Mild Solution and Exact Controllability of Semilinear Singular Distributed Parameter Systems

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***Abstract*—Mild solution and exact controllability of semilinear singular distributed parameter system are discussed in Hilbert space, some results are obtained by functional analysis and generalized operator semigroup. First, we study the classical solution concerning the homogeneous linear singular distributed parameter system by generalized operator semigroup. Second, the existence and uniqueness for the mild solution of semilinear singular distributed parameter system is proved. Third, a new result concerning the exact controllability of linear singular distributed parameter system is obtained. At last, the exact controllability for the semilinear singular distributed parameter system is discussed. This research is theoretical important for studying the controllability of nonlinear singular distributed parameter systems.**

Keywords-*mild solution; exact controllability; semilinear singular distributed parameter system; generalized operator semigroup.*

I. INTRODUCTION

One of the most important problem for the study of infinite dimensional system is exact controllability (see, for example [1-14]). The exact controllability of infinite dimensional linear systems was introduced in [1]. The exact controllability concerning the infinite dimensional nonlinear systems were discussed in [2-10]. The exact controllability of the infinite dimensional linear singular systems were discussed in [11-14]. It is regrettably that the exact controllability of the infinite dimensional nonlinear singular systems was not discussed. The main purpose of this paper is to obtain the sufficient conditions concerning the exact controllability of the following nonlinear singular distributed parameter system:

$$\begin{cases} \dfrac{dEx(t)}{dt} = Ax(t) + Bu(t) \\ \qquad\qquad + F(t, x(t), u(t)), 0 \le t \le T, \\ x(t) = x_0, \end{cases} \tag{1}$$

where $x(t) \in X, u(t) \in U$, X and U are two Hilbert spaces, $E: X \to X$ is a bounded linear operator, $A: D(A) \subset X \to X$ is a linear operator, $B: U \to X$ is a bounded linear operator, control function $u(t)$ belongs to $L^2([0,T], U)$, $F: [0,T] \times X \times U \to X$ is a proper nonlinear function. In section 2, the classical solution of the following linear singular distributed parameter system

$$\begin{cases} \dfrac{dEx(t)}{dt} = Ax(t), 0 \le t \le T, \\ x(0) = x_0, \end{cases} \tag{2}$$

is discussed by generalized operator semigroup. In section 3, the existence and uniqueness concerning the mild solution of the following semilinear singular distributed parameter system

$$\begin{cases} \dfrac{dEx(t)}{dt} = Ax(t) + Bu(t) \\ \qquad\qquad + F(t, x(t)), 0 \le t \le T, \\ x(t) = x_0, \end{cases} \tag{3}$$

are proved, where $F: [0,T] \times X \to X$ is a proper nonlinear function. In section 4, a new result concerning the exact controllability of the following linear singular distributed parameter system

$$\begin{cases} \dfrac{dEx(t)}{dt} = Ax(t) + Bu(t), 0 \le t \le T, \\ x(t) = x_0, \end{cases} \tag{4}$$

is obtained. In section 5, the exact controllability for the semilinear singular distributed parameter system (1) is discussed, some sufficient conditions are obtained.

In the following, $B(X)$ denotes the set of all bounded linear operators on X , $\|\cdot\|$ the norm and $<\cdot,\cdot>$ the inner product in X ,

$$L^2([0,T]; U) = \left\{ u : u \in U, (\int_0^t \|u(t)\|^2)^{1/2} < \infty \right\},$$

$$C([0,T]; X) = \left\{ x : [0,T] \to X, x(t) \text{ is continuous} \right\},$$

B^* the adjoint operator of B, $R(B) = \mathrm{range}(B)$.

II. GENERALIZED OPERATOR SEMIGROUP

In this section, first of all we introduce the generalized operator semigroup and generator, and then the classical solution of system (2) is discussed by generalized operator semigroup theory.

Definition 1[15] Suppose $\{S(t): t \geq 0\}$ is a one parameter family of bounded linear operators in Hilbert space X, and $E \in B(X)$. If

$$S(t+s) = S(t)ES(s), \quad t \geq 0, s \geq 0,$$

then $\{S(t): t \geq 0\}$ is called a generalized operator semigroup induced by E, or generalized operator semigroup for short.

