

Fixed airflow distribution of ventilation network with maximum flow algorithm

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Abstract—Whether there is a feasible flow of ventilation network that meets the demand of fixed airflow and the balance of airflow, when the number, quantities of fixed branches are all uncertain. It is called fixed airflow distribution problem, which was transformed into flow distribution problem. All branches have the upper and lower capacities bounds. The upper and lower capacities of fixed airflow branches were set to their values, while the upper capacities of other branches were set to infinity, and lower capacities were set to zero. Two virtual sources-sink nodes was added to the network, while the fixed branches were splitted into two new branches connected with the virtual source-sink nodes. The classical max flow algorithm was adopted to solve the new network. There was a solution of fixed airflow distribution when the new network had a feasible flow and each splitted branches reached its capacity. At last, the method was analysed by means of two examples.

Keywords—ventilation network; fixed airflow; independent circuit; max flow; the upper and lower capacities

I. INTRODUCTION

The first step of ventilation network solution is to distribute airflows that satisfied with the flow conservation law. If there are fixed airflow branches, the traditional methods just consider that fixed airflow branches are all in the remaining tree [1]. But in practical application, fixed airflow branches do not always fall in the remaining tree due to the restrictions of ventilation network topology. A self-adaption method was proposed to estimate the resistance of some tunnels [2]. The key of this method lies in how to determine the existence of the airflow distribution solution, sink the demand of fixed airflow and the balance of airflow, while the parameters of fixed airflow branches such as position, quantity are uncertain.

These problems above are all related to the fixed airflow distribution algorithm. In this paper, a new algorithm was proposed on the basis of max flow algorithm with the upper and lower bounds, compared with the algorithm based on argument path.

II. RESISTANCE INVERSION BASED ON FIXED AIRFLOW

Ventilation resistance measurement is an important technical means to obtain ventilation network basic data such as resistance and airflow. Errors exist in all measuring; therefore some resistances are far from the true values [3]. A self-adaption method was proposed to correct the resistance of some tunnels. Some airflow of tunnels with

large errors is fixed artificially, and then we can correct inversely the resistance of these tunnels.

For a single source-sink ventilation network with n branches and m nodes, we handpick a spanning tree to determine $r=n-m+1$ independent cycles and r linearly independent branches in the remaining tree. So the airflow of any branch can be represented by the linear function of these independent branches [4].

$$q_i = \sum_{s=1}^r c_{si} q_{ys} \quad (i=1, \dots, n) \quad (1)$$

In the meanwhile, r independent cycles are satisfied with the resistance balance law:

$$f_s = \sum_{i=1}^n c_{si} r_i q_i |q_i| - h'_s = 0, \quad (s=1, 2, \dots, r) \quad (2)$$

Where c_{si} is the direction coefficient of i th branch in the s th cycle, q_{ys} is the airflow of the s th independent branch, m^3/s , q_i is the airflow of the i th branch, m^3/s , r_i is the resistance of the i th branch, kg/m^7 , h'_s is the additional resistance of the s th independent cycle, Pa , and f_s is the resistance sum of the s th independent cycle, Pa .

We can suppose that k branches with fixed airflow are known. In order to facilitate the description, branches will be renumbered. Fixed airflow branches are $1, 2, \dots, k$ and other branches are $k+1, k+2, \dots, n$. So the resistance of independent cycles with fixed airflow branches can be detailed as flows:

$$\begin{aligned} f_s &= \sum_{i=1}^n c_{si} r_i q_i |q_i| - h'_s \\ &= \sum_{i=1}^k c_{si} r_i q_i |q_i| - (h'_s - \sum_{i=k+1}^n c_{si} r_i q_i |q_i|) \\ &= \sum_{i=1}^k c_{si} r_i q_i |q_i| - g_s \end{aligned} \quad (3)$$

Where g_s is the resistance sum of branches that do not contain fixed airflow branches in the s th independent cycle, Pa . If the i th branch is in the s th cycle, $c_{si} = \pm 1$, otherwise $c_{si} = 0$.

