About Extension of Differential Realization of the Countable Beam of Nonlinear Processes Input-Output in a Hilbert Space

Rusanov V.A., Lakeyev A.V.
Institute for System Dynamics and Control Theory,
Siberian Branch of RAS
Irkutsk, Russia
e-mail: v.rusanov@mail.ru, lakeyev@icc.ru

Linke Yu.È.
Irkutsk State Transport University
Irkutsk, Russia
e-mail: linkeyurij@gmail.com

***Abstract*—The study of algebraic extension of a countable family of controlled nonlinear dynamic processes having differential realization in the class of ordinary quasi-linear differential equations (with software-positional control and without) in a separable Hilbert space was conducted.**

Keywords-nonlinear processes "input–output", nonlinear differential realization, nonstationary $(A,B,B^\#)_2$-model.

Further $(X,\|\cdot\|_X)$, $(Y,\|\cdot\|_Y)$, $(Z,\|\cdot\|_Z)$ – real separable Hilbert spaces (pre-Hilbert [1, p. 64] define norms $\|\cdot\|_X$, $\|\cdot\|_Y$, $\|\cdot\|_Z$), $U:=X\times Y\times Z$ – Hilbert space with the norm $\|(x,y,z)\|_U:=(\|x\|_X^2 + \|y\|_Y^2 + \|z\|_Z^2)^{1/2}$, $L(Y,X)$ – Banach space with the operator norm $\|\cdot\|_{L(Y,X)}$ of all linear continuous operators from the space Y to X (similar $(L(X,X), \|\cdot\|_{L(X,X)})$ and $(L(Z,X), \|\cdot\|_{L(Z,X)})$), $T := [t_0, t_1]$ – segment of the real line R with the Lebesque measure μ and $\wp_\mu$ – σ-algebra of all μ-measurable subsets of the interval T. If below $(B, \|\cdot\|)$ – some Banach space, then as usual through $L_2(T,\mu,B)$ we will denote Banach quotient space of classes μ-equivalence of all integrable maps f: $T\to B$ of Bochner [1, p. 132] with the norm $(\int_T\|f(\tau)\|^2\mu(d\tau))^{1/2}$. In addition everywhere further $AC(T,X)$ – linear set of all absolutely continuous on T functions (with respect to μ measure) with values in the space X, moreover $\Pi:= AC(T,X)\times L_2(T,\mu,Y)\times L_2(T,\mu,Z)$.

Now we will distinguish for consideration controlled differential models of the form

$$dx(t)/dt = Ax(t) + Bu(t) + B^\#u^\#(x(t)), \qquad (1)$$

where $(x,u,u^\#(x))\in\Pi$, x – Carathéodory solution (C-solution), u and $u^\#(x)$ – software and positional control, $(A,B,B^\#)\in L_2(T,\mu,L(X,X))\times L_2(T,\mu,L(Y,X))\times L_2(T,\mu,L(Z,X))$; in purposes of terminological convenience triple of vector-functions $(x,u,u^\#(x))$ we will also call C-solution of equation (1) and triple of operator-functions $(A,B,B^\#)$, adhering the terminology from [2, 3] we will call $(A,B,B^\#)_2$-model of differential system (1).

The task of elementary (singleton) extension of differential realization of the beam of dynamic processes: for a given(possibly nonlinear) law $x\mapsto u^\#(x)$: $AC(T,X)\to L_2(T,\mu,Z)$ and fixed families N, N^* of processes "input–output" such that $N, N^*\subset\{(x,u,q)\in\Pi: (x,u,q)=(x,u,u^\#(x))\}$, $1 \le \text{Card } N \le \aleph_0$ (aleph-zero), Card $N^* = 1$, $N^* \not\subset N$, where N, N^* have differential realizations (1) to determine analytical conditions under which $N\cup N^*$ – family of C-solutions of some equation (1).

We endow the space $H_2:= L_2(T,\mu,X)\times L_2(T,\mu,Y)\times L_2(T,\mu,Z)$ with the topology of the norm

$$(\int_T \|(g(\tau),w(\tau),q(\tau))\|_U^2\, \mu(d\tau))^{1/2},\ (g,w,q)\in H_2;$$

H_2 – Hilbert space [1, p. 39]; we differ the element $(x,u,u^\#(x))\in\Pi$ in the notations as class of equivalence (i.e. element H_2) from the specific representative (vector-function) $(x(\cdot),u(\cdot),u^\#(x(\cdot)))$ from this class.

