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# A Family of Ratio Estimators in Stratified Random Sampling Utilizing Auxiliary Attribute Along Side the Nonresponse Issue

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**ABSTRACT**

This article proposes some estimators based on an adaptation of the estimators developed by Bahl and Tuteja [1], Diana [2], Koyuncu and Kadilar [3], Koyuncu and Kadilar [4], Shabbir and Gupta [5], and Koyuncu [6] utilizing available supplementry attributes. Further, a new family of estimators is also developed by adapting Koyuncu [7] and Koyuncu [6]. Under stratified sampling scheme along side the nonresponse issue, the expressions for the the mean square errors (MSEs) of the adapted and proposed estimators have been determined. These theoretical findings are demonstrated by a numerical illustration with original data set.

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## 1. INTRODUCTION

In survey sampling, utilization of the auxiliary information results in significant improvement in the accuracy/precision of estimators of the population mean. When auxiliary information is accessible, the analysts need to use it in the strategy for estimation to acquire the most productive estimator. Ratio method is utilized to get more efficient estimates for the population mean by utilizing the correlation between study and auxiliary variables. Hansen *et al.* [8] and Kaur [9] proposed some estimators utilizing auxiliary information under stratified random sampling. Diana [2] also developed a class of estimators of the population mean by examining their mean square errors (MSEs) upto the $k^{th}$ order approximation. After that Koyuncu and Kadilar [10] extended the idea of Diana [2]. Sisodia and Dwivedi [11] introduced a ratio estimator by using coefficient of variation of an auxiliary variable. Singh and Kakran [12] proposed another ratio estimator by using known coefficient of kurtosis of an auxiliary variable. Upadhyaya and Singh [13] also suggested a ratio type estimator by using coefficient of variation and kurtosis of an auxiliary variable. The estimators of Sisodia and Dwivedi [1], Singh and Kakran [12], and Upadhyaya and Singh [13] belong to simple random sampling scheme. Kadilar and Cingi [14] give the stratified versions of these abovementioned estimators. Kadilar and Cingi [15] and Koyuncu and Kadilar [4] developed some estimators by extending the idea of Prasad [16].

All of the above discussion shows that much of literature is available on the estimation of population mean under stratified random sampling using available auxiliary information. Recently, Koyuncu [7] introduced, a family of estimators under stratified random sampling scheme using known parameters of auxiliary attribute. So we extend the work of Koyuncu [7], by introducing the estimators of Bahl and Tuteja [1], Diana [2], Koyuncu and Kadilar [3], Koyuncu and Kadilar [4], Shabbir and Gupta [5] for the situation when auxiliary attribute is available. Note that these estimators were developed when auxiliary variable is known, but in some situations it is not possible to obtain information on auxiliary variable, in these situations we look for auxiliary character and, if available, use it in estimation stage. So we are introducing these estimators for the case when auxiliary character (attribute) is known. Also, we propose a new class of estimators in this filed by taking motivation from Koyuncu [7] and Koyuncu [6]. Further we also extend the work for nonresponse issue, when nonresponse present in variate of interest $Y$. For detail study see Chaudhary *et al.* [17].

Remaining part of this paper is composed as follow. In Sections 2 and 3, we quickly present preliminaries and reviewed estimators when auxiliary information is available. Adapted estimators are presented in Section 4. In Section 5, a new class of estimators is proposed for stratified random sampling along with their minimum MSE. Section 6, is devoted for nonresponse issue. Analytical examinations are

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appeared in Section 7 to represent that the proposed estimator are dominant than the existing ones. Finally, in order to check the performance of existing estimators with the proposed estimators, a numerical illustration has been carried out using real (natural) population in Section 8.

## 2. NOTATIONS

Suppose $\mathfrak{U} = \{\mathfrak{U}_1, \mathfrak{U}_2, ..., \mathfrak{U}_N\}$ be the finite population containing $N$ units. Let $\phi$ is an auxiliary character (attribute) and $Y$ is the study variable taking values $y_{\mathfrak{h}i}$ and $\phi_{\mathfrak{h}i}$, respectively, in the unit $(i = 1, 2, ..., N)$ in the $\mathfrak{h}$th stratum consisting of $N_{\mathfrak{h}}$ units such that $\sum_{\mathfrak{h}=1}^{\mathfrak{L}} N_{\mathfrak{h}} = N$. Let $n_{\mathfrak{h}}$ be the size of the sample drawn from the $\mathfrak{h}$th stratum by using simple random sampling without replacement scheme such that $\sum_{\mathfrak{h}=1}^{\mathfrak{L}} n_{\mathfrak{h}} = n$. Further, we consider that $\phi_{\mathfrak{h}}$ takes only binary values (0, 1) as

$\phi_i = 1$, if the ith unit of the population possesses attribute $\phi$,

$\phi_i = 0$, otherwise.

Let $A = \sum_{i=1}^{N} \phi_i$, $a = \sum_{i=1}^{n} \phi_i$, $A_{\mathfrak{h}} = \sum_{i=1}^{N_{\mathfrak{h}}} \phi_{\mathfrak{h}i}$, $a_{\mathfrak{h}} = \sum_{i=1}^{n_{\mathfrak{h}}} \phi_{\mathfrak{h}i}$ denote the total number of units in the population, population stratum $\mathfrak{h}$ and sample stratum $\mathfrak{h}$ possessing an auxiliary attribute $\phi$, respectively. The corresponding population and sample, $\mathfrak{h}$-th proportions are $P_{\mathfrak{h}} = \frac{A_{\mathfrak{h}}}{N}$ and $\widehat{p_{\mathfrak{h}}} = \frac{a_{\mathfrak{h}}}{n}$, respectively.

