Study on the most important function of evaporation duct Model A

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Abstract—**The non-dimensional gradient function and stability amendment function of evaporation duct models are very important in calculating atmospheric modified refractivity profiles. It is proved that stability amendment function of Model A used in unstable conditions is not the integrated form of non-dimensional function of the model used in unstable conditions, and the right modified refractivity formula of Model A is given. The difference between actual evaporation duct strength，atmospheric modified refractivity and Model A results is studied by using the data measured in Chinese sea areas during recent years, the conclusions offer some help for understanding Model A deeply and forecasting the maximum radar detection range in the condition of the evaporation duct.**

Keywords- evaporation duct model； non-dimensional gradient function； stability amendment function； evaporation duct

I. INTRODUCTION

Non-standard electromagnetic wave propagation occurs when the atmospheric refractivity is modified by changes in temperature gradient，pressure or water vapour content. Over the oceans, a kind of atmospheric layer called evaporation duct often occurs, which can duct the transmitted energy along the ground greatly extending the normal range of radar. More and more researchers attach importance to evaporation duct to predict its worldwide distribution[1,2]. At present, the most widely used prediction way is the evaporation duct model, and people can obtain evaporation duct height, strength and vertical modified refractivity profiles using the model. Currently, there are a lot kinds of evaporation duct models, such as PJ[3,4], Model A[5,6] (USA), Pseudo-refractivity[7] (China), MGB[8-10] (Europe). Model A was proposed by Babin from JHU/APL (The Johns Hopkins University Applied Physics Laboratory). Model A uses the work of Godfrey and Beljaars(1991) to extend the validity of MOS (Monin-Obukhov Similarity) theory to low wind speeds, and it determines the Obukhov

length iteratively without using the bulk Richardson number. The model uses a modified version of the TOGA COARE (Tropical Ocean Global Atmosphere Coupled Ocean Atmosphere Response Experiment) bulk flux algorithm to determine Monin-Obukhov parameters[11].

However, the calculation methods of evaporation duct strength and vertical modified refractivity profiles in all literature about Model A are not introduced. The two quantities are also very important, especially the vertical modified refractivity profiles. Considering the importance of the two quantities, the calculation methods are studied in this paper, and the differences between actual data and results of the methods are also analyzed.

II. REVIEW OF THE EVAPORATION DUCT HEIGHT CALCULATION METHOD OF MODEL A

Model A's duct height result is used to calculate its duct strength and modified refractivity profiles, so Model A's calculation methods of evaporation duct height must be introduced.

Model A uses 6m air temperature, relative humidity, wind speed, pressure and the sea surface temperature as input. Water vapor pressure e is related to specific humidity q by:

$$e = \frac{qp}{\varepsilon + (1-\varepsilon)q} \tag{1}$$

Where ε is the ratio of the gas constant for dry air to that of water vapor (0.62197), and refractivity N is given by:

$$N = \frac{77.6p}{T}(1 + \frac{4810q}{T(\varepsilon + (1-\varepsilon)q)}) \tag{2}$$

Next we can obtain

$$\begin{cases} \dfrac{\partial N}{\partial z} = A + B\dfrac{\partial \theta}{\partial z} + C\dfrac{\partial q}{\partial z} \\[2mm] \dfrac{\partial \theta}{\partial z} = \dfrac{\theta_* \phi_\theta}{kz} \\[2mm] \dfrac{\partial q}{\partial z} = \dfrac{q_* \phi_q}{kz} \end{cases} \qquad (3)$$

The values of A, B, C are introduced in the relative literature. z is the height, θ_* and q_* are the Monin-Obukhov scaling parameters, ϕ_θ is the non-dimensional gradient function for temperature, and ϕ_q for moisture ($\phi_\theta = \phi_q$), k is constant (0.4).Model A uses TOGA COARE bulk flux algorithm to calculate the values of θ_*, q_*, ϕ_θ and ϕ_q. The bulk flux algorithm can download via the World Wide Web at http://www.coaps.fsu.edu:80/coare/flux_algor/.