We will denote through G_E arbitrary (but fixed and numbered) algebraic basis in E:=Span N and let $\{(x^*,u^*,u^\#(x^*))\} := N^*$, while $(x^*,u^*,u^\#(x^*))\notin E$. It is obvious that at any point $t\in T$ expansion in the Hilbert space of U vector $(x^*(t),u^*(t),u^\#(x^*(t)))$ is possible on the projection in Span $\{(x(t),u(t),u^\#(x(t)))_i$: $(x,u,u^\#(x))_i \in G_E$, $i=1, 2, \ldots\}$, which is denoted by $(x^*\_(t),u^*\_(t),u^\#\_(x^*(t)))$ and addition $(x^*_\perp(t),u^*_\perp(t), u^\#_\perp(x^*(t))):=(x^*(t),u^*(t),u^\#(x^*(t))) - (x^*\_(t),u^*\_(t),u^\#\_(x^*(t)))$.

Lemma 1. *Vector-functions*

$$t\mapsto(x^*\_(t),u^*\_(t),u^\#\_(x^*(t))): T\to U,$$
$$t\mapsto(x^*_\perp(t),u^*_\perp(t),u^\#_\perp(x^*(t))): T\to U$$

μ*-measurable.*

(By the separability of U weak and strong measurabilities coincide [1, p. 130]). □

Lemma 2. *Representation* $(x^*,u^*,u^\#(x^*))=(x^*\_,u^*\_,u^\#\_(x^*))+(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))$ *doesn't depend on the choice of algebraic basis G_E, while*

$$(x^*\_,u^*\_,u^\#\_(x^*)),\ (x^*_\perp,u^*_\perp,u^\#_\perp(x^*))\in H_2. \ \square$$

We denote through Ω_E and $\Omega^*_\perp$ circuits in the space H_2 respectively to linear manifolds Span$\{\chi\cdot(x,u,u^\#(x))$: $\chi\in F$, $(x,u,u^\#(x))\in E\}$ and Span$\{\chi\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))$: $\chi\in F\}$, where $F\subset L(T,\mu,R)$ – family of equivalence classes (mod μ) of all characteristic functions induced by elements of σ-algebra $\wp_\mu$.

Lemma 3. *Subspaces Ω_E, $\Omega^*_\perp$ are orthogonal, i.e.* $\Omega_E\perp\Omega^*_\perp$. □

Remark 1. Everywhere further for two closed subspaces from the space H_2, such that their intersection is $\{0\}\subset H_2$, and the vector sum is closed in H_2 we agree to denote the sign of their vector addition through $\oplus$, in particular, Theorem 14.C [4, p. 28] and Lemma 3 make note $\Omega_E \oplus \Omega^*_\perp$ correctly.

We ask the question: what are the analytical conditions imposed on the sets of controlled dynamic processes N and $\{(x^*,u^*,u^\#(x^*))\}$, "extended" family of processes $N \cup \{(x^*,u^*,u^\#(x^*))\}$ has a differential realization (1)? On one of the ways of geometric solution of this problem is construction of characteristic feature (see below Theorem 1) defining equality

$$\Omega_E + \Omega^* = \Omega_E \oplus \Omega^*_\perp, \tag{2}$$

where Ω^* – closure in the space H_2 of linear manifold $\mathrm{Span}\{\chi\cdot(x^*,u^*,u^\#(x^*)) \colon \chi\in F\}$, while a particular form of equation (2), namely, of the type

$$\Omega_E \oplus \Omega^* = \Omega_E \oplus \Omega^*_\perp, \tag{3}$$

positively responds to the aforesaid issue about the realization of the expanded beam $N\cup\{(x^*,u^*,u^\#(x^*))\}$ in the context of approach to geometric solution of the task of expansion of differential realization based on the Theorem 14.C [4, p. 28] and theorem (3) [3] below Theorem 2 detects one characteristic property of equality (3).