If the generalized operator semigroup $S(t)$ satisfies

$$\lim_{t \to 0^+} \| S(t) - S(0) \| = 0,$$

then it is called uniformly continuous;

If the generalized operator semigroup $S(t)$ satisfies

$$\lim_{t \to 0^+} \| S(t)x - S(0)x \| = 0, \text{ for arbitrary } x \in X,$$

then it is called strongly continuous on X.

Property 1[15] If the generalized operator semigroup $S(t)$ is strongly continuous on X, then

(i) there exist constants $M \geq 1$ and $\omega \geq 0$ such that

$$\| S(t) \| \leq Me^{\omega t}, \quad t \geq 0,$$

i.e. $S(t)$ is exponentially bounded.

(ii) for arbitrary $x \in X$ and $t \geq 0$,

$$\lim_{h \to 0} \| S(t+h)x - S(t)x \| = 0$$

holds true.

Definition 2 The generator A of generalized operator semigroup $S(t)$ is defined as

$$Ax = \lim_{h \to 0^+} \frac{ES(h)Ex - ES(0)Ex}{h} \text{ for every } x \in D(A),$$

where

$$D(A) = \{ x \in X : S(0)Ex = x, \lim_{h \to 0^+} \frac{ES(h)Ex - ES(0)Ex}{h} \text{ exists} \}$$

Definition 3 $x(t): [s,T] \to X$ is called a classical solution of system (2) on $[0,T]$, if

(i) $x(0) = x_0$.

(ii) $x(t)$ is strong continuous on $[0,T]$.

(iii) $Ex(t)$ is strongly continuously differentiable on $[0,T]$ and satisfies

$$\frac{d}{dt}(Ex(t)) = Ax(t) \text{ for } t \in [0,T].$$

Theorem 1 If A is a generator of the strongly continuously generalized operator semigroup $S(t)$, and $x_0 \in D(A)$, then $S(t)Ex_0 \in D(A)$ for $t \geq 0$, and $x(t) = S(t)Ex_0$ is a classical solution of system (2).

Proof Let $h > 0$. Then

$$\begin{aligned} ES(t)Ax_0 &= ES(t) \lim_{h \to 0^+} \frac{1}{h}(ES(h)E - ES(0)E)x_0 \\ &= \lim_{h \to 0^+} \frac{1}{h}(ES(t+h)E - ES(t)E)x_0 \\ &= \lim_{h \to 0^+} \frac{1}{h}(ES(h)E - ES(0)E)S(t)Ex_0 \\ &= AS(t)Ex_0, \end{aligned}$$

i.e.

$$ES(t)Ax_0 = AS(t)Ex_0,$$

and

$$S(0)ES(t)Ex_0 = S(t)Ex_0,$$

i.e. $S(t)Ex_0 \in D(A)$.

Otherwise, from the above we have that

$$\frac{d^+ES(t)Ex_0}{dt} = AS(t)Ex_0,$$

and $x(t) = S(t)Ex_0$ is strong continuous on $[0,T]$.

In the following, we prove that

$$\frac{d^-ES(t)Ex_0}{dt} = AS(t)Ex_0.$$

In fact, for $0 < t \leq T$, let $h > 0$ be sufficient small such that $t - h > 0$. Since $\| S(t-h) \| \leq Me^{\omega t}$, thus

$$\begin{aligned} &\left\| \frac{ES(t)Ex_0 - ES(t-h)Ex_0}{h} - AS(t)Ex_0 \right\| \\ &\leq \left\| ES(t-h)[\frac{ES(h)Ex_0 - ES(0)Ex_0}{h} - Ax_0] \right\| \\ &\quad + \left\| ES(t-h)Ax_0 - AS(t)Ex_0 \right\| \\ &\leq M_1 e^{\omega T} \left\| \frac{ES(h)Ex_0 - ES(0)Ex_0}{h} - Ax_0 \right\| \\ &\quad + \left\| ES(t-h)Ax_0 - ES(t)Ax_0 \right\| \to 0 (h \to 0). \end{aligned}$$

Hence

$$\frac{d^-ES(t)Ex_0}{dt} = AS(t)Ex_0,$$

i.e. $x(t) = S(t)Ex_0$ is a classical solution of system (2).