Let $\alpha_{si} = c_{si}q_i|q_i|$, equation (3) can be simplified as:

$$f_s = \sum_{i=1}^k \alpha_{si}r_i - g_s \quad (4)$$

In order to ensure that independent cycles are satisfied with the resistance balance law, let $f_s = 0$. It is supposed that the number of independent cycles with fixed airflow branches is l , so the above equation can be developed as a linear equations:

$$\begin{cases} \alpha_{11}r_1 + \alpha_{12}r_2 + \cdots + \alpha_{1k}r_k = g_1 \\ \alpha_{21}r_1 + \alpha_{22}r_2 + \cdots + \alpha_{2k}r_k = g_2 \\ \cdots \\ \alpha_{l1}r_1 + \alpha_{l2}r_2 + \cdots + \alpha_{lk}r_k = g_l \end{cases} \quad (5)$$

The matrix coefficient $A^{l \times k}$ is denoted as follows:

$$A = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1k} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{l1} & \alpha_{l2} & \cdots & \alpha_{lk} \end{bmatrix} \quad (6)$$

Let $x = (r_1, r_2, \cdots, r_k)^T$, $b = (g_1, g_2, \cdots, g_l)^T$, then:

$$Ax = b \quad (7)$$

According to the theory of linear algebra, the solution of linear equations (7) is denoted as follows [5]:

- 1) When $r(A) = r(A, b) = k$, equation (7) has a unique solution, the wind resistance of fixed airflow branch is also the only one.
- 2) When $r(A) = r(A, b) < k$, there are infinitely many solutions of equation (7).
- 3) When $r(A) \neq r(A, b)$, there is no solution.

III. FIXED AIRFLOW BALANCE ALGORITHM BASED ON AUGMENTING PATH

From the perspective of network flow, fixed airflow balance problem can also be treated as flow distribution problem. The capacities of fixed airflow branches are equal to its values while the capacities of other branches are infinite. A fixed airflow balance algorithm based on augmenting path was proposed [2]. It is similar to the max-flow algorithm based on augmenting path. It continually finds augmenting paths and augmenting flow among all paths until there is no such augmenting path in the network.

For the flow network $G = (V, E)$ with a single source-sink node, each branch (u, v) in E has a capacity c , and c satisfies the condition $c(u, v) \geq 0$. The capacities of the fixed airflow branches are equal to their fixed its values, while the capacities of other branches are all infinite. Each branch (u, v) is given airflow $f(u, v)$. The key of the augmenting path algorithm is building the residual network G' , each branch (u, v) in the original network G is mapped as a forward branch (u, v) and a backward branch (v, u) . The capacities of the two branches can be denoted as follows [6]:

$$\begin{cases} c'(u, v) = c(u, v) - f(u, v) \\ c'(v, u) = f(u, v) \end{cases} \quad (10)$$

Let P be an augmenting path, δ be an augmenting airflow. δ is denoted as $\delta = \min_{(u,v) \in P} \{c'(u, v)\}$. We can

search an augmenting path P in the residual network G' , add δ to the quantities of the forward branches in P , and reduce δ from the quantities of the backward branches in P . So the cycle repeats until there is no P can be found in G' .

Fig. 1 shows a ventilation network example taken from literature [2] and TABLE 2 shows the two different augmenting processes and results. The max-flow of the two different augmenting processes is both 120. Obviously, the search order of augmenting path makes a direct influence on the final results.

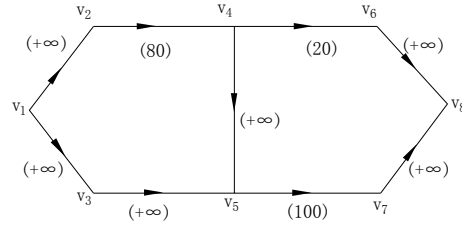


Figure 1. Ventilation network with fixed airflow (from: reference[2], P137)

TABLE I. TWO AUGMENTING PROCESSES OF VENTILATION NETWORK AS SHOWN IN FIG. 1

Augmenting process proposed in reference [2]	A new augmenting process
1) augmenting path $P_1 = \{v_1, v_2, v_4, v_6, v_8\}$, augmenting airflow 20, saturated branch (v_4, v_6)	1) augmenting path $P_1 = \{v_1, v_3, v_5, v_7, v_8\}$, augmenting airflow 100, saturated branch (v_5, v_7)
2) augmenting path $P_2 = \{v_1, v_2, v_4, v_5, v_7, v_8\}$, augmenting airflow 60, saturated branch (v_2, v_4)	2) augmenting path $P_2 = \{v_1, v_2, v_4, v_6, v_8\}$, augmenting airflow 20, saturated branch (v_4, v_6)
3) augmenting path $P_3 = \{v_1, v_3, v_5, v_7, v_8\}$, augmenting airflow 40, saturated branch (v_5, v_7)	3) no augmenting paths can be found
Conclusions: all fixed airflow branches are saturated and the max-flow value is 120.	Conclusions: fixed airflow branch (v_2, v_4) is not saturated, the residual airflow is 60, and the max-flow value is 120.