Further $T_0 := \{t\in T\colon (x^*_\perp(t),u^*_\perp(t),u^\#_\perp(x^*(t))) = 0\}$, $\nu^*_\perp,\nu^*$ – Lebesque replenishments of measures

$$\int_S \|(x^*_\perp(\tau),u^*_\perp(\tau),u^\#_\perp(x^*(\tau)))\|^2_U\, \mu(d\tau),\ S\in \wp_\mu,$$

$$\int_S \|(x^*(\tau),u^*(\tau),u^\#(x^*(\tau)))\|^2_U\, \mu(d\tau),\ S\in \wp_\mu.$$

Theorem 1. $\Omega_E + \Omega^* = \Omega_E \oplus \Omega^*_\perp$ *only if*

$$\mathrm{L}_2(T,\nu^*_\perp,R) = \chi_\perp\cdot \mathrm{L}_2(T,\nu^*,R),$$

where $\chi_\perp$ *– characteristic function of the set* $T\backslash T_0$.

Proof of Theorem 1 we reduce to the establishment of Lemmas 4 and 5.

Lemma 4. $\Omega_E + \Omega^* \subset \Omega_E \oplus \Omega^*_\perp$.

Proof. Let $\omega'\in\Omega^*$, then according to Lemma 4 [3] will be

$$\omega' = \lambda'\cdot(x^*,u^*,u^\#(x^*)) =$$
$$= \lambda'\cdot(x^*\_,u^*\_,u^\#\_(x^*)) + \lambda'\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*)),$$

where $\lambda'\in \mathrm{L}_2(T,\nu^*,R)$. Further, since for each function $\lambda\in \mathrm{L}_2(T,\nu^*,R)$ we have

$$\lambda^2(t)\ \|(x^*(t),u^*(t),u^\#(x^*(t)))\|^2_U \ge \lambda^2(t)\ \|(x^*_\perp(t),u^*_\perp(t),u^\#_\perp(x^*(t)))\|^2_U,$$

then the following embedding of functional spaces is true

$$\mathrm{L}_2(T,\nu^*,R) \subset \mathrm{L}_2(T,\nu^*_\perp,R),$$

where $\lambda'\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))\in\Omega^*_\perp$ (based on the analytical structure of the subspace $\Omega^*_\perp$, given in Lemma 4 [3]. Thus, by the arbitrariness of the choice of the element $\omega'\in\Omega^*$, the lemma will be proved as soon as we discover:

$$\lambda'\cdot(x^*\_,u^*\_,u^\#\_(x^*))\in\Omega_E.$$

For this it is sufficient to show (Corollary [1, p. 109]) that $<\lambda'\cdot(x^*\_,u^*\_,u^\#\_(x^*)),\ \omega''>_{H2} = 0$, where $<\cdot,\cdot>_{H2}$ – scalar product in H_2, for all $\omega''\in H_2$, such that $<\omega'',\ \omega>_{H2} = 0$, for any $\omega\in \mathrm{Span}\{\chi\cdot(x,u,u^\#(x)) \colon \chi\in F,\ (x,u,u^\#(x))\in E\}$, which is equivalent to install:

$$\omega''(t) \perp \mathrm{Span}\{(x(t),u(t),u^\#(x(t)))_i \colon (x,u,u^\#(x))_i\in G_E,\ i=1, 2, \ldots\}$$

μ-almost everywhere in T, here $\perp$ – relation of orthogonality in the structure of space U.

We expand vector-function $\omega''(\cdot)$ in each point $t\in T$ in the sum of

$$\omega''\_(t) + \omega''_\perp(t) := \omega''(t),$$

where $\omega''\_(t)\in \mathrm{Span}\{(x(t),u(t),u^\#(x(t)))_i \colon (x,u,u^\#(x))_i\in G_E,\ i=1, 2, \ldots\}$ and $\omega''_\perp(t)$ – is orthogonal to $\mathrm{Span}\ \{(x(t),u(t),u^\#(x(t)))_i \colon (x,u,u^\#(x))_i\in G_E,\ i=1, 2, \ldots\}$. Then if $\omega''\_\neq 0$, there exists such set $S^*\in \wp_\mu$, $\mu(S^*)>0$, that $\omega''\_(t) \neq 0$, $\forall t\in S^*$, while in the basis G_E there is such vector $(x,u,u^\#(x))_i$, that is $(x(t),u(t),u^\#(x(t)))_i \neq 0$ μ-almost everywhere in S^*; otherwise for μ-almost all $t\in S^*$ equalities will be "realized"

$$\mathrm{Span}\{(x(t),u(t),u^\#(x(t)))_i \colon (x,u,u^\#(x))_i\in G_E,\ i=1, 2, \ldots\} = \{0\},$$

and therefore $\omega''\_ = 0$ should be performed in this position.