Moreover $\bar{y}_{\mathfrak{st}} = \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}} \bar{y}_{\mathfrak{h}}$, where $\bar{y}_{\mathfrak{h}} = \frac{1}{n_{\mathfrak{h}}} \sum_{i=1}^{n_{\mathfrak{h}}} y_{\mathfrak{h}i}$; $\bar{Y} = \bar{Y}_{st} = \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}} \bar{Y}_h$ and $\mathfrak{W}_{\mathfrak{h}} = \frac{N_{\mathfrak{h}}}{N}$ is the stratum weight. We can define similar expressions for $p$ also. Further $\psi_s = \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}} S_{\phi\mathfrak{h}}$, $\psi_c = \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}} C_{\phi\mathfrak{h}}$, $\psi_b = \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}} \beta_2 (\phi\mathfrak{h})$ and $\psi_r = \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}} \rho_{\phi\mathfrak{h}}$.

For finding MSE of the proposed and existing estimators, we use the following notation

$$e_{o\mathfrak{st}} = \frac{\bar{y}_{\mathfrak{st}} - \bar{Y}}{\bar{Y}}, \quad E\left(e_{o\mathfrak{st}}^2\right) c = \frac{\sum_{\mathfrak{h}=1}^{L} \mathfrak{W}_{\mathfrak{h}}^2 f'_{\mathfrak{h}} S_{yh}^2}{\bar{Y}^2} = \delta_{2.0}, \quad E\left(e_{o\mathfrak{st}}\right) = 0,$$

$$e_{1\mathfrak{st}} = \frac{p_{\mathfrak{st}} - P}{P}, \quad E\left(e_{1\mathfrak{st}}^2\right) = \frac{\sum_{\mathfrak{h}=1}^{L} \mathfrak{W}_{\mathfrak{h}}^2 f'_{\mathfrak{h}} S_{xh}^2}{\bar{X}^2} = \delta_{0.2}, \quad E\left(e_{1\mathfrak{st}}\right) = 0,$$

$$f' = \left(\frac{1}{n_h} - \frac{1}{N_h}\right), \quad E\left(e_{o\mathfrak{st}} e_{1\mathfrak{st}}\right) = \frac{\sum_{\mathfrak{h}=1}^{L} \mathfrak{W}_{\mathfrak{h}}^2 f'_{\mathfrak{h}} S_{xyh}^2}{\bar{Y}^2} = \delta_{1.1}$$

and

$$\delta_{a.b} = \sum_{\mathfrak{h}=1}^{L} \mathfrak{W}_{\mathfrak{h}}^{a+b} E\left[\frac{(y_{\mathfrak{h}i} - \bar{Y}_{\mathfrak{h}})^a (p_{\mathfrak{h}i} - P_{\mathfrak{h}})^b}{\bar{Y}^a P^b}\right].$$

Now, we describe some existing estimators with their MSEs in the next section.

## 3. EXISTING ESTIMATORS

The variance of the sample mean $\bar{y}_{\mathfrak{st}}$ in stratified random sampling with out replacement is given by

$$V\left(\bar{y}_{\mathfrak{st}}\right) = \bar{Y}^2 \delta_{2.0}. \tag{1}$$

The stratified version of customary ratio estimator for mean is given by

$$\hat{\bar{y}}_{R\mathfrak{st}} = \frac{\bar{y}_{\mathfrak{st}}}{p_{\mathfrak{st}}} P. \tag{2}$$

The MSE of customary ratio estimator given in Eq. (2) for the first order approximation is given below

$$\text{MSE}\left(\hat{\bar{y}}_{R\mathfrak{st}}\right) = \bar{Y}^2 \left[\delta_{2.0} + \delta_{0.2} - 2\delta_{1.1}\right]. \tag{3}$$

The traditional Regression estimator is given by,

$$\hat{\bar{y}}_{st(lr)} = \bar{y}_{\mathfrak{st}} + b_{\mathfrak{st}} \left(P - p_{\mathfrak{st}}\right). \tag{4}$$

The MSE of $\hat{\bar{y}}_{st(lr)}$ is

$$\text{MSE}\left(\hat{\bar{y}}_{st(lr)}\right) = \overline{Y}^2 \delta_{2.0}\left(1 - \rho_{st}^2\right), \tag{5}$$

where $\rho_{\mathfrak{st}} = \frac{\delta_{1.1}}{\sqrt{\delta_{2.0}}\sqrt{\delta_{0.2}}}$.

Utilizing auxiliary attributes, Koyuncu [7] introduced the following family of estimators under stratified random sampling as follows:

$$\hat{\bar{y}}_{kk(\mathfrak{st})} = \left[w_{c1}\bar{y}_{\mathfrak{st}} + w_{c2}\left(P - p_{\mathfrak{st}}\right)\right]\left(\frac{a_{\mathfrak{st}}P + b_{\mathfrak{st}}}{a_{\mathfrak{st}}p_{\mathfrak{st}} + b_{\mathfrak{st}}}\right), \tag{6}$$

where $a\,(\neq 0)$ and $b$ be any known characteristics of auxiliary information.

Some family members of Koyuncu [7] are shown in Table 1.

