New Approach to Financial Time Series Forecasting - Quantum Minimization Regularizing BWGC and NGARCH Composite Model

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Abstract

A hybrid BPNN-weighted GREY-C3LSP prediction (BWGC) is used for resolving the overshooting phenomenon significantly; however, it may lose the localization once volatility clustering occurs. Thus, we propose a compensation to deal with the time-varying variance in the residual errors, that is, incorporating a non-linear generalized autoregressive conditional heteroscedasticity (NGARCH) into BWGC, and quantum minimization (QM) is employed to regularize the smoothing coefficients for both BWGC and NGARCH to effectively improve model's robustness as well as to highly balance the generalization and the localization.

Keywords: BPNN-weighted GREY-C3LSP prediction, non-linear generalized autoregressive conditional heteroscedasticity, quantum minimization.

1. Introduction

Grey model (GM) [1] has been widely applicable for the purpose of short-term forecasting. Literatures [2] have claimed that the prediction using grey model can achieve better accuracy than traditional ones, e.g., Holt-Winters smoothing model and Box-Jenkins method [3]. The overshooting phenomenon [4] results in some big residual errors in grey model applications [5] as shown in Fig. 1. In order to eliminate this phenomenon, compensation is considered to offset the overshooting results by employing a cumulated 3-point least squared linear prediction (C3LSP) [4] that produces the underestimated outputs at the same period. A back-propagation neural net (BPNN) [6] is also introduced to explore the appropriate weights for combining both models linearly and this combination denotes BWGC [5]. BWGC hopefully can yield the satisfactory results in the forecasting as shown in Fig. 2. However, the volatility clustering phenomenon [7] or the fat tail effect [8] deteriorates the performance of BWGC model some as shown in Fig. 4. In order to lessen the volatility clustering phenomenon or the fat tail effect, nonlinear type of generalized autoregressive conditional hereoscedasticity (NGARCH) [9] [10] may incorporate into BWGC model, and then the combination of BWGC and NGARCH is regularized optimally by quantum minimization.

2. Models

2.1. Grey Model

The following steps briefly describe GM(1,1|α) modeling.

$$x^{(1)}(k)=\sum_{j=1}^{k}x^{(0)}(j),\quad k=1,2,\ldots,n \tag{1}$$

$x^{(0)}(k)$: sampled nonnegative data sequence

$$x^{(0)}(k)+az^{(1)}(k)=b,\ \ k=2,3,\ldots,n \tag{2}$$

$$z^{(1)}(k)=\alpha x^{(1)}(k)+(1-\alpha)x^{(1)}(k-1),\ \ 0\le\alpha\le 1 \tag{3}$$

$z^{(1)}(k)$: the background value

$$Y_N=\begin{bmatrix}x^{(0)}(2)\\x^{(0)}(3)\\x^{(0)}(4)\\ \vdots \\x^{(0)}(n)\end{bmatrix},\ B=\begin{bmatrix}-z^{(1)}(2)&1\\-z^{(1)}(3)&1\\-z^{(1)}(4)&1\\ \vdots & \vdots\\-z^{(1)}(n)&1\end{bmatrix},\ \hat{a}=\begin{bmatrix}a\\b\end{bmatrix} \tag{4}$$

$$Y_N=B\hat{a} \tag{5}$$

$$\frac{d\hat{x}^{(1)}(k)}{dk}+a\hat{x}^{(1)}(k)=b \tag{6}$$

$$\begin{aligned}\hat{x}^{(0)}(k)&=\hat{x}^{(1)}(k)-\hat{x}^{(1)}(k-1)\\&=(x^{(0)}(1)-\frac{b}{a})(e^{-a(k-1)}-e^{-a(k-2)}),\quad k=2,3,\ldots\end{aligned} \tag{7}$$

2.2. Cumulated 3-point Least Square Prediction Model

A cumulated 3-point least squared prediction (C3LSP) [4] is constructed as follows:

$$s^{(1)}(k) = scale \cdot s^{(0)}(k) + offset, \quad k = 1,2,3 \tag{8}$$

$s^{(0)}(k)$: given data.

$scale$: an user-defined scaling factor.