IV. FIXED AIRFLOW BALANCE ALGORITHM BASED ON MAX-FLOW ALGORITHM

The augmenting path algorithm only took into account the upper bound of the branch capacity. In this paper, both the lower and the upper bound of the branch capacity were taken into account. The upper and lower capacity of the fixed airflow branches is equal to its quantities. So the fixed airflow distribution problem was transformed into determining whether there is a feasible flow of the network limited by the upper and lower capacity bounds. All the flow problems can be solved by the max-flow algorithm.

The classical max-flow algorithm only considers satisfying the upper capacity bound [7]. Besides, the airflow of the branch must satisfy the lower capacity bound in the network that has the upper and lower bounds. In the premise that the flow network has upper and lower bounds, we built an extra network without the source-sink nodes, splitted the lower capacity into a new branch, and

resort to the max-flow algorithm for determining the feasible flow.

A. Definition of flow network with the upper and lower bounds and the source-sink nodes

Different from the flow network definition of the traditional max-flow problem, quantities of the branches in flow network *with* upper and lower bounds must be greater than or equal to a certain lower bound. For the flow network $G = (V, E)$ with a single source-sink node, s is the Source, and t is the sink node. Each branch (u, v) in E has a lower capacity bound $b(u, v)$ and an upper capacity bound $c(u, v)$, and they satisfy the equations: $b(u, v) \geq 0$ and $c(u, v) \geq 0$. Each branch (u, v) is denoted by a flow $f(u, v)$. If f satisfies two conditions as follows, it is a feasible flow of G .

(1) Flow balance conditions: i is one of the nodes in the network except the Source s and the sink node t , and it must satisfy the equation:
$$\sum_{(u,i) \in E} f(u, i) = \sum_{(i,v) \in E} f(i, v)$$

(2) Capacity bounds conditions: any branch (u, v) in E must satisfy the equation: $b(u, v) \leq f(u, v) \leq c(u, v)$.

B. Feasible flow in the network with the upper and lower bounds and without the source-sink nodes

From the perspective of network flow, the flow is a circulation flow, and each node's inflow and outflow are equal, satisfying the flow conservation law[8]. Considering a flow network C with the upper and lower bounds and without the source-sink nodes, it demands that any node should satisfy the airflow balance and the capacity bounds, and find a feasible flow.

The flow of the branch is defined by $f(u, v)$, which must be greater than or equal to the lower capacity.

$$f(u, v) = b(u, v) + g(u, v) \quad (11)$$

we can ensure that $f(u, v) \geq b(u, v)$, if $g(u, v) \geq 0$.

$$g(u, v) \leq c(u, v) - b(u, v) \quad (12)$$

Let $c'(u, v) = c(u, v) - b(u, v)$. We can construct a new flow network $C' = (V, E)$, $g(u, v)$ is the flow of the branch. In new network C' , and $c'(u, v)$ is the capacity of the branch.

In order to deal with the lower bounds $b(u, v)$, equation (11) was put into the airflow balance expression.

$$\sum_{(u,i) \in E} [b(u, i) + g(u, i)] = \sum_{(i,v) \in E} [b(i, v) + g(i, v)] \quad (13)$$

$$\sum_{(i,v) \in E} g(i, v) - \sum_{(u,i) \in E} g(u, i) = \sum_{(u,i) \in E} b(u, i) - \sum_{(i,v) \in E} b(i, v) \quad (14)$$

The equation right end is denoted by $m(i)$, which is the sum of the inflows minus the sum of the outflows of the i th node's lower bound.

$$m(i) = \sum_{(u,i) \in E} b(u, i) - \sum_{(i,v) \in E} b(i, v) \quad (15)$$

Excluding the excess flow $m(i)$ from the network C' in order to make the nodes in the network C' satisfy the airflow conservation conditions:

$$\sum_{(i,v) \in E} g(i, v) = \sum_{(u,i) \in E} g(u, i) \quad (16)$$

$m(i) > 0$ means that the inflow of lower capacity is greater than the flow of lower capacity, and make excess flow $m(i)$ transfer to a new branch (s', i) . The capacity of the new branch is:

$$c'(s', i) = m(i) \quad (17)$$

$m(i) < 0$ mean that the inflow of lower capacity is less than the flow of lower capacity, and make excess flow transfer to a new branch (i, t') . The capacity of the new branch is:

$$c'(i, t') = -m(i) \quad (18)$$

In the network C' , if any branch's flow was saturated, a feasible flow g must be corresponding with a feasible flow f of the original network C . On the contrary, each feasible flow f of the network C must be corresponding with the saturated flow that all branches $g(s', i)$ and $g(i, t')$ reached their capacities.