**Table 1** Some family members of Koyuncu [7].

| $\hat{\bar{y}}_{kk(\mathfrak{st})}$ | $a_{\mathfrak{st}}$ | $b_{\mathfrak{st}}$ | $\nu_j$ |
|---|---|---|---|
| $\hat{\bar{y}}_{kk1(\mathfrak{st})}$ | 1 | $\psi_r$ | $\nu_3$ |
| $\hat{\bar{y}}_{kk2(\mathfrak{st})}$ | 1 | $\psi_c$ | $\nu_2$ |
| $\hat{\bar{y}}_{kk3(\mathfrak{st})}$ | $\psi_r$ | $\psi_c$ | $\nu_4$ |
| $\hat{\bar{y}}_{kk4(\mathfrak{st})}$ | $\psi_c$ | $\psi_r$ | $\nu_5$ |

$$\text{MSE}\left(\hat{\bar{y}}_{kk(\mathfrak{st})}\right) = \overline{Y}^2 + w_{c1}^2 A_{kk} + w_{c2}^2 B_{kk} + 2w_{c1}w_{c2}C_{kk} - 2w_{c1}D_{kk} - 2w_{c2}E_{kk}. \tag{7}$$

The minimum MSE of $\hat{\bar{y}}_{kk(\mathfrak{st})}$ for

$$w_{c1}^{(opt)} = \frac{B_{kk}D_{kk} - C_{kk}E_{kk}}{A_{kk}B_{kk} - C_{kk}^2} \text{ and } w_{c2}^{(opt)} = \frac{-C_{kk}D_{kk} + A_{kk}E_{kk}}{A_{kk}B_{kk} - C_{kk}^2}$$

is given by

$$\text{MSE}_{min}\left(\hat{\bar{y}}_{kk(\mathfrak{st})}\right) = \overline{Y}^2 - \frac{B_{kk}D_{kk}^2 + A_{kk}E_{kk}^2 - 2C_{kk}D_{kk}E_{kk}}{A_{kk}B_{kk} - C_{kk}^2}, \tag{8}$$

where

$$A_{kk} = \overline{Y}^2\left[1 + \delta_{2.0} + 3\nu_j^2\delta_{0.2} - 4\nu_j\delta_{1.1}\right],$$

$$B_{kk} = P^2\delta_{0.2},$$

$$C_{kk} = P\overline{Y}\left[2\nu_j\delta_{0.2} - \delta_{1.1}\right],$$

$$D_{kk} = \overline{Y}^2\left[1 + \nu_j^2\delta_{0.2} - \nu_j\delta_{1.1}\right],$$

$$E_{kk} = P\overline{Y}\nu_j\delta_{0.2}.$$

## 4. ADAPTED ESTIMATORS

By adapting Bahl and Tuteja [1], we propose the following exponential estimator under stratified sampling using auxiliary attributes

$$\hat{\bar{y}}_{BT\mathfrak{st}} = \bar{y}_{\mathfrak{st}} exp\left[\frac{P - p_{\mathfrak{st}}}{P + p_{\mathfrak{st}}}\right]. \tag{9}$$

The MSE of $\hat{\bar{y}}_{BT\mathfrak{st}}$ for the first order approximation are given below:

$$\text{MSE}\left(\hat{\bar{y}}_{BT\mathfrak{st}}\right) = \overline{Y}^2\left[\delta_{2.0} + \frac{1}{4}\delta_{0.2} - \delta_{1.1}\right]. \tag{10}$$

By adapting Diana [2], we propose the following family of estimators under stratified sampling using auxiliary attributes

$$\hat{\bar{y}}_{di} = \bar{y}_{\mathfrak{st}}\left(\frac{p_{\mathfrak{st}}}{P}\right)^{m_1}\left[w_d + (1 - w_d)\left(\frac{p_{\mathfrak{st}}}{P}\right)^{m_2}\right]^{m_3}, \tag{11}$$

where $m_1$, $m_2$, $m_3$, and $w_d$ can take values finitely.

Koyuncu and Kadilar [3] constructed the generalized version of Diana [2]. By adapting Koyuncu and Kadilar [3], we constructed the generalized version of $\hat{\hat{y}}_{di}$, utilizing available auxiliary attributes as

$$\hat{\hat{y}}_{kd} = \bar{y}_{\mathfrak{st}} \left(\frac{p_{\mathfrak{st}}}{P}\right)^{m_4} \left[w_{kd} + (1 - w_{kd}) \left(\frac{a_{\mathfrak{st}} p_{\mathfrak{st}} + b_{\mathfrak{st}}}{a_{\mathfrak{st}} P + b_{\mathfrak{st}}}\right)^{m_5}\right]^{m_6}, \tag{12}$$

where $m_4$, $m_5$, $m_6$, and $w_{kd}$ can take values finitely.

It is interesting to note that $\hat{\hat{y}}_{kd}$, $\hat{\hat{y}}_{di}$, and $\hat{\hat{y}}_{st(lr)}$ have equal MSE.

By adapting Koyuncu and Kadilar [4], we developed the following ratio estimators under stratified sampling, utilizing auxiliary attributes as

$$\hat{\hat{y}}_{ki(\mathfrak{st})} = k\bar{y}_{\mathfrak{st}} \left[\frac{aP + b}{\alpha \left(a_{\mathfrak{st}} p_{\mathfrak{st}} + b_{\mathfrak{st}}\right) + (1 - \alpha) \left(a_{\mathfrak{st}} P + b_{\mathfrak{st}}\right)}\right]^M \; for \, (i = 1, 2, ..., 6), \tag{13}$$

where $M = 1$ for ratio type estimators, and $\alpha = 1$. The MSE of $\hat{\hat{y}}_{ki(\mathfrak{st})}$ is given by

$$\text{MSE}\left(\hat{\hat{y}}_{ki(\mathfrak{st})}\right) = \bar{Y}^2 \left[k^2\delta_{2.0} + g\alpha^2\nu_j^2\delta_{0.2} - 2M\alpha\nu_j\delta_{1.1}\left(2k^2 - k\right) + (k - 1)^2\right],$$

where

$$g = k^2\left(2M^2 + M\right) - k\left(M^2 + M\right).$$

The optimum value of k is

$$k^{opt} = \frac{A_k}{2B_k}.$$

Some family members of Koyuncu and Kadilar [4] are shown in Table 2.