$offset$: an user-defined shift factor or offset.

$$s^{(2)}(k) = \sum_{j=1}^{k} s^{(1)}(j), \quad k = 1,2,3 \tag{9}$$

$s^{(1)}(j)$: representing three successive sampled data denoted as $s^{(1)}(k-3)$, $s^{(1)}(k-2)$, and $s^{(1)}(k-1)$ before predicting the single step-ahead output $\hat{s}^{(1)}(k)$.

$$s^{(2)}(k) = c_1 k + c_0 \,,\ k = 1,2,3 \tag{10}$$

This turns out to be a normal equation [11],

$$\mathbf{S} = \mathbf{KC} \,, \tag{11}$$

and its approximate optimal parameter estimation [11] is

$$\mathbf{C} = (\mathbf{K}^T \mathbf{K})^{-1} \mathbf{K}^T \mathbf{S} \,. \tag{12}$$

$\mathbf{S} = [s^{(2)}(1)\,,\, s^{(2)}(2),\, s^{(2)}(3)]^T$, $\mathbf{K} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}$, and $\mathbf{C} = [c_1, c_0]^T$.

$$\hat{s}^{(2)}(k+1) = c_1(k+1) + c_0 \,,\ k = 3 \tag{13}$$

$$\hat{s}^{(1)}(k+1) = \hat{s}^{(2)}(k+1) - s^{(2)}(k) \,,\ k = 3 \tag{14}$$

$$\hat{s}^{(0)}(k+1) = \frac{1}{scale}(\hat{s}^{(1)}(k+1) - offset) \tag{15}$$

2.3. ARMAX/NGARCH Composite Model

The ARMAX [12] encompass autoregressive (AR), moving-average (MA), and regression (X) models, in any combinations as expressed below.

$$y(t) = K_1 + \sum_{i=1}^{r} R_i y(t-i) + e(t) + \sum_{j=1}^{m} M_j e(t-j) + \sum_{k=1}^{N_x} \beta_k X(t,k) \tag{16}$$

The NGARCH(p,q) [10] consists of nonlinear conditional time-varying variances and Gaussian innovations. Its mathematical formula is shown as follows.

$$h^2(t) = K_2 + \sum_{i=1}^{p} G_i h^2(t-1) + \sum_{j=1}^{q} A_j h^2(t-j)\left[\frac{e(t-j)}{\sqrt{h^2(t-j)}} - C_j\right]^2 \tag{17}$$

with constraints

$\sum_{i=1}^{p} G_i + \sum_{j=1}^{q} A_j < 1$; $K_2 > 0$; $G_i \geq 0,\ i = 1,\dots,p$; $A_j \geq 0,\ j = 1,\dots,q$

3. Quantum Minimization

Quantum minimization that makes optimization task work out associated with probability of success at least 1/2 within an unsorted database is realized by quantum minimum searching algorithm [13]. A quantum exponential searching algorithm [14] is called by quantum minimum searching algorithm to be as a subroutine to serve a fast database searching engine.

3.1. Quantum Exponential Searching Algorithm

We assume in this section that the number t of solutions is known and that it is not zero. Let $A = \{i \mid F(i) = 1\}$ and $B = \{i \mid F(i) = 0\}$.

Step 1: For any real numbers k and l such that $tk^2 + (N-t)l^2 = 1$, redefin

$$|\,\Psi(k,l)\rangle = \sum_{i \in A} k\,|\,i\rangle + \sum_{i \in B} l\,|\,i\rangle\,.$$

A straightforward analysis of Grover's algorithm [15] shows that one iteration transforms $|\Psi(k,l)\rangle$ into

$$\left|\Psi\left(\frac{N-2t}{N}k + \frac{2(N-t)}{N}l, \frac{N-2t}{N}l - \frac{2t}{N}k\right)\right\rangle.$$

Step 2: This gives rise to a recurrence similar to the iteration transforms in Grover's algorithm, whose solution is that the state $|\Psi(k_j, l_j)\rangle$ after j iterations is given by

$$k_j = \frac{1}{\sqrt{t}}\sin((2j+1)\theta) \ \ and \ \ l_j = \frac{1}{\sqrt{N-t}}\cos((2j+1)\theta)\,.$$

where the angle θ is chosen so that $\sin^2\theta = t/N$ and $0 < \theta \leq \pi/2$.

3.2. Quantum Minimum Searching Algorithm

We second give the minimum searching algorithm [13] in which the minimum searching problem is to find the index i such that $T[i]$ is minimum where $T[0,\dots,N-1]$ is to be an unsorted table of N items, each holding a value from an ordered set.

Step 1: Choose threshold index $0 \leq i \leq N-1$ uniformly at random.