Then we have:

$$\frac{\partial N}{\partial z} = A + B\frac{\theta_* \phi_\theta}{kz} + C\frac{q_* \phi_q}{kz} = A + B\frac{\theta_* \phi_\theta}{kz} + C\frac{q_* \phi_\theta}{kz}$$

$$= A + \frac{\phi_\theta}{kz}(B\theta_* + Cq_*) \qquad (4)$$

At the evaporation duct height, $\partial N / \partial z = -0.157$ and we then have:

For stable and neutral conditions, the analytical evaporation duct height d is given:

$$d = \frac{-(B\theta_* + Cq_*)}{k(A+0.157) + \dfrac{5}{L}(B\theta_* + Cq_*)} \qquad (5)$$

For unstable conditions, the functional form of ϕ_θ is such that the following must be solved iteratively, since ϕ_θ is a function of d/L, and the iteration continues until the new value of d is within 0.0001m of the old value.

$$d = \frac{-(B\theta_* + Cq_*)\phi_\theta}{k(A+0.157)} \qquad (6)$$

III. THE EVAPORATION DUCT STRENGTH , VERTICAL MODIFIELD REFRACTIVITY PROFILES CALCULATION METHOD OF MODEL A

According to the evaporation duct theory, the duct strength is the difference between the modified refractivity at sea surface and at duct height, so the formulas of vertical modified refractivity profiles must be obtained firstly.

In the light of Eq.4 and the refractivity result at the evaporation duct height, we can obtain:

$$-0.157 = A + \frac{\phi_\theta(d/L)}{kd}(B\theta_* + Cq_*)$$

$$\rightarrow \frac{(-0.157 - A)d}{\phi_\theta(d/L)} = \left(\frac{B\theta_*}{k} + \frac{Cq_*}{k}\right) \qquad (7)$$

Combining Eq.7 with Eq.4:

$$\frac{\partial N}{\partial z} = A + \frac{\phi_\theta(z/L)}{z}\frac{(-0.157 - A)d}{\phi_\theta(d/L)} \qquad (8)$$

According to the modified atmospheric refractivity formula:

$$M = N + 0.157z \qquad (9)$$

We can obtain:

$$\frac{\partial M}{\partial z} = \frac{\partial N}{\partial z} + 0.157$$

$$= A + \frac{\phi_\theta(z/L)}{z}\frac{(-0.157 - A)d}{\phi_\theta(d/L)} + 0.157 \qquad (10)$$

The vertical modified refractivity profile formula can be acquired using the integral of Eq.10. In the process of solving, Eq.11 must be used:

$$\int_0^z \frac{\phi_\theta - 1}{z'}dz' \qquad (11)$$

In order to study conveniently, we use the stability amendment function:

$$\psi_\theta = \int_0^z \frac{1 - \phi_\theta}{z'}dz' \qquad (12)$$

Where ψ_θ is the stability amendment function for temperature. Babin assume the following expressions for the non-dimensional gradient and stability amendment functions.

For the stable case,

$$\phi_\theta = 1 + 5(z/L) \qquad (13)$$

$$\psi_\theta = -5(z/L) \qquad (14)$$

For the unstable case,

$$f_f = \frac{1}{1 + (z/L)^2} \qquad (15)$$

$$z_{p\theta} = (1 - 16z/L)^{\frac{1}{2}} \qquad (16)$$

$$z_{pg} = (1 - 12.87z/L)^{\frac{1}{3}} \qquad (17)$$

$$\psi_{\theta k} = 2\ln((1 + z_{p\theta})/2) \qquad (18)$$

$$\psi_k = 1.5\ln\frac{(z_{pg}^2 + z_{pg} + 1)}{3} \qquad (19)$$

$$- \sqrt{3}\arctan((2z_{pg} + 1)/\sqrt{3}) + \frac{\pi}{\sqrt{3}}$$

$$\psi_\theta = f_f \psi_{\theta k} + (1 - f_f)\psi_k \qquad (20)$$

$$\phi_\theta = \frac{f_f}{z_{p\theta}} + \frac{(1-f_f)}{z_{pg}} \qquad (21)$$

For the stable case, we can use Eq.13 and 14 to obtain the vertical modified refractivity profiles, then the duct strength can be calculated by using the difference between the modified refractivity at sea surface and at duct height.

