM	1934.1	59	42.6

Source：Han Furong，Xu Yan. Enterprise Bionics. Enterprise Management Publishing House，2001：176

B. Japanese Commercial Age Pattern

Japanese researchers divided listed companies into two parts: longer and shorter, and extracted 20 sample enterprises from each sect, calculating an enterprise's commercial age through these business index data. The specific methods are as follows:

1) Calculation of specific data of three indicators

The basis of calculating commercial age is the index data of each enterprise. So Japanese scholars designed three indicators to reflect the commercial age of an enterprise.

The formula of each indicator:

Average growth rate of sales（x1）=($\sqrt[5]{\text{A recent years of sales/Sales of five years ago}}$ —1）×100%

Average age of employees(x2)= Average age of employees×（－1）

Euipment age

$$(x_3)=\frac{\ln\left[\text{Gross assets depreciation object}/\left(\begin{array}{l}\text{Gross assets depreciation object}+\\ \text{Stylish fold the accumulative amount of fixed assets}\end{array}\right)\right]}{\ln\left[\text{Gross assets depreciation object}/\left(\text{Gross assets depreciation object}+\text{real depreciation}\right)\right]}\times(-1)$$

Being Multiplied by (-1) is to convert the two inverse indicators into positive indicators. The average age of employees and the equipments age can be calculated according to the data in a recent year, and the depreciation of equipment can be extracted according to straight-line depreciation method.

2)Standardization of Index

Standardization in accordance with the following formula: $x'_{ij}=\frac{x_{ij}-\bar{x}_j}{s_j}\qquad (i=1,2..n,\ j=1,2..m)$

n is the number of simples and m is the number of variables.

There into： $\bar{x}_j=\frac{1}{n}\sum_{i=1}^{n}x_{ij}$ (*j*=1,2…m)

$$s_j=\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{ij}-\bar{x}_j)^2}\qquad (j=1,2\ldots m)$$

When one certain index exceeds ± 3 times the standard deviation of the index of the sample enterprises, the value is the standard deviation that is ±3 times the average value.

3)Calculation of Integrated Value of the Three Indexes

Converting the standardized value of three indexes into integrated score according to the following formula, and then get the score of each sample enterprises Zi.

$$Z_i=0.30486+0.10413\times x'_{i1}+0.14263\times x'_{i2}+0.14201\times x'_{i3}$$

The overall average score value of sample enterprises is $\bar{Z}$：

$$\bar{Z}=0.30486+0.10413\times\bar{x}_1+0.14263\times\bar{x}_2+0.14201\times\bar{x}_3$$

4)Conversion from integrated score into enterprise age

When $Z_i>\bar{Z}$, enterprise commercial age SL_i is:

$$SL_i=40-\frac{Z_i-\bar{Z}}{Z_{\max}-\bar{Z}}\times 20$$

Z_{min} is the value of the enterprise that gets the highest score among the sample enterprises

When $Z_i<\bar{Z}$， enterprise commercial age SL_i is ：

$$SL_i=40+\frac{\bar{Z}-Z_i}{\bar{Z}-Z_{\min}}\times 40$$

Z_{mi} is the value of the enterprise that gets the highest score among the sample enterprises

Through the conversion, enterprise commercial age is limited in the interval 20-80.

C. Evaluation and Analysis of the Commercial Age Model in Japan

Above all, the original intension of constructing model of Japanese commercial age is to calculate enterprise commercial age through the conversion of data. It's in essence the comprehensive evaluation of enterprise vitality, on the basis of which realizing the conversion from evaluation score to age through linear interpolation. The technical route is shown in figure3:

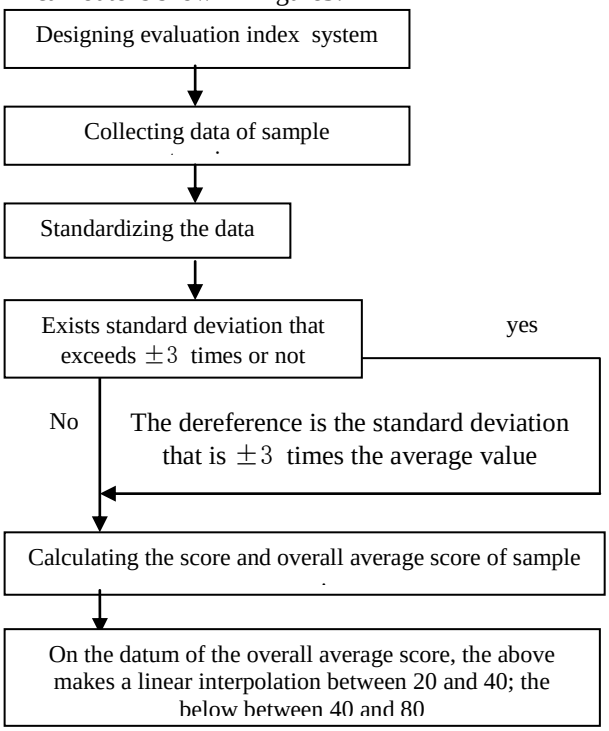


Figure 3. the Technical Route of Japan Commercial Age Model

The problems are:

1) Defect in Designing Index System

This index system uses only 3 indexes, without enough strength, to reflect enterprise's state of operation. Besides, in the process of designation the definition of indexx3 is ambiguous, and the formula just laboriously defines the result as enterprise age not the real equipment age.

2) Data Standardization Problems

The use of standardization does a man-made change in data, that is, when the data index exceeds ±3 times the standard deviation of the sample enterprises indicators the value is the standard deviation ±3 times mean value.

3) Vague Definition of Age Conversion Formula

For each enterprise the constant term 0.30486 is the same in score calculation formula, so that whether the project exists or not will not influence enterprise commercial age. From its original index definition, all three indicators are designed to be positive indicators, that is , the bigger the index value, the better it is. Under the premise that all indexes are positive ones, the current operating state of the enterprise with the highest score value should be better, but the commercial age of the enterprise is defined 20 years of age; in contrast, the commercial age of the enterprise with the lowest score value is defined 80 years of age.

V. Revised Model of Commercial Age

Here is the revised mathematical model of commercial age against the problems exists in the model. The following is the concrete steps.

A. Setting Up Index System

The revised index system consists of four indicators: Enterprise sales growth rate, Average age of employees, Capital return, Strength put into research .Calculation of each index in the index system is as follows:

Enterprise sale growth rate （x_1）=(enterprise sales for this year—enterprise sales for last year)/enterprise sales for this year×100%

Average age of employees (x_2)= average age of employees in this year×(-1)

Strength put into research and development （x_3）= research and development expenses/amount of profit for this year×100%

Capital return（x_4）=total profit/net value of fixed assets ×100%

The four indicators reflect the operating state of an enterprise from four aspects—growth, younger degree of employees, strength put into technological development and economic benefit , and this can basically meet the requirement of commercial age.

B. Ensuring the Weight among Indicators

The rationality of the weight among indicators can influence the veracity and rationality of the result to a large extent. Here adopt the analytic hierarchy process (AHP) to determine the weight among indicators（w1,w2,w3,w4）, and basic principle of AHP will not be specified.

C. Using principal component analysis method to evaluate the vitality of enterprises

The principle component analysis can simplify this case and find several multi-stresses, which are not related to each other, to represent the original numerous variables and to reflect the information of original variables as more as possible by Dimension reduction method.

The principal component analysis eliminates the influence of the related factors between indicators. The calculation steps of the principal componential analysis are as follows:

1) Standardizing the Data

Centralizing the column at first , then standardizing through standard deviation:

$$x_{ik}' = \frac{x_{ik} - \bar{x}_k}{s_k} \qquad (i=1,2,\cdots,n;k=1,2,\cdots,p)$$

Here: $\bar{x}_k = \frac{1}{n}\sum_{i=1}^{n} x_{ik}$ $(k=1,2,\cdots,p)$

$$s_k^2 = \frac{1}{n-1}\sum_{j=1}^{n}(x_{ik}-\bar{x}_k)^2 \qquad (k=1,2\cdots p)$$

After standardizing the data, put the weight obtained from AHP into standardized data.

2) Calculating the Covariance Coefficient Matrix

$$r_{ij} = \frac{\sum_{k=1}^{n}(x'_{ki} \cdot x'_{kj})}{n-1} \qquad (i, j=1, 2, \cdots, p)$$

3) Counting the Eigenvalue:

Corresponding to the correlation coefficient matrix R , the P Eigenvalues $\lambda_1 > \lambda_2 > ... > \lambda_p \geq 0$ of the characteristic equation $|R - \lambda I| = 0$ can be calculated by Jacobi Method， the corresponding eigenvectors of the eigenvalue λ_i is：

$$C_i = (C_1^i, C_2^i, \cdots C_p^i) \quad (i=1, 2, \cdots, p)$$

Constructing new factors：

$$Z_i = \sum_{k=1}^{p}(C_k^i \times X'_k) \qquad (k=1, 2, \cdots, p)$$

4) 4）Calculating the Score：

If the rate at which the sum of variance of the first m principal components Z_1、Z_2、$\cdots Z_m$（$m<p$）to the total variance is ：

$$a = \frac{\sum_{i=1}^{m}\lambda_i}{\sum_{i=1}^{p}\lambda_i}$$

When a is approximate to 1(such as $a \geqslant 0.85$) the first m factors can be selected so that the number of the factors will be reduced from p to m, which has the effect of filtering factors.