**Table 2** Some family members of Koyuncu and Kadilar [4].

| $\hat{\hat{y}}_k$ | $\nu_j$ |
|---|---|
| $\hat{\hat{y}}_{k1(\mathfrak{st})}$ | $\nu_1 = \frac{P}{P+\psi_s}$ |
| $\hat{\hat{y}}_{k2(\mathfrak{st})}$ | $\nu_2 = \frac{P}{P+\psi_c}$ |
| $\hat{\hat{y}}_{k3(\mathfrak{st})}$ | $\nu_3 = \frac{P}{P+\psi_r}$ |
| $\hat{\hat{y}}_{k4(\mathfrak{st})}$ | $\nu_4 = \frac{\psi_r P}{\psi_r P+\psi_c}$ |
| $\hat{\hat{y}}_{k5(\mathfrak{st})}$ | $\nu_5 = \frac{\psi_c P}{\psi_c P+\psi_r}$ |
| $\hat{\hat{y}}_{k6(\mathfrak{st})}$ | $\nu_6 = \frac{\psi_r P}{\psi_r P+\psi_s}$ |

The minimum mean squared error of $\hat{\hat{y}}_{ki(\mathfrak{st})}$ is

$$\text{MSE}_{min}\left(\hat{\hat{y}}_{ki(\mathfrak{st})}\right) = \bar{Y}^2\left[1 - \frac{A_k^2}{4B_k}\right], \tag{14}$$

where

$$A_k = \left(M^2 + M\right)\alpha^2\nu_j^2\delta_{2.0} - 2M\alpha\nu_j\delta_{1.1} + 2,$$

$$B_k = \delta_{0.2} + \left(2M^2 + M\right)\alpha^2\nu_j^2\delta_{0.2} - 4M\alpha\nu_j\delta_{1.1} + 1.$$

By adapting Shabbir and Gupta [5], we introduce an exponential ratio estimator as

$$\hat{\hat{y}}_{sg(\mathfrak{st})} = \left[w_{sg1}\bar{y}_{st} + w_{sg2}\left(P - p_{st}\right)\right] exp\left(\frac{\bar{A} - \bar{a}_{st}}{\bar{A} + \bar{a}_{st}}\right), \tag{15}$$

where

$$\bar{a} = p_{st} + NP, \bar{A} = P + NP.$$

The MSE of $\hat{\hat{y}}_{sg(\mathfrak{st})}$ is

$$\text{MSE}\left(\hat{\hat{y}}_{sg(\mathfrak{st})}\right) = \bar{Y}^2 + w_{sg1}^2 A_{sg} + w_{sg2}^2 B_{sg} + 2w_{sg1}w_{sg2}C_{sg} - 2w_{sg1}D_{sg} - w_{sg2}E_{sg}. \tag{16}$$

The minimum MSE of $\hat{\hat{y}}_{sg(\mathfrak{st})}$ for

$$w_{sg1}^{(opt)} = \frac{B_{sg}D_{sg} - \frac{C_{sg}E_{sg}}{2}}{A_{sg}B_{sg} - C_{sg}^2} \text{ and } w_{sg2}^{(opt)} = \frac{-C_{sg}D_{sg} + \frac{A_{sg}E_{sg}}{2}}{A_{sg}B_{sg} - C_{sg}^2}$$

is given by

$$\text{MSE}_{min}\left(\hat{\hat{y}}_{sg(\mathfrak{st})}\right) = \overline{Y}^2 - \frac{B_{sg}D_{sg}^2 + \frac{A_{sg}E_{sg}^2}{4} - C_{sg}D_{sg}E_{sg}}{A_{sg}B_{sg} - C_{sg}^2}, \tag{17}$$

where

$$A_{sg} = \overline{Y}^2\left[1 + \delta_{2.0} + \frac{\delta_{0.2}}{(1+N)^2} - \frac{2\delta_{1.1}}{(1+N)}\right],$$

$$B_{sg} = P^2\delta_{0.2},$$

$$C_{sg} = P\overline{Y}\left[\frac{\delta_{0.2}}{(1+N)} - \delta_{1.1}\right],$$

$$D_{sg} = \overline{Y}^2\left[1 + \frac{3\delta_{0.2}}{8(1+N)^2} - \frac{\delta_{1.1}}{2\,(1+N)}\right],$$

$$E_{sg} = P\overline{Y}\frac{\delta_{0.2}}{(1+N)}.$$

By adapting Koyuncu [6], we introduce exponential family of ratio estimators as

$$\hat{\hat{y}}_{nk(\mathfrak{st})} = \left[w_{1nk}\bar{y}_{st} + w_{2nk}\left(\frac{p_{\mathfrak{st}}}{P}\right)^{\gamma'}\right] exp\left[\frac{U_{\mathfrak{st}}\left(P - p_{\mathfrak{st}}\right)}{U_{\mathfrak{st}}\left(P + p_{\mathfrak{st}}\right) - 2V_{\mathfrak{st}}}\right]. \tag{18}$$

Some family members of Koyuncu [6] are shown in Table 3.

**Table 3** Some family members of Koyuncu [6].

| $\hat{\hat{y}}_{nk}$ | $U_{\mathfrak{st}}$ | $V_{\mathfrak{st}}$ | $\gamma'$ |
|---|---|---|---|
| $\hat{\hat{y}}_{nk1(\mathfrak{st})}$ | $\psi_b$ | $\psi_c$ | 1 |
| $\hat{\hat{y}}_{nk2(\mathfrak{st})}$ | $\psi_b$ | $\psi_c$ | 2 |
| $\hat{\hat{y}}_{nk3(\mathfrak{st})}$ | $\psi_c$ | $\psi_b$ | 1 |
| $\hat{\hat{y}}_{nk4(\mathfrak{st})}$ | $\psi_c$ | $\psi_b$ | 2 |
| $\hat{\hat{y}}_{nk5(\mathfrak{st})}$ | $\psi_r$ | $\psi_c$ | 1 |
| $\hat{\hat{y}}_{nk6(\mathfrak{st})}$ | $\psi_r$ | $\psi_c$ | 2 |