For the unstable case, there is a question that Model A's evaporation duct height is not equal to the height where the modified atmospheric refractivity is the smallest in the vertical refractivity profiles. For example, if sea surface temperature 29 ^{o}C , 6m atmospheric pressure 1008.6hPa, 6m wind speed 0.8m/s, 6m relative humidity (RH) is 81%, 6m air temperature 28.8 ^{o}C, and Model A's evaporation duct height is 7.1m, the height where the modified atmospheric refractivity is the smallest in the vertical refractivity profiles is 9.1m (Fig.1)
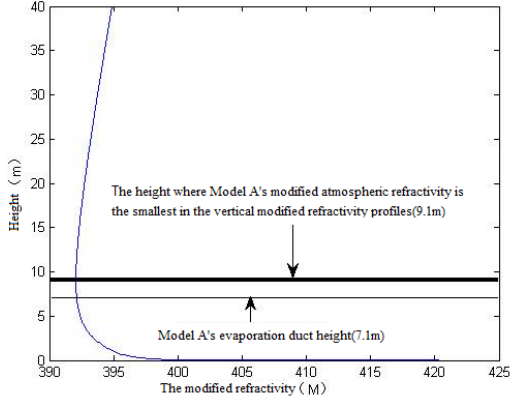

Figure 1. The simulation results analysis

IV. THE ANALYSIS OF THE REASON

According to the solving process of Model A's modified atmospheric refractivity, the reason could be the non-dimensional gradient and stability amendment functions. So we analyze Eq.20, 21 and 12 using computer simulation.

If L is -2.2 (for the unstable condition), z (the altitude) is changed from 0 to 40m with increments of 0.1m, the results of left side and right side of Eq.12 are shown in Fig.2.

If L is -200 (for the stable condition), z (the altitude) is changed from 0 to 40m with increments of 0.1m, the results of left side and right side of Eq.12 are shown in Fig.3.
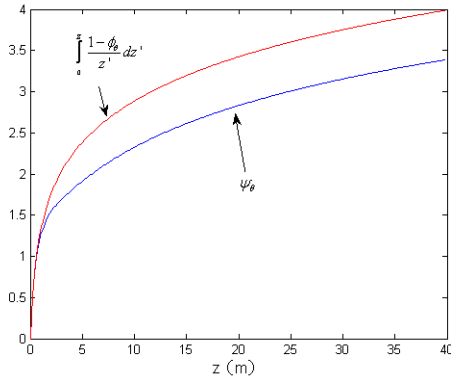

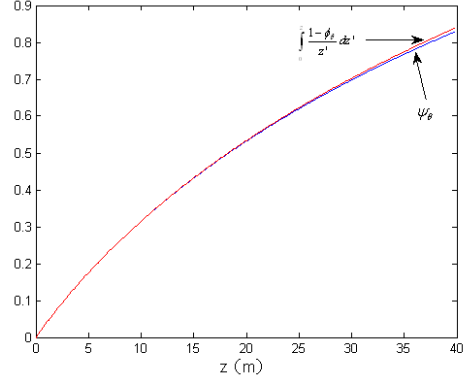Figure 2. The results analysis (L is -2.2)

Figure 3. The results analysis (L is -200)

According to Fig.2 and 3, in unstable conditions, the left and right side of Eq.12 is not equal, and with the increase of the altitude, the difference between the left side and right side will be bigger. If the stability condition is close to neutral condition, the result of both sides of Eq.12 is approximately equal. Based on the analysis results, in unstable conditions, Eq.12 cannot be used.