Now we denote through S^*_+ and $S^*\_$ subsets (partition) S^* equal

$$S^*_+ = \{t\in S^* \colon <\omega''\_(t),(x(t),u(t),u^\#(x(t)))_i>_U \ge 0\},$$
$$S^*\_ = \{t\in S^* \colon <\omega''\_(t),(x(t),u(t),u^\#(x(t)))_i>_U < 0\}.$$

It is obvious that at least one of the sets S^*_+ or $S^*\_$ has a nonzero measure. Let S^*_+ acts as such set. Then $\chi_+\cdot(x,u,u^\#(x))_i\in \mathrm{Span}\{\chi\cdot(x,u,u^\#(x)) \colon \chi\in F,\ (x,u,\ u^\#(x))\in E\}$ and $<\omega''\_,\ \chi_+\cdot(x,u,u^\#(x))_i>_{H2}>0$, where χ_+ – characteristic function of a set S^*_+. It is clear that we obtain $<\omega'',\chi_+\cdot(x,u,u^\#(x))_i>_{H2} > 0$ whereby we arrive at a contradiction with the conditions defined above the construction of the functional ω''. □

The above proof provides a useful clarification:

Corollary 1. $\mathrm{L}_2(T,\nu^*,R) \subset \mathrm{L}_2(T,\nu^*_\perp,R)$. □

Lemma 5. $\Omega_E + \Omega^* \supset \Omega_E \oplus \Omega^*_\perp \Leftrightarrow \mathrm{L}_2(T,\nu^*_\perp,R) = \chi_\perp\cdot \mathrm{L}_2(T,\nu^*,R)$.

Proof. ($\Rightarrow$). Let $\lambda_\omega \in \mathrm{L}_2(T,\nu^*_\perp,R)$ and $\omega := \lambda_\omega\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))$, where (Lemma 4 [3]) $\omega\in\Omega_E \oplus \Omega^*_\perp$, means (assumption $\Rightarrow$) $\omega\in\Omega_E + \Omega^*$. Then by $\omega\in\Omega_E \oplus \Omega^*_\perp$ vector ω has an expansion of (unique) form $\omega = \omega' + \lambda_\omega\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))$, where $\omega' = 0\in \Omega_E$, at this effect $\omega\in\Omega_E + \Omega^*$ representation is true:

$$\omega = \omega''+\lambda^*\cdot(x^*,u^*,u^\#(x^*)) =$$
$$= \omega''+\lambda^*\cdot(x^*\_,u^*\_,u^\#\_(x^*))+\lambda^*\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*)),$$

where $\omega''\in\Omega_E$, $\lambda^*\in \mathrm{L}_2(T,\nu^*,R)$. Since (reasonings are similar to the proof of Lemma 4 [3]) the inclusions take place $\lambda^*\cdot(x^*\_,u^*\_,u^\#\_(x^*))\in\Omega_E$, $\lambda^*\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))\in\Omega^*_\perp$, then $\omega' = \omega''+\lambda^*\cdot(x^*\_,u^*\_,u^\#\_(x^*))$ и $\lambda_\omega\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*)) = \lambda^*\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))$. Thus, taking into account the presence of a linear isometry between $\mathrm{L}_2(T,\nu^*_\perp,R)$ and $\Omega^*_\perp$ (Lemma 4 [3]) will be $\lambda_\omega = \chi_\perp\cdot\lambda^*$, where in the end by the arbitrariness of the choice of function λ_ω, we obtain $\mathrm{L}_2(T,\nu^*_\perp,R) \subset \chi_\perp\cdot \mathrm{L}_2(T,\nu^*,R)$ or taking into account Corollary 1 $\mathrm{L}_2(T,\nu^*_\perp,R) = \chi_\perp\cdot \mathrm{L}_2(T,\nu^*,R)$.