The MSE of $\hat{\hat{y}}_{nk(\mathfrak{st})}$ is

$$\text{MSE}\left(\hat{\hat{y}}_{nk(\mathfrak{st})}\right) = \overline{Y}^2 + w_{1nk}^2A_{nk} + w_{2nk}^2B_{nk} + 2w_{1nk}w_{2nk}C_{nk} - 2w_{1nk}D_{nk} - 2w_{2nk}E_{nk}. \tag{19}$$

The minimum MSE of $\hat{\hat{y}}_{nk(\mathfrak{st})}$ for

$$w_{1nk}^{(opt)} = \frac{B_{nk}D_{nk} - C_{nk}E_{nk}}{A_{nk}B_{nk} - C_{nk}^2} \text{ and } w_{2nk}^{(opt)} = \frac{-C_{nk}D_{nk} + A_{nk}E_{nk}}{A_{nk}B_{nk} - C_{nk}^2}$$

is given by

$$\text{MSE}_{min}\left(\hat{\hat{y}}_{nk(\mathfrak{st})}\right) = \overline{Y}^2 - \frac{B_{nk}D_{nk}^2 + A_{nk}E_{nk}^2 - 2C_{nk}D_{nk}E_{nk}}{A_{nk}B_{nk} - C_{nk}^2}. \tag{20}$$

where

$$A_{nk} = \overline{Y}^2 \left[1 + \delta_{2.0} + \theta_{nk}^2 \delta_{0.2} - 2\theta_{nk}\delta_{1.1}\right],$$

$$B_{nk} = \left[1 + \left(a_{nk}^2 + 2c_{nk}\right)\delta_{0.2}\right],$$

$$C_{nk} = \overline{Y}\left[1 + \left(\frac{3}{8}\theta_{nk}^2 - \frac{1}{2}a_{nk}\theta_{nk} + c_{nk}\right)\delta_{0.2} + \left(a_{nk} - \frac{1}{2}\theta_{nk}\right)\delta_{1.1}\right],$$

$$D_{nk} = \overline{Y}^2 \left[1 - \frac{3}{8}\theta_{nk}^2\delta_{0.2} - \frac{1}{2}\theta_{nk}\delta_{1.1}\right],$$

$$E_{nk} = \overline{Y}\left[1 + c_{nk}\delta_{0.2}\right].$$

Further

$$a_{nk} = \gamma - \frac{1}{2}\theta_{nk},\ b_{nk} = \frac{\gamma(\gamma - 1)}{2},\ c_{nk} = b_{nk} - \frac{1}{2}\gamma\theta_{nk} + \frac{3}{8}\theta_{nk}^2,\ \theta_{nk} = \frac{PU_{\mathfrak{st}}}{PU_{\mathfrak{st}} + V_{\mathfrak{st}}}.$$

## 5. PROPOSED CLASS OF ESTIMATORS

Taking motivation from Koyuncu [7], we propose the following class of estimators as

$$\hat{\bar{y}}_{N\mathfrak{st}} = \psi'_1 \left[\frac{a_{\mathfrak{st}}P + b_{\mathfrak{st}}}{\beta\left(a_{\mathfrak{st}}p_{\mathfrak{st}} + b_{\mathfrak{st}}\right) + (1-\beta)\left(a_{\mathfrak{st}}P + b_{\mathfrak{st}}\right)}\right]^g, \tag{21}$$

where

$$\psi'_1 = \left[w_1\left\{\frac{\bar{y}_{\mathfrak{st}}}{2}\left(\frac{P}{p_{\mathfrak{st}}} + \frac{p_{\mathfrak{st}}}{P}\right)\right\} + w_2\left(\frac{p_{\mathfrak{st}}}{P}\right)^\gamma\right].$$

We can write $\hat{\bar{y}}_{N\mathfrak{st}}$ as

$$\hat{\bar{y}}_{N\mathfrak{st}} = w_1\overline{Y}\left\{1 + e_{o\mathfrak{st}} - ae_{1\mathfrak{st}} + \left(b + \frac{1}{2}\right)e_{1\mathfrak{st}}^2 - ae_{o\mathfrak{st}}e_{1\mathfrak{st}}\right\} + w_2\left\{1 + (\gamma - a)e_{1\mathfrak{st}} + ce_{1st}^2\right\}, \tag{22}$$

where

$$a = g\beta v',\ b = \frac{g(g+1)}{2}\beta^2 v'^2,\ c = \left(b - a\gamma + \frac{\gamma(\gamma-1)}{2}\right),\ and v' = \frac{a_{\mathfrak{st}}P}{a_{\mathfrak{st}}P + b_{\mathfrak{st}}}.$$

The bias of $\hat{\bar{y}}_{N\mathfrak{st}}$ is

$$B\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) = w_1\overline{Y}\left\{1 + \left(b + \frac{1}{2}\right)\delta_{0.2} - a\delta_{11}\right\} + w_2\left(1 + c\delta_{0.2}\right) - \overline{Y}. \tag{23}$$

The MSE of $\hat{\bar{y}}_{N\mathfrak{st}}$ is

$$\mathrm{MSE}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) = L_1 + w_1^2\phi_{A1} + w_2^2\phi_{B1} + 2w_1w_2\phi_{C1} - 2w_1\phi_{D1} - w_2\phi_{E1}, \tag{24}$$

where

$$L_1 = \overline{Y}^2,$$

$$\phi_{A1} = \overline{Y}^2\left\{1 + \delta_{2.0} + \left(a^2 + 2b + 1\right)\delta_{0.2} - 2a\delta_{11}\right\},$$

$$\phi_{B1} = \left\{1 + \left((\gamma - a)^2 + 2c\right)\delta_{0.2}\right\},$$

$$\phi_{C1} = \overline{Y}\left\{1 + \left(c - a(\gamma - a) + b + \frac{1}{2}\right)\delta_{0.2} + (\gamma - 2a)\delta_{1.1}\right\},$$

$$\phi_{D1} = \overline{Y}^2 \left\{1 + \left(b + \frac{1}{2}\right)\delta_{0.2} - a\delta_{1.1}\right\},$$

$$\phi_{E1} = 2\overline{Y}\{1 + c\delta_{1.1}\}.$$

Some family members of proposed class are shown in Table 4.