Step 2: Repeat the following stages (2a and 2b) and interrupt it when the total running time is more than $22.5\sqrt{N} + 1.4\lg^2 N$. Then go to stage (2c).

(a) Initialize the memory as $\sum_j \frac{1}{\sqrt{N}}|\,j\rangle|\,i\rangle$. Mark every item j for which $T[j] < T[i]$.

(b) Apply the quantum exponential searching algorithm of [14].

(c) Observe the first register: let i' be the outcome. If $T[i'] < T[i]$, then set threshold index i to i' .

Step 3: Return i

This process is repeated until the probability that the threshold index selects the minimum is sufficiently large.

4. Regularizing BWGC/NGARCH Model

A model is with the output $\hat{u}^{(0)}(k)$ as expressed below.

$$\hat{u}^{(0)}(k) = w_1\hat{x}^{(0)}(k) + w_2\hat{s}^{(0)}(k)\,, \quad w_1 + w_2 = 1 \tag{18}$$

In Eq. (18), $\hat{x}^{(0)}(k)$ and $\hat{s}^{(0)}(k)$ stand for the outputs from GM and C3LSP, respectively, denoted as GREY-C3LSP. A BPNN [16] is employed to train and simulate the weight values of w_1 and w_2.

Moreover, NGARCH has been incorporated into BWGC to deal with volatility clustering and the formulated equation is expressed below.

$$\hat{y}_{final\ output}{}^{(0)}(k) = f(\hat{u}^{(0)}(k), \hat{h}^{(0)}(k)) \tag{19}$$

where $\hat{h}^{(0)}(k)$ represents NARCH output and $\hat{u}^{(0)}(k)$ stands for BWGC output. In order to simplify Eq. (19) and speed-up computation, a linear form [17] is suggested to fast computation as follows.

$$\hat{y}_{final\ output}{}^{(0)}(k) = \lambda_1\hat{u}^{(0)}(k) + \lambda_2\hat{h}^{(0)}(k)\,, \quad \lambda_1 + \lambda_2 = 1 \tag{20}$$

Two coefficients λ_1 and λ_2 represent the regularization for $\hat{u}^{(0)}(k)$ and $\hat{h}^{(0)}(k)$, respectively. Quantum minimization is used for optimizing Eq. (20) and hopefully reducing both the empirical risk and the structural risk simultaneously.

5. Experimental Results

Training data are sampled from observed historical monthly stock price index for a period of 47 months from January 1997 to November 2000. Next, the second phase, the prior validation, goes on several single-step-look-ahead prediction by the regularized QM-BWGC/GNARCH model. This single-step-look-ahead forecast on international stock price indices dated from December 2000 to February 2002 for 15 monthly index's values that referred to the stock price indices at four markets [18] —New York D.J. Industrials Index, Taipei TAIEX Index, Tokyo Nikkei Index, and Seoul Composite Index. Four forecasts (GM, C3LSP, BWGC, and QM-BWGC/GNARCH) are illustrated as shown in Figure 1, 2, 3, and 4. Two criterions mean-square-error (MSE) and mean-absolute-percent-error (MAPE) will be employed to compare the performance for each model listed in Table 1 and 2.

6. Concluding Remarks

The crucial problem of the overshoot and volatility clustering effects has been resolved simultaneously by a quantum-minimized BWGC/NGARCH model. This is because NGARCH can used to deal with the inherent volatility clustering and quantum minimization is applied for regularizing NGARCH into BWGC so as to highly balance the generalization and localization of model and to successfully achieve the best performance.

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8. References

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Table 1

The performance evaluation follows the criteria: (a) mean square error (MSE)* and (b) mean absolute percent error (MAPE) on international stock price indices forecasting up to 15 monthly index's values from Dec. 2000 to Feb. 2002.

(a)

Methods	NY-D.J. Industrials Index	Taipei TAIEX Index	Tokyo Nikkei Index	Seoul Composite Index	Average MSE
GM	0.0043	0.0599	0.0195	0.0904	0.0435
C3LSP	0.0032	0.0610	0.0254	0.1060	0.0489
4LR	0.0042	0.0725	0.0236	0.0946	0.0487
HW	0.0058	0.1197	0.0379	0.1452	0.0772
BJ	0.0107	0.1150	0.0188	0.1856	0.0825
RBFNN	0.0023	0.0486	0.0188	0.1183	0.0470
GRNN	0.0021	0.0477	0.0181	0.1067	0.0437
NGARCH	0.0036	0.0682	0.0171	0.0718	0.0402
BWGC	0.0021	0.0420	0.0150	0.0587	0.0295
QM-BWGCNG	0.0018	0.0417	0.0139	0.0570	0.0286