($\Leftarrow$). Let $\omega\in\Omega_E \oplus \Omega^*_\perp$. Then $\omega = \omega'+\lambda_\omega\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*))$, where $\omega'\in\Omega_E$, $\lambda_\omega\in \mathrm{L}_2(T,\nu^*_\perp,\ R)$. Since (assumption $\Leftarrow$) $\lambda_\omega\in\chi_\perp\, \mathrm{L}_2(T,\nu^*,R)$, then we have a bunch of equalities

$$\omega' + \lambda_\omega\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*)) = \omega' + \lambda_\omega\cdot(x^*_\perp,u^*_\perp,u^\#_\perp(x^*)) +$$
$$+ \lambda_\omega\cdot(x^*\_,u^*\_,u^\#\_(x^*)) - \lambda_\omega\cdot(x^*\_,u^*\_,u^\#\_(x^*)) =$$
$$= \omega' - \lambda_\omega\cdot(x^*\_,u^*\_,u^\#\_(x^*)) + \lambda_\omega\cdot(x^*,u^*,u^\#(x^*)),$$

therefore, $\omega\in\Omega_E + \Omega^*$ taking into account $(\omega' - \lambda_\omega\cdot(x^*\_,u^*\_,u^\#\_(x^*)))\in\Omega_E$, $\lambda_\omega\cdot(x^*,u^*, u^\#(x^*))\in\Omega^*$. □

Now we present a variant of characteristic conditions of equality (3).

Theorem 2. *If we implement* $T_0 = \varnothing$ (mod μ) *offer is valid*:

$$\Omega_E\oplus\Omega^* = \Omega_E\oplus\Omega^*_\perp \Leftrightarrow L_2(T,\nu^*_\perp,R) = L_2(T,\nu^*,R).$$

Proof. That is $\Omega_E + \Omega^* = \Omega_E\oplus\Omega^*_\perp \Leftrightarrow L_2(T,\nu^*_\perp,R) = L_2(T,\nu^*,R)$ – a direct statement of Theorem 1. On the other hand, confirmation of equality $\Omega_E\cap\Omega^* = \{0\}\subset H_2$ follows from the assumption $\{t\in T: (x^*_\perp(t),u^*_\perp(t),u^\#_\perp(x^*(t))) = 0\} = \varnothing$ (mod μ) and Corollary of Mazur's Theorem [1, p. 109]. □

Theorem 1 (given the finding of Lemma 5) and Theorem 2 attracting Theorem 14.C [4, p. 28] and Theorem 3 [3] do a fair conclusion:

Corollary 2. *The following three properties are equivalent*:

$$L_2(T,\nu^*_\perp,R)\subset\chi_\perp L_2(T,\nu^*,R)\Leftrightarrow$$
$$\Leftrightarrow L_2(T,\nu^*_\perp,R) = \chi_\perp L_2(T,\nu^*,R)\Leftrightarrow$$
$$\Leftrightarrow\Omega_E\oplus\Omega^*_\perp = \Omega_E + \Omega^*,$$

and if $T_0 = \varnothing$ (mod μ), *then any signified property turns the beam* $N\cup\{(x^*,u^*,u^\#(x^*))\}$ *into the set of dynamic processes with the differential realization* (1). □

Remark 2. Corollary 2 allows to call Theorem 2 as "direct theorem" about elementary algebraic extension of differential realization while hypothesis: $T_0 = \varnothing$ (mod μ), $N\cup\{(x^*,u^*,u^\#(x^*))\}$ has a realization (1) $\Rightarrow L_2(T,\nu^*_\perp,R)\subset L_2(T,\nu^*,R)$ in general case isn't confirmed that the following example illustrates.