**Table 4** Some family members of proposed class.

| $\hat{\hat{y}}_{N\mathfrak{st}}$ | $\gamma$ | g | $a_{\mathfrak{st}}$ | $b_{\mathfrak{st}}$ | $\beta$ |
|---|---|---|---|---|---|
| $\hat{\hat{y}}_{N1\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | $\psi_b$ | $\psi_c$ | 1 |
| $\hat{\hat{y}}_{N2\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | $\psi_c$ | $\psi_b$ | 1 |
| $\hat{\hat{y}}_{N3\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_r$ | 1 |
| $\hat{\hat{y}}_{N4\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_b$ | 1 |
| $\hat{\hat{y}}_{N5\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_c$ | 1 |
| $\hat{\hat{y}}_{N6\mathfrak{st}}$ | $\frac{1}{3}$ | 1 | $\psi_c$ | $\psi_r$ | 1 |
| $\hat{\hat{y}}_{N7\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_b$ | 1 |
| $\hat{\hat{y}}_{N8\mathfrak{st}}$ | 1 | 2 | $\psi_c$ | $\psi_r$ | 1 |
| $\hat{\hat{y}}_{N9\mathfrak{st}}$ | 1 | 2 | $\psi_r$ | $\psi_c$ | 1 |

By partially differentiating MSE of $\hat{\hat{y}}_{N\mathfrak{st}}$ w.r.t $w_1$ and $w_2$, we get the optimum values of $w_1$, $w_2$, that is,

$$w_1^{opt} = \left[\frac{\phi_{B1}\phi_{D1} - \frac{\phi_{C1}\phi_{E1}}{2}}{\phi_{A1}\phi_{B1} - \phi_{C1}^2}\right],$$

and

$$w_2^{opt} = \left[\frac{-\phi_{C1}\phi_{D1} + \frac{\phi_{A1}\phi_{E1}}{2}}{\phi_{A1}\phi_{B1} - \phi_{C1}^2}\right].$$

By putting $w_1^{opt}$, $w_2^{opt}$ in MSE $\left(\hat{\hat{y}}_{N\mathfrak{st}}\right)$ and get minimum MSE of $\hat{\hat{y}}_{N\mathfrak{st}}$, that is,

$$\text{MSE}_{min}\left(\hat{\hat{y}}_{N\mathfrak{st}}\right) = \left[\overline{Y}^2 L - \frac{A_N}{B_N}\right]. \tag{25}$$

where $A_N = \left[\phi_B\phi_D^2 + \frac{\phi_A\phi_E^2}{4} - \phi_C\phi_D\phi_E\right]$ and $B_N = \left[\phi_A\phi_B - \phi_C^2\right]$.

## 6. NONRESPONSE

Hansen and Hurwitz [18] estimator in case of nonresponse under stratified sampling scheme for $\mathfrak{h}^{th}$ stratum is as follows:

$$\overline{y}'_{\mathfrak{h}} = \frac{n_{\mathfrak{h}1}\overline{y}_{\mathfrak{h}1} + n_{\mathfrak{h}2}\overline{y}_{\mathfrak{h}2}}{n_{\mathfrak{h}}},$$

where $n_{\mathfrak{h}1}$ is the sample mean of response group and $n_{\mathfrak{h}2} = \frac{n_{\mathfrak{h}22}}{l}$ is the mean of subsample of nonresponse group in the $\mathfrak{h}^{th}$ stratum.

Using Hansen and Hurwitz [18] subsampling scheme, an unbaised estimator of population mean $\overline{Y}$ under stratified sampling scheme is as follows:

$$\overline{y}'_{\mathfrak{st}} = \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}}\overline{y}'_{\mathfrak{h}}.$$

The variance of $\bar{y}'_{\mathfrak{st}}$ is defined as

$$V\left(\bar{y}'_{\mathfrak{st}}\right) = \bar{Y}^2\delta_{2.0} + \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}}^2 w S^2_{(2)y_{\mathfrak{h}}}, \tag{26}$$

where $w = \frac{N_{(2)\mathfrak{h}}(l-1)}{N_{\mathfrak{h}} n_{\mathfrak{h}}}$.

The customary ratio estimator under nonresponse is given by

$$\hat{\bar{y}}'_{R\mathfrak{st}} = \frac{\bar{y}'_{\mathfrak{st}}}{p_{\mathfrak{st}}} P. \tag{27}$$

The MSE of $\hat{\bar{y}}'_{R\mathfrak{st}}$ is given by

$$\text{MSE}\left(\hat{\bar{y}}'_{R\mathfrak{st}}\right) = \bar{Y}^2\left[\delta'_{2.0} + \delta_{0.2} - 2\delta_{1.1}\right], \tag{28}$$

where $\delta'_{2.0} = \delta_{2.0} + \sum_{\mathfrak{h}=1}^{\mathfrak{L}} \mathfrak{W}_{\mathfrak{h}}^2 \frac{wS^2_{y_{\mathfrak{h}2}}}{\bar{Y}^2}$.