(b)

Methods	NY-D.J. Industrials Index	Taipei TAIEX Index	Tokyo Nikkei Index	Seoul Composite Index	Average MAPE
GM	0.0615	0.1105	0.0586	0.1090	0.0849
C3LSP	0.0513	0.1170	0.0643	0.1028	0.0839
4LR	0.0635	0.1136	0.0645	0.1048	0.0866
HW	0.0704	0.1455	0.0865	0.1325	0.1087
BJ	0.0773	0.1473	0.0540	0.1329	0.1029
RBFNN	0.0413	0.1056	0.0580	0.1178	0.0807
GRNN	0.0412	0.0891	0.0515	0.0941	0.0690
NGARCH	0.0603	0.1098	0.0601	0.1058	0.0840
BWGC	0.0482	0.0951	0.0576	0.0902	0.0728
QM-BWGCNG	0.0401	0.0898	0.0512	0.0860	0.0668

Note: Method abbreviation
1. GM- GM(1,1|α) Model
2. C3LSP- Cumulated 3-Point Least Squared Linear Model
3. 4LR- 4 Points Linear Regression
4. HW-Holt-Winters Smoothing Model
5. BJ-Box-Jenkins Forecasting Model
6. RBFNN- Radial Basis Function Neural Net
7. GRNN- General Regression Neural Net
8. NGARCH- ARMAX/NGARCH Composition Model
9. BWGC- BPNN-Weighted GREY-CLSP Model
10. QM-NGBWGC- QM-NGARCH/BWGC Model

*: normalized MSE

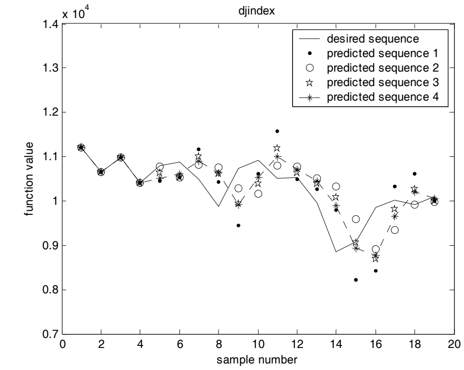

Fig. 1. The forecasts of New York D.J. Industrials Index.

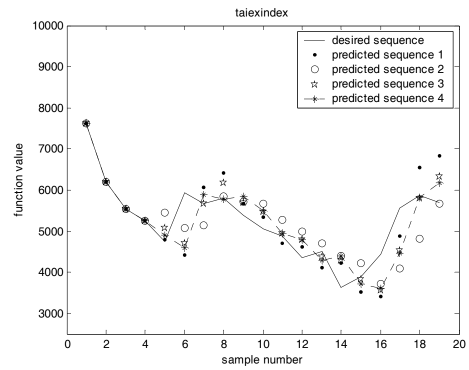

Fig. 2. The forecasts of Taipei TAIEX Index.

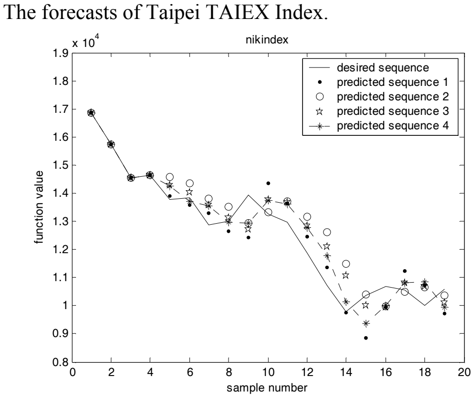

Fig. 3. The forecasts of Tokyo Nikkei Index.

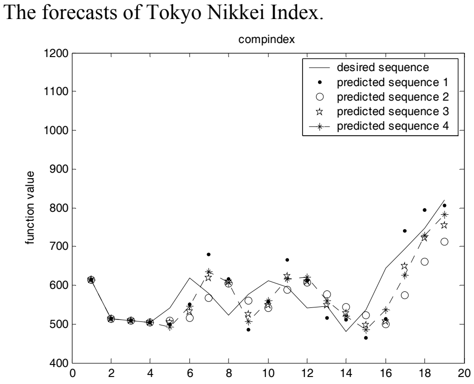

Fig. 4. The forecasts of Seoul Composite Index.