Theorem 2 Let A be a generator of the strongly continuously generalized operator semigroup $S(t)$, there exist $\beta > 0$ such that

$$\lambda \in \rho(E,A) = \left\{\mu : \mu \in R, (\mu E - A)^{-1}\text{exist}\right\}$$

when $\lambda > \beta$, and

$$\left\|(\lambda E - A)^{-1}\right\| \le \frac{M}{\lambda - \beta}.$$

If $x_0 \in D(A)$, and the classical solution $x(t)$ of system (2) satisfies that $x(t) \in D(A)$, then $x(t)$ is unique.

Proof It is only need to prove that the classical solution $x(t) \equiv 0$ of the system

$$\begin{cases} \dfrac{dEx(t)}{dt} = Ax(t), 0 \le t \le T, \\ x(0) = 0. \end{cases} \quad (5)$$

In fact, suppose $\varepsilon > 0$, since $x(t)$ is a classical solution of system (5), then

$$Ex(t-\varepsilon) - Ex(t) = -\varepsilon Ax(t) + x_\varepsilon,$$

where $x_\varepsilon \in X$ satisfies $\|x_\varepsilon\| = o(\varepsilon)(\varepsilon \to 0^+)$. Thus

$$\begin{aligned} Ex(t-\varepsilon) &= (E - \varepsilon A)x(t) + x_\varepsilon \\ &= \varepsilon(\frac{1}{\varepsilon}E - A)x(t) + x_\varepsilon. \end{aligned}$$

Pick ε satisfying $0 < \varepsilon < 1$ and $\beta < 1/\varepsilon$, then

$$1/\varepsilon \in \rho(E,A)$$

and

$$\left\|(\frac{1}{\varepsilon}E - A)^{-1}E\right\| \le \frac{M\|E\|}{(1/\varepsilon) - \beta} = \frac{\varepsilon M\|E\|}{1 - \varepsilon\beta}.$$

Therefore

$$\begin{aligned} x(t) = \varepsilon^{-1}(\varepsilon^{-1}E - A)^{-1}Ex(t-\varepsilon) \\ -\varepsilon^{-1}(\varepsilon^{-1}E - A)^{-1}x_\varepsilon, \end{aligned}$$

and

$$\begin{aligned} Ax(t) = \varepsilon^{-1}A(\varepsilon^{-1}E - A)^{-1}Ex(t-\varepsilon) \\ -\varepsilon^{-1}A(\varepsilon^{-1}E - A)^{-1}x_\varepsilon. \end{aligned}$$

Since

$$\begin{aligned} \left\|A(\varepsilon^{-1}E - A)^{-1}\right\| &\le \varepsilon^{-1}\left\|E(\varepsilon^{-1}E - A)^{-1}\right\| + 1 \\ &\le \frac{M\|E\|}{1 - \beta\varepsilon} + 1 \le M_0, \end{aligned}$$

where constant M_0 is independent with ε, thus

$$\begin{aligned} \|Ax(t)\| &\le \varepsilon^{-1}M_0\|E\|\|x(t-\varepsilon)\| + \varepsilon^{-1}M_0\|x_\varepsilon\| \\ &= \varepsilon^{-1}M_0\|E\|\|x(t-\varepsilon)\| + o(1). \end{aligned}$$

Prolongate $x(t)$, let $x(t) = 0$ for arbitrary $t < 0$. Since $x(t)$ is a classical solution of system (5), we have

$$Ex(t) = \int_0^t Ax(s)ds,$$

and from $x(t) \in D(A)$, we obtain $S(0)Ex(t) = x(t)$. Therefore

$$x(t) = S(0)\int_0^t Ax(s)ds.$$

Hence

$$\begin{aligned} \|x(t)\| &\le \|S(0)\|\int_0^t \|Ax(s)\|ds \\ &\le \|S(0)\|\varepsilon^{-1}M_0\|E\|\int_0^t \|x(s-\varepsilon)\|ds + o(1) \\ &= \|S(0)\|\varepsilon^{-1}M_0\|E\|\int_0^{t-\varepsilon} \|x(s)\|ds + o(1) \\ &\le \|S(0)\|\varepsilon^{-1}M_0\|E\|\int_0^t \|x(s)\|ds + o(1), \end{aligned}$$

i.e. $x(t) \equiv 0$.

III. SEMILINEAR SINGULAR DISTRIBUTED PARAMETER SYSTEM

In this section, the existence and uniqueness concerning the mild solution of the semilinear singular distributed parameter system (3) are discussed by the generalized operator semigroup.