The traditional Regression estimator under nonresponse is given by

$$\hat{\bar{y}}'_{st(lr)} = \bar{y}'_{\mathfrak{st}} + b_{\mathfrak{st}}\left(P - p_{\mathfrak{st}}\right). \tag{29}$$

The MSE of $\hat{\bar{y}}'_{st(lr)}$ is

$$\text{MSE}\left(\hat{\bar{y}}'_{st(lr)}\right) = \bar{Y}^2\delta'_{2.0}\left(1 - \rho^2_{st}\right), \tag{30}$$

where $\rho_{\mathfrak{st}} = \frac{\delta_{1.1}}{\sqrt{\delta_{2.0}}\sqrt{\delta_{0.2}}}$.

Koyuncu [7] family of estimators for nonresponse is as follows:

$$\hat{\bar{y}}'_{kk(\mathfrak{st})} = \left[w_{c1}\bar{y}'_{\mathfrak{st}} + w_{c2}\left(P - p_{\mathfrak{st}}\right)\right]\left(\frac{a_{\mathfrak{st}}P + b_{\mathfrak{st}}}{a_{\mathfrak{st}}p_{\mathfrak{st}} + b_{\mathfrak{st}}}\right). \tag{31}$$

Some family members of Koyuncu and Kadilar [7] in case of nonresponse are shown in Table 5.

**Table 5** Some family members of Koyuncu and Kadilar [7] in case of nonresponse.

| $\hat{\bar{y}}'_{kk(\mathfrak{st})}$ | $a_{\mathfrak{st}}$ | $b_{\mathfrak{st}}$ | $\nu_j$ |
|---|---|---|---|
| $\hat{\bar{y}}'_{kk1(\mathfrak{st})}$ | 1 | $\psi_r$ | $\nu_3$ |
| $\hat{\bar{y}}'_{kk2(\mathfrak{st})}$ | 1 | $\psi_c$ | $\nu_2$ |
| $\hat{\bar{y}}'_{kk3(\mathfrak{st})}$ | $\psi_r$ | $\psi_c$ | $\nu_4$ |
| $\hat{\bar{y}}'_{kk4(\mathfrak{st})}$ | $\psi_c$ | $\psi_r$ | $\nu_5$ |

The minimum MSE of $\hat{\bar{y}}'_{kk(\mathfrak{st})}$ is

$$\text{MSE}_{min}\left(\hat{\bar{y}}'_{kk(\mathfrak{st})}\right) = \bar{Y}^2 - \frac{B_{kk}D^2_{kk} + A'_{kk}E^2_{kk} - 2C_{kk}D_{kk}E_{kk}}{A'_{kk}B_{kk} - C^2_{kk}}, \tag{32}$$

where

$$A'_{kk} = \bar{Y}^2\left[1 + \delta'_{2.0} + 3\nu_j^2\delta_{0.2} - 4\nu_j\delta_{1.1}\right].$$

## 6.1. Adapted Estimators Under Nonresponse

$\hat{\bar{y}}_{BT\mathfrak{st}}$ in case of nonresponse is as follows:

$$\hat{\bar{y}}'_{BT\mathfrak{st}} = \bar{y}'_{\mathfrak{st}} exp\left[\frac{P - p_{\mathfrak{st}}}{P + p_{\mathfrak{st}}}\right]. \quad (33)$$

The MSE of $\hat{\bar{y}}'_{BT\mathfrak{st}}$ for the first order approximation is given below:

$$\text{MSE}\left(\hat{\bar{y}}'_{BT\mathfrak{st}}\right) = \bar{Y}^2\left[\delta'_{2.0} + \frac{1}{4}\delta_{0.2} - \delta_{1.1}\right]. \quad (34)$$

$\hat{\bar{y}}_{di} = \bar{y}_{\mathfrak{st}}$ in case of nonresponse is as follows:

$$\hat{\bar{y}}'_{di} = \bar{y}'_{\mathfrak{st}c}\left(\frac{p_{\mathfrak{st}}}{P}\right)^{m_1}\left[w_d + (1 - w_d)\left(\frac{p_{\mathfrak{st}}}{P}\right)^{m_2}\right]^{m_3}. \quad (35)$$

$\hat{\bar{y}}_{kd}$ for nonresponse is as follows:

$$\hat{\bar{y}}'_{kd} = \bar{y}'_{\mathfrak{st}}\left(\frac{p_{\mathfrak{st}}}{P}\right)^{m_4}\left[w_{kd} + (1 - w_{kd})\left(\frac{a_{\mathfrak{st}}p_{\mathfrak{st}} + b_{\mathfrak{st}}}{a_{\mathfrak{st}}P + b_{\mathfrak{st}}}\right)^{m_5}\right]^{m_6}. \quad (36)$$

In case of nonresponse, $\hat{\bar{y}}'_{kd}$, $\hat{\bar{y}}'_{di}$, and $\hat{\bar{y}}'_{st(lr)}$ also have equal MSE.

$\hat{\bar{y}}'_{ki(\mathfrak{st})}$ estimator for nonresponse situation is as follows:

$$\hat{\bar{y}}'_{ki(\mathfrak{st})} = k\bar{y}_{\mathfrak{st}}\left[\frac{aP + b}{\alpha\left(a_{\mathfrak{st}}p_{\mathfrak{st}} + b_{\mathfrak{st}}\right) + (1 - \alpha)\left(a_{\mathfrak{st}}P + b_{\mathfrak{st}}\right)}\right]^M . for\,(i = 1, 2, ..., 6.) \quad (37)$$

Some family members of Koyuncu and Kadilar [4] under nonresponse are shown in Table 6.