Example 1. Let $X = Y = R$, $T = [-1, 1]$, $u^\#(\cdot) = 0$ and

$$N=\{t\mapsto(e^t,0,0): t\in T\},$$
$$\{(x^*,u^*,u^\#(x^*))\}=\{t\mapsto(e^t+t^2/2,t,0): t\in T\};$$

it is obvious that $T_0 = \varnothing$ (mod μ) and the beam $N\cup\{(x^*,u^*,u^\#(x^*))\}$ has a realization (1); we note that $T_0\neq\varnothing$. Then $L_2(T,\nu^*,R) = L_2(T,\mu,R)$ and $L_2(T,\nu^*_\perp,R)$, $\nu^*_\perp = \int\tau^2\mu(d\tau)$, because $(x^*_\perp(t),u^*_\perp(t), u^\#_\perp(x^*(t))) = (0,t,0)$. It is clear that $1/t\in L_2(T,\nu^*_\perp,R)$, $1/t\notin L_2(T,\mu,R)$, where $L_2(T,\nu^*_\perp,R)\not\subset L_2(T,\nu^*,R)$; hence by Lemma 5 we also conclude that $\Omega_E\oplus\Omega^*_\perp\not\subset\Omega_E+\Omega^*$.

Next statement shows that the construction similar to Example 1 can't be realized in the functional class $AC(T,X)\times\{0\}\times\{0\}\subset\Pi$, i.e. for free trajectories (C-solutions) it can be said that for $N\subset AC(T,X)\times\{0\}\times\{0\}$ Corollary 3 in a known sense is opposite to Corollary 2 (see above Remark 2).

Corollary 3. *If* $N\subset AC(T,X)\times\{0\}\times\{0\}$, Card $N<\infty$ *and* $N\cup\{(x^*,0,0)\}$ – *set of trajectories with the realization* (1) *with* $u = 0$, $u^\# = 0$, *then the following relations are true*:

$$T_0 = \varnothing,$$
$$L_2(T,\nu^*_\perp,R) = L_2(T,\nu^*,R),$$
$$\Omega_E\oplus\Omega^* = \Omega_E\oplus\Omega^*_\perp.$$

Proof. It is easy to see that $T_0 = \varnothing$, because otherwise there exists a period of time $t_*\in T$, which $x^*(t_*)=\Sigma\alpha_i x_{(i)}(t_*)$, where all constants α_i, except the finite number are zero, $x_{(i)}$ – the first component of the triple $(x,0,0)_i\in G_E$. Consequently, the trajectory $x^*(\cdot)$ has a representation $\Sigma\alpha_i x_{(i)}(\cdot)$ by the uniqueness of solution, extending at time t_* through the point $x^*(t_*)$, for the differential system (1) with $(A,0,0)_2$-model, corresponding to a set of dynamic processes $N\cup\{(x^*,0,0)\}$; that is contrary to its earlier condition $(x^*,0,0)\notin E$.

Further, because of the continuity of the trajectory $x^*(\cdot)$ and the compactness of the interval T, there exist such real constants c_1, $c_2>0$, that equalities are true

$$\inf\{\|x^*(t)\|_X: t\in T\} = c_1,\quad \sup\{\|x^*(t)\|_X: t\in T\} = c_2,$$

similarly (including $T_0=\varnothing$, Card $N<\infty$), for some c_3, $c_4>0$ will be

$$\inf\{\|x^*_\perp(t)\|_X: t\in T\} = c_3,\quad \sup\{\|x^*_\perp(t)\|_X: t\in T\} = c_4.$$

Consequently, the classes of real-valued functions summable with square on T on measures $\nu^*_\perp=\int\|x^*(\tau)\|^2_X\,\mu(d\tau)$ and $\nu^*_\perp=\int\|x^*_\perp(\tau)\|^2_X\,\mu(d\tau)$, or in other words $L_2(T,\nu^*,R) = L_2(T,\nu^*_\perp,R)$, and hence (see Theorem 2) $\Omega_E\oplus\Omega^* = \Omega_E\oplus\Omega^*_\perp$. □

If we look at Theorem 2 under foreshortening of unmanaged trajectories of a differential system (1), we can see that the analyst of output condition $L_2(T,\nu^*,R) = L_2(T,\nu^*_\perp,R)$ in the proof of Corollary 3 enables us to strengthen this theorem to the characteristic feature of elementary algebraic extension of differential realization of a finite beam of unmanaged implementation processes $N\subset AC(T,X)\times\{0\}\times\{0\}$.

Theorem 3. *In the family of free K-solutions the problem of singleton expansion of differential realization of the finite beam of trajectories is solvable if and only if* $T_0 = \varnothing$.

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