Definition 4 The mild solution of the semilinear singular distributed parameter system (3) is the continuous solution on $[0,T]$ of the following integral equation

$$x(t) = S(t)Ex_0 + \int_0^t S(t-s)F(s,x(s))ds. \quad (6)$$

Theorem 3 Suppose that there exists constant $L > 0$ such that

$$\|F(t,x_1) - F(t,x_2)\| \le L\|x_1 - x_2\|, x_1, x_2 \in X. \quad (7)$$

Then, for arbitrary $x_0 \in X$, there exists an unique mild solution of the system (3) on $[0,T]$.

Proof Let $x_0 \in X$. Define P on $C([0,T];X)$ as following:

$$(Px)(t) = S(t)x_0 + \int_0^t S(t-s)F(s,x(s))ds. \quad (8)$$

According to (7), we can prove that P is a mapping from $C([0,T];X)$ to itself, and for arbitrary

$$x_1, x_2 \in C([0,T];X),$$

we have

$$\begin{aligned} &\|(Px_1)(t) - (Px_2)(t)\| \\ &= \left\|\int_0^t [S(t-s)F(s,x_1(s)) - S(t-s)F(s,x_2(s))]ds\right\| \end{aligned}$$

$$\leq M_F \int_0^t \|F(s, x_1(s)) - F(s, x_2(s))\| ds$$
$$\leq M_F L \int_0^t \|x_1(s) - x_2(s)\| ds$$
$$\leq M_F Lt \|x_1(\cdot) - x_2(\cdot)\|_C, \quad (9)$$

where $M_F = \max_{0 \leq t \leq T} \|S(t)\|$, $\|\cdot\|_C$ denotes the norm in space $C([0,T]; X)$. Using (8) and (9) again and again, we can obtain

$$\|(P^n x_1)(t) - (P^n x_2)(t)\|$$
$$\leq \frac{(M_F Lt)^n}{n!} \|x_1(\cdot) - x_2(\cdot)\|_C.$$

Thus

$$\|(P^n x_1)(\cdot) - (P^n x_2)(\cdot)\|_C$$
$$\leq \frac{(M_F Lt)^n}{n!} \|x_1(\cdot) - x_2(\cdot)\|_C.$$

When n is sufficiently large, we have $\frac{(M_F Lt)^n}{n!} < 1$. Therefore P^n is a contraction mapping, and there exists a uniquely fixed point $x^*(\cdot)$ such that $P^n(x^*) = x^*$. Since

$$P^n(Px^*) = P^{n+1}(x^*) = Px^*,$$

thus Px^* is also a fixed point of P^n. Because the fixed point of P^n is unique, we have $Px^* = x^*$. Therefore x^* satisfy integral equation (6). Hence $x^*(\cdot)$ is a uniquely mild solution of equation (3) on $[0,T]$.

IV. CONTROLLABILITY OF LINEAR SINGULAR DISTRIBUTED PARAMETER SYSTEM

In this section, some results concerning the exact controllability of the linear singular distributed parameter system (4) are given. In the following, we suppose that for all $x_0 \in X$ and $u \in L^2([0,T], X)$ the system (4) only has a uniquely mild solution given by the following formula,

$$x(t) = S(t)Ex_0 + \int_0^t S(t-s)Bu(s)ds, t \in [0,T] \quad (10)$$

for all $x_0 \in X$ and $u \in L^2([0,T], U)$.

Definition 5 System (4) is called exactly controllable if for all $x_0, x_1 \in X$ there exists a control $u \in L^2([0,T], U)$ such that the corresponding solution of (10) satisfies $x(T) = x_1$.

Consider the following bounded linear operator, $G_B : L^2([0,T], U) \to X$,

$$G_B u = \int_0^T S(T-s)Bu(s)ds. \quad (11)$$

Define operator

$$L_B = G_B G_B^* : X \to X,$$

where $G_B^* : X \to L^2([0,T], U)$ is given by

$$(G_B^* x)(s) = B^* S^*(T-s)x.$$

Then

$$L_B x = \int_0^T S(T-s)BB^* S^*(T-s)xds. \quad (12)$$

Lemma 1[11,13] System (4) is exactly controllable on $[0,T]$ if and only if there exists $\gamma > 0$ such that for all $x \in X$, one of the following conditions hold true:

(i) $< L_B x, x > \geq \gamma \|x\|_X^2$;

(ii) $\int_0^T \|B^* S^*(T-s)x\|_U^2 ds \geq \gamma \|x\|_U^2$.