The score of sample i:

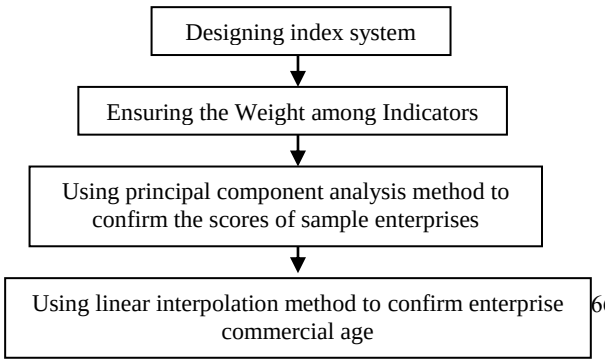
$$Y_i = \sum_{k=1}^{m}(b_k \times Z_{ki}) \qquad (i=1, 2\cdots p)$$

Z_{ki} is the factor score，and b_i is the rate at which the eigenvalue i to all eigenvalues：

$$b_i = \frac{\lambda_i}{\sum_{i=1}^{p}\lambda_i}$$

D. Using linear interpolation method to achieve the conversion of enterprise score to commercial age

Indicators used here are positive ones so that the commercial age corresponding to the enterprise with the highest scores should be the best. It is assumed that the commercial age corresponding to the highest value of each indicator is 30 years old, and the minimum is 1 year old. In other words, the maximum age of the four indicators is 30 years old; the minimum age is 1 year old. The overall commercial age of enterprise is limited between the ages of 1-30, and within this range, the older the age, the stronger vitality an enterprise has.



Converting, as following, the index data based on above assumption

$$F_i = \frac{Y_i - Y_{i\min}}{Y_{i\max} - Y_{i\min}} \times 29 + 1$$

Here： Y_i ——The score of enterprise i；

Y_{imin} ——the value of sample enterprise with the lowest score；

Y_{imax} ——the value of sample enterprise with the lowest score；

F_i —— the commercial age of sample enterprise i。

Technical line of the improved commercial age calculation model is as shown in Figure 5

VI. CONCLUSIONS

Productivity emphasizes that necessary labor time of products should be lower than social necessary labor time on average; while trading emphasizes that costs of internal management and coordination should be lower than that of the external market transaction. Whether from the perspective of production or trading, enterprise, after all, has life circle. Enterprise age studies the growth strategy of the enterprise in the various stages of the life cycle from the perspective of business survival process.

REFERENCES

[1] Zhang Jinshi. Understanding of Life Expectancy Estimates in "Life table". Journal of Wuhan Education Institute, 1997, (6) : 52—55.

[2] Han Furong , Xu Yanmei .Enterprise Bionics. Enterprise Management Press, 2001.

[3] Li Ye. Correction Model and Thinking of Enterprise Life Circle. Southern Economic Journal 2000, (2) : 47—50.

[4] Anne Beatt , Anne Gron,Bjorn Jorgensen.Corporate risk management:evidence from Produet liability[J].Journal of Finaneial Intermediation,14(2005)152 一 178

[5] Alan Kirman,Jan Tuinstra Bounded rationality,heterogeneity and market dynamies Journal of Eeonomie Dynamics and Control,Volume29,Issue4,APril2005:595 一 600

[6] Alan Kirman,Jan Tuinstra Bounded rationality,heterogeneity and market dynamies Journal of Eeonomie Dynamics and Control, Volume29, Issue4, APril2005:595 一 600

[7] Cui Shijuan, Wang Zhiqiu , Enterprise dynamic capability research were reviewed[J], Journal of shenzhen university,2008(2)

[8] Herbert Simon, Rational choice behavior model[C], The capital economic and trade university press,2002:186-193

[9] Li Weian, Modern enterprise vitality theory and evaluation[M], China financial economic publishing house,2002

[10] Gu Ligang, Corporate life analysis[J], Journal of Beijing university of technology,2002(4)