C. Feasible flow in the network with the upper and lower bounds and with the source-sink node

By means of the method above, we would like to construct an extra network G' as shown in Fig .2, adding a virtual source source s' and a virtual sink t' . Then the branches with the upper and lower bounds were splitted into three new branches according to equation (17) and (18). The first and second branches are (s', v) and (u, t') . The third branch is (u, v) , the capacity of that is equal to $c'(u, v) = c(u, v) - b(u, v)$. At last, the source and the sink are connected by a new branch (t, s) , making the network be of no source-sink.

So the task is to find a feasible flow from s' to t' . If all the outflow branches of s' are saturated, there is a feasible flow in the original network G , which is satisfied with the demand of fixed air flow and the airflow balance.

The classic maximum flow algorithm was adopted to find a feasible flow. There are many max-flow algorithms such as Ford-Fulkerson method, Shortest Augmenting Path (SAP), Edmonds-Karp Algorithm, Dinic Algorithm, Improved SAP Algorithm, Maximum Capacity Path Algorithm, Capacity Scaling Algorithm, Push-Relabel Algorithm and so on. Those maximum flow algorithms are very mature and perfect. Please reference the literature [9, 10].

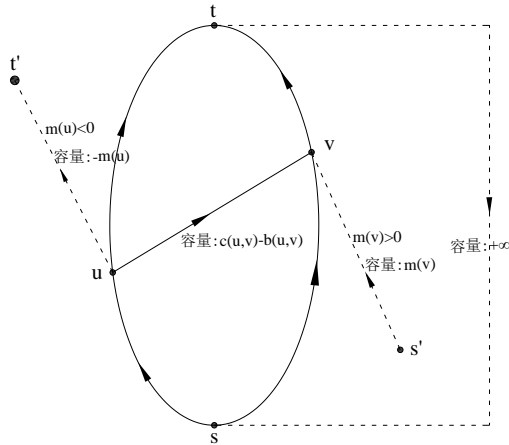


Figure 2. Schematic diagram of additional network G'

V. 4 EXAMPLE ANALYSIS

The example as previously shown in Fig .1 was a single source-sink network, the source and sink respectively was v_1 and v_8 . The upper and lower bounds of the branch capacities were shown in TABLE II. The new branches were added into the extra network according to equation (17) and (18), shown in TABLE III. The final extra network was shown in Fig .3. The capacities of all the branches in the extra network were shown in TABLE IV.

The maximum flow was used to find a feasible flow from s' to t' , then the result was shown in TABLE V. It could conclude that all the ourflow branches of the virtual source node s' were saturated, so there was a feasible solution of air flow, satisfying with both the demand of fixed airflow and the flow balance. The result of fixed air flow distribution was shown in TABLE VI.

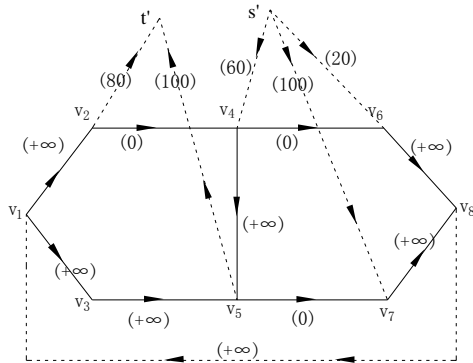


Figure 3. The extra network

TABLE II. THE UPPER AND LOWER BOUNDS OF BRANCH CAPACITY IN VENTILATION NETWORK

Source	Target	Bounds	Source	Target	Bounds
v_2	v_4	80-80	v_3	v_5	0- $+\infty$
v_4	v_6	20-20	v_4	v_5	0- $+\infty$
v_5	v_7	100-100	v_6	v_8	0- $+\infty$
v_1	v_2	0- $+\infty$	v_7	v_8	0- $+\infty$
v_1	v_3	0- $+\infty$			