We rewrite the non-dimensional gradient function：

$$\begin{cases} \phi_\theta = \dfrac{f_f}{z_{p\theta}} + \dfrac{(1-f_f)}{z_{pg}} \\[2mm] \quad + \dfrac{2(z/L)^2 \left[2\ln\left(\dfrac{1+y_t}{2}\right) - \psi_k \right]}{\left[1+\left(z/L\right)^2 \right]^2} \\[2mm] y_t = \left(1 - 16\dfrac{z}{L}\right)^{1/2} \end{cases} \qquad (22)$$

Comparing Eq.21 with Eq.22, we can find that Babin ignored the third term of Eq.22 in Model A, and that is the reason why Model A's evaporation duct height is not equal to the height where the modified atmospheric refractivity is the smallest in the vertical modified refractivity profiles. Therefore, if we need Model A's right vertical modified refractivity profile, we must use the integral of Eq.11.

V. THE ANALYSIS OF EXPERIMENT DATA

Experiments were carried out to study the difference between Model A's duct strength, vertical modified refractivity profiles and actual data.
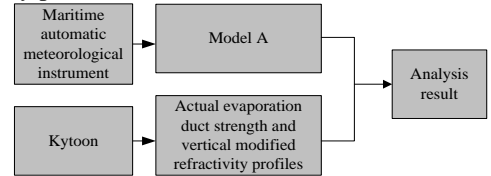

Figure 4. The flow chart for the experiment

The flow chart for the experiment is shown in Fig.4, A maritime automatic meteorological instrument is fixed in a experimental boat to get Model A's input data, and then we can obtain evaporation duct height, strength, and vertical

modified refractivity profiles. The kytoon is used to get actual vertical modified refractivity profiles and strength. Then, the analysis results between two kinds of values are given in Table I and II.

TABLE I.   DUCT STRENGTH DIFFERENCE (ACTUAL DATA MINUS MODEL RESULTS) VERSUS ASTD (AIR-SEA TEMPERATURE DIFFERENCE)

ASTD（℃）		
[-4.9,-0.6]	Difference mean（M）	0.66
	The root-mean-square errors（M）	3.32
[-0.5,0.5]	Difference mean（M）	2.53
	The root-mean-square errors（M）	2
[0.6,1]	Difference mean（M）	2.47
	The root-mean-square errors（M）	1.56

TABLE II.   THE MODIFIED REFRACTIVITY DIFFERENCE WITH HEIGHT (ACTUAL DATA MINUS MODEL RESULTS) VERSUS ASTD (AIR-SEA TEMPERATURE DIFFERENCE)

	height	5m	10m	15m	20m
ASTD<0	Difference mean（M）	0.75	-0.14	0.29	0.80
	The root-mean-square errors（M）	2.57	2.71	2.80	3.12
	height	25m	30m	35m	
	Difference mean（M）	1.03	0.85	1.01	
	The root-mean-square errors（M）	3.02	2.81	2.95	
ASTD≥0	height	5m	10m	15m	20m
	Difference mean（M）	-0.33	-0.87	-0.56	-0.25
	The root-mean-square errors（M）	2.13	2.24	2.11	2.26
	height	25m	30m	35m	
	Difference mean（M）	-0.14	0.09	0.29	
	The root-mean-square errors（M）	2.44	2.54	2.70	

In the Table I, the results show that the evaporation duct strength difference mean results in [-0.5，0.5] and [0.6，1] are bigger than in [-4.9，-0.6]. The more ASTD is, the less the root-mean-square errors are.

In the Table II, the results show that if the height increases, the modified refractivity difference mean and root-mean-square errors values will be bigger. Meanwhile, the results also show that the difference mean and root-mean-square errors values in ASTD<0 is a little bigger than in ASTD≥0.

VI.   CONCLUSION

Through in-depth analysis of Model A, the modified refractivity formula of Model A has been obtained, and for Model A, Eq.12 is not established. Using experimental data measured in Chinese sea areas recent years, the difference between actual data and Model A's evaporation duct strength is analyzed, the difference between actual data and Model A's modified refractivity is also analyzed. The analysis results show the duct strength difference mean in unstable condition is smallest, the root-mean-square errors in this condition is biggest. Meanwhile, for different stability conditions, the difference mean and root-mean-square errors would be bigger with the addition of the height.

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