Theorem 4 System (4) is exactly controllable on $[0,T]$ if and only if L_B is invertible. Moreover, the control $u \in L^2([0,T], U)$ satisfying $x(T) = x_1$ is given by the following formula,

$$u(t) = B^* S^*(T-t) L_B^{-1}(x_1 - S(T)Ex_0). \quad (13)$$

Proof Necessity. Suppose the system (4) is exactly controllable. According to Lemma 1, we have that

$$< L_B x, x > \geq \gamma \|x\|_X^2, \ x \in X. \quad (14)$$

This implies that L_B is one to one. In the following, we prove that L_B is surjective, i.e., $R(L_B) = X$. In fact, if $R(L_B)$ is strictly contained in X, from Cauchy Schwarz's inequality and (14), we have

$$\|L_B x\| \geq \gamma \|x\|_X, x \in X.$$

This implies that $R(L_B)$ is closed. Using Hahn Banach's Theorem, there exists $x_0 \in X$ and $x_0 \neq 0$ such that

$$< L_B x, x_0 >= 0, \forall x \in X.$$

Especially, let $x = x_0$, from (14) we obtain

$$0 =< L_B x_0, x_0 > \geq \gamma \|x_0\|_X^2.$$

Thus $x_0 = 0$. This is a contradiction. Hence L_B is a bijection. From the open mapping Theorem we have that L_B^{-1} is a bounded linear operator.

Sufficiency. Suppose L_B is invertible, then given $x_1 \in X$, we can prove that there exists

$$u \in L^2([0,T], U)$$

such that $x(T) = x_1$. This control can be taken as the form of (13). In fact,

$$x(T) = S(T)Ex_0 + \int_0^T S(T-s)BB^*S^*(T-s)L_B^{-1}(x_1 - S(T)Ex_0)ds = S(T)Ex_0 + L_B L_B^{-1}(x_1 - S(T)Ex_0) = x_1$$

Corollary 1 If the system (4) is exactly controllable, then the operator defined as follows

$$S_B : X \to L^2([0,T],U)$$
$$S_B x = G_B^* L_B^{-1} x$$

or

$$(S_B x)(s) = B^* S^*(T-s) L_B^{-1} x \qquad (15)$$

is a right inverse of G_B, i.e., $G_B \circ S_B = I$.

V. CONTROLLABILITY OF NONLINEAR SINGULAR DISTRIBUTED PARAMETER SYSTEM

In this section, some results concerning the exact controllability of the nonlinear singular distributed parameter system (1) are given in Hilbert space. In the following, we suppose that for all $x_0 \in X$ and $u \in L^2([0,T],X)$ the system (1) only has a uniquely mild solution given by the following formula,

$$x(t) = S(t)Ex_0 + \int_0^t S(t-s)Bu(s)ds + \int_0^t S(t-s)F(s,x(s),u(s))ds . \qquad (16)$$

Definition 6 System (1) is called exactly controllable on $[0,T]$ if for all $x_0, x_1 \in X$, there exists control $u \in L^2([0,T],X)$ such that the corresponding solution of (16) satisfies $x(T) = x_1$.

Define the operator

$$G_{BF} : L^2([0,T],X) \to X$$

as following

$$G_{BF}u = \int_0^T S(T-s)Bu(s)ds + \int_0^T S(T-s)F(s,x(s),u(s))ds = G_B u + \int_0^T S(T-s)F(s,x(s),u(s))ds , \quad (17)$$

where $x(t) = x(t,u)$ is the solution of (16) corresponding to control u. Then it is obvious that the following result holds true.

Theorem 5 System (1) is exactly controllable if and only if $R(G_{BF}) = X$.