TABLE III. THE UPPER AND LOWER BOUNDS OF FIXED BRANCH CAPACITY AFTER BEING SPLITTED

Node	The sum of inflows	The sum of the outflows	$m(u)$	Add the branch and capacity
v_2	0	80	-80	$v_2 \rightarrow t'$, $c'=80$
v_4	80	20	60	$s' \rightarrow v_4$, $c'=60$
v_5	0	100	-100	$v_5 \rightarrow t'$, $c'=100$
v_6	20	0	20	$s' \rightarrow v_6$, $c'=20$
v_7	100	0	100	$s' \rightarrow v_7$, $c'=100$

TABLE IV. BRANCH CAPACITIES OF THE EXTRA NETWORK

Source	Target	Capacity	Source	Target	Capacity
s'	v_4	60	v_1	v_2	$+\infty$
s'	v_6	20	v_1	v_3	$+\infty$
s'	v_7	100	v_3	v_5	$+\infty$
v_2	t'	80	v_4	v_5	$+\infty$
v_5	t'	100	v_6	v_8	$+\infty$
v_8	v_1	$+\infty$	v_7	v_8	$+\infty$

TABLE V. THE FEASIBLE FLOW OF EXTRA NETWORK BY MEANS OF THE MAXIMUM FLOW ALGORITHM

Source	Target	Capacity	Flows	Whether saturated
s'	v_4	60	60	Yes
s'	v_6	20	20	Yes
s'	v_7	100	100	Yes
v_2	t'	80	80	Yes
v_5	t'	100	100	Yes
v_8	v_1	$+\infty$	120	No
v_1	v_2	$+\infty$	80	No
v_1	v_3	$+\infty$	40	No
v_3	v_5	$+\infty$	40	No
v_4	v_5	$+\infty$	60	No
v_6	v_8	$+\infty$	20	No
v_7	v_8	$+\infty$	100	No

TABLE VI. THE FIXED AIR FLOW DISTRIBUTION

Source	Target	Airflow	Source	Target	Airflow
v_2	v_4	80	v_3	v_5	40
v_4	v_6	20	v_4	v_5	60
v_5	v_7	100	v_6	v_8	20
v_1	v_2	80	v_7	v_8	100
v_1	v_3	40	-	-	-

Let's modify the network shown in Fig .1, the fixed air flow of the branch (v_5, v_7) is 50. Obviously, there was not a feasible solution for fixed air flow distribution. The extra network was shown in Fig.4 and the result of the max-flow algorithm was shown in TABLE VII. We found that the branch (s', v_4) was not saturated, so there was not a reasonable solution to meet the demand of fixed air flows and the airflow balance.

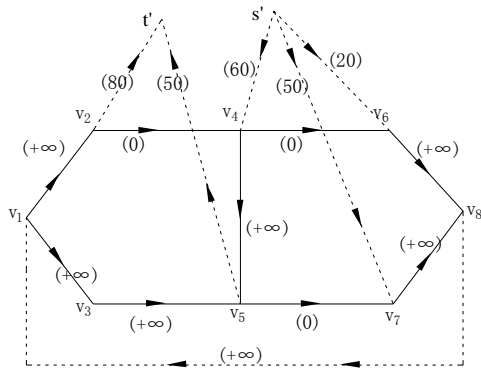


Figure 4. The extra network

TABLE VII. THE FEASIBLE FLOW OF EXTRA NETWORK BY MEANS OF THE MAXIMUM FLOW ALGORITHM

Source	Target	Capacity	Flows	Whether saturated
s'	v ₄	60	50	No
s'	v ₆	20	20	Yes
s'	v ₇	50	50	Yes
v ₂	t'	80	20	No
v ₅	t'	50	50	Yes
v ₈	v ₁	+∞	70	No
v ₁	v ₂	+∞	20	No
v ₁	v ₃	+∞	0	No
v ₃	v ₅	+∞	0	No
v ₄	v ₅	+∞	50	No
v ₆	v ₈	+∞	20	No
v ₇	v ₈	+∞	50	No

VI. CONCLUSION

The algorithm of fixed airflow distribution based on augmenting path is similar with the classical maximum flow algorithm. The drawback of this algorithm is that only the upper bound of branch capacity is taken into account. So the search order of augmented paths directly influences the result of fixed air distribution.

The fixed air flow distribution problem was transformed into finding a feasible flow, which can be solved by means of the max-flow algorithm with the upper and lower bounds. In order to use the classic max-flow algorithm, we made a little change to the flow network, adding two virtual source-sink nodes, splitting the excess flow into new branches. Then the conditions of applying the classic max-flow algorithm were also reached. The classic max-flow algorithm is very mature and quick, so the fixed air flow distribution problem can be well solved.

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