Hence, in order to prove that system (1) is exactly controllable, we must prove that the condition of Theorem 5 holds true. For this reason, we need to suppose that the linear singular distributed parameter system (1) is exactly controllable. In such case, according to Corollary 1, the operator S_B defined by equation (15) is the right inverse of G_B. If let $\tilde{G}_{BF} = G_{BF} \circ S_B$, we can obtain the operator equation concerning the exactly controllable as follows:

$$\tilde{G}_{BF}\, \eta = G_{BF} \circ S_B \eta = \eta + \int_0^T S(T-t)F(s, x_\eta(s), (S_B\eta)(s))ds , \quad (18)$$

where $x_\eta(\cdot)$ is the solution of equation (10) corresponding to control

$$u(t) = (S_B\eta)(t) = B^*S^*(T-s)L_B^{-1}\eta, t \in [0,T].$$

Therefore, if we define operator $K_F : X \to X$ as

$$K_F \eta = \int_0^T S(T-s)F(s, x_\eta(s), (S_B\eta)(s))ds , \quad (19)$$

then equation (18) can be written as

$$\tilde{G}_{BF}\, \eta = \eta + K_F\eta = (I + K_F)\eta, \eta \in X . \qquad (20)$$

In order to obtain the main theorem, first of all the following well known Lemma is given:

Lemma 2[8] Suppose X is a Banach space and $K : X \to X$ is a Lipchitz function with a Lipchitz constant $k < 1$, $G(x) = x + Kx$. Then G is an homemorphis whose inverse is a Lipchitz function with a Lipchitz constant $(1-k)^{-1}$.

Theorem 6 If system (4) is exactly controllable on $[0,T]$ and operator K_F is a globally Lipchitz function with a Lipchitz constant $k < 1$, then nonlinear singular distributed parameter system (1) is exactly controllable on $[0,T]$, and the control steering $x(T) = x_1$ is

$$u(t) = B^*S^*(T-t)L_B^{-1}(I + K_F)^{-1}(x_1 - S(T)Ex_0) .$$

Proof In fact, according to Lemma 2, we have that $(I + K_F)$ is an homemorphis whose inverse is a Lipchitz function with a Lipchitz constant $(1-k)^{-1}$. From (18) and (20), $R(G_{BF}) = R(\tilde{G}_{BF}) = X$ holds true. Therefore system (1) is exactly controllable on $[0,T]$ from Theorem 5. Since

$$u(t) = B^*S^*(T-t)L_B^{-1}(I + K_F)^{-1}(x_1 - S(T)Ex_0),$$

according to (16), we obtain

$$x(T) = S(T)Ex_0 + \int_0^T [S(T-s)BB^*S^*(T-s) \cdot$$

$$L_B^{-1}(I+K_F)^{-1}(x_1-S(T)Ex_0)]ds$$

$$+K_F(I+K_F)^{-1}(x_1-S(T)Ex_0)$$

$$=S(T)Ex_0+(I+K_F)^{-1}(x_1-S(T)Ex_0)$$
$$+(I+K_F)^{-1}K_F(x_1-S(T)Ex_0)$$
$$=S(T)Ex_0+x_1-S(T)Ex_0=x_1.$$

Hence Theorem 6 holds true.

Theorem 7 If system (4) is exactly controllable on $[0,T]$, operator K_F is linear and $K_F \geq 0$, then nonlinear singular distributed parameter system (1) is exactly controllable on $[0,T]$, and the control steering $x(T)=x_1$ is

$$u(t)=B^*S^*(T-t)L_B^{-1}(I+K_F)^{-1}(x_1-S(T)Ex_0).$$

Proof It is obvious that $\tilde{G}_{BF}=(I+K_F)$ is one to one, and

$$\left\|\tilde{G}_{BF}\, x\right\|=\|x\|, x\in X. \tag{21}$$

This implies that $R(\tilde{G}_{BF})$ is a closed set. In the following, we prove that $\tilde{G}_{BF}$ is surjective, i.e., $R(\tilde{G}_{BF})=X$. For the purpose of contradiction, suppose $R(\tilde{G}_{BF})$ is strictly contained in X, from Hahn Banach's Theorem there exists $x_0\in X$ and $x_0\neq 0$ such that

$$<\tilde{G}_{BF}\, x,x_0>=<x+K_Fx,x_0>=0,\forall x\in X.$$

In particular, let $x=x_0$, then we obtain

$$<\tilde{G}_{BF}\, x_0,x_0>=<x_0,x_0>+<K_Fx_0,x_0>=0.$$

Thus $x_0=0$. This is in contradiction with $x_0\neq 0$. Therefore, $\tilde{G}_{BF}$ is a bijection and according to open mapping Theorem we obtain that $\tilde{G}_{BF}^{-1}=(I+K_F)^{-1}$ is a bounded linear operator. The following proof is similar to Theorem 6. Hence Theorem 7 holds true.

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