**Table 6** Some family members of Koyuncu and Kadilar [4] under nonresponse.

| $\hat{\bar{y}}'_k$ | $\nu_j$ |
|---|---|
| $\hat{\bar{y}}'_{k1(\mathfrak{st})}$ | $\nu_1 = \frac{P}{P+\psi_s}$ |
| $\hat{\bar{y}}'_{k2(\mathfrak{st})}$ | $\nu_2 = \frac{P}{P+\psi_c}$ |
| $\hat{\bar{y}}'_{k3(\mathfrak{st})}$ | $\nu_3 = \frac{P}{P+\psi_r}$ |
| $\hat{\bar{y}}'_{k4(\mathfrak{st})}$ | $\nu_4 = \frac{\psi_r P}{\psi_r P+\psi_c}$ |
| $\hat{\bar{y}}'_{k5(\mathfrak{st})}$ | $\nu_5 = \frac{\psi_c P}{\psi_c P+\psi_r}$ |
| $\hat{\bar{y}}'_{k6(\mathfrak{st})}$ | $\nu_6 = \frac{\psi_r P}{\psi_r P+\psi_s}$ |

The minimum mean squared error of $\hat{\bar{y}}'_{ki(\mathfrak{st})}$ is

$$\text{MSE}_{min}\left(\hat{\bar{y}}'_{ki(\mathfrak{st})}\right) = \bar{Y}^2\left[1 - \frac{A'^2_k}{4B_k}\right], \quad (38)$$

where

$$A'_k = \left(M^2 + M\right)\alpha^2\nu_j^2\delta'_{2.0} - 2M\alpha\nu_j\delta_{1.1} + 2.$$

$\hat{\bar{y}}_{sg(\mathfrak{st})}$ in case of nonresponse is as follows:

$$\hat{\bar{y}}'_{sg(\mathfrak{st})} = \left[w_{sg1}\bar{y}'_{st} + w_{sg2}\left(P - p_{st}\right)\right] exp\left(\frac{\bar{A} - \bar{a}_{st}}{\bar{A} + \bar{a}_{st}}\right). \quad (39)$$

The minimum MSE of $\hat{\bar{y}}'_{sg(\mathfrak{st})}$ is

$$\text{MSE}_{min}\left(\hat{\bar{y}}'_{sg(\mathfrak{st})}\right) = \bar{Y}^2 - \frac{B_{sg}D^2_{sg} + \frac{A'_{sg}E^2_{sg}}{4} - C_{sg}D_{sg}E_{sg}}{A'_{sg}B_{sg} - C^2_{sg}}, \quad (40)$$

where

$$A'_{sg} = \overline{Y}^2\left[1 + \delta_{2.0} + \frac{\delta_{0.2}}{(1+N)^2} - \frac{2\delta_{1.1}}{(1+N)}\right].$$

Family members of Koyuncu [6] under nonresponse are shown in Table 7.

**Table 7** Family members of Koyuncu [6] under nonresponse.

| $\hat{\bar{y}}'_{nk}$ | $U_{\mathfrak{st}}$ | $V_{\mathfrak{st}}$ | $\gamma'$ |
|---|---|---|---|
| $\hat{\bar{y}}'_{nk1(\mathfrak{st})}$ | $\psi_b$ | $\psi_c$ | 1 |
| $\hat{\bar{y}}'_{nk2(\mathfrak{st})}$ | $\psi_b$ | $\psi_c$ | 2 |
| $\hat{\bar{y}}'_{nk3(\mathfrak{st})}$ | $\psi_c$ | $\psi_b$ | 1 |
| $\hat{\bar{y}}'_{nk4(\mathfrak{st})}$ | $\psi_c$ | $\psi_b$ | 2 |
| $\hat{\bar{y}}'_{nk5(\mathfrak{st})}$ | $\psi_r$ | $\psi_c$ | 1 |
| $\hat{\bar{y}}'_{nk6(\mathfrak{st})}$ | $\psi_r$ | $\psi_c$ | 2 |

$\hat{\bar{y}}_{nk(\mathfrak{st})}$ family of estimators under nonresponse as

$$\hat{\bar{y}}'_{nk(\mathfrak{st})} = \left[w_{1nk}\bar{y}'_{st} + w_{2nk}\left(\frac{p_{\mathfrak{st}}}{P}\right)^{\gamma'}\right] exp\left[\frac{U_{\mathfrak{st}}\left(P - p_{\mathfrak{st}}\right)}{U_{\mathfrak{st}}\left(P + p_{\mathfrak{st}}\right) - 2V_{\mathfrak{st}}}\right]. \tag{41}$$

The minimum MSE of $\hat{\bar{y}}'_{nk(\mathfrak{st})}$ is

$$\text{MSE}_{min}\left(\hat{\bar{y}}'_{nk(\mathfrak{st})}\right) = \overline{Y}^2 - \frac{B_{nk}D^2_{nk} + A'_{nk}E^2_{nk} - 2C_{nk}D_{nk}E_{nk}}{A'_{nk}B_{nk} - C^2_{nk}}, \tag{42}$$

where

$$A'_{nk} = \overline{Y}^2\left[1 + \delta'_{2.0} + \theta^2_{nk}\delta_{0.2} - 2\theta_{nk}\delta_{1.1}\right].$$

## 6.2. Proposed Class of Estimators Under Nonresponse

Proposed class of estimators in case of nonresponse is as follows:

$$\hat{\bar{y}}'_{N\mathfrak{st}} = \psi''_1\left[\frac{a_{\mathfrak{st}}P + b_{\mathfrak{st}}}{\beta\left(a_{\mathfrak{st}}p_{\mathfrak{st}} + b_{\mathfrak{st}}\right) + (1-\beta)\left(a_{\mathfrak{st}}P + b_{\mathfrak{st}}\right)}\right]^g, \tag{43}$$

where

$$\psi''_1 = \left[w_1\left\{\frac{\bar{y}'_{\mathfrak{st}}}{2}\left(\frac{P}{p_{\mathfrak{st}}} + \frac{p_{\mathfrak{st}}}{P}\right)\right\} + w_2\left(\frac{p_{\mathfrak{st}}}{P}\right)^{\gamma}\right].$$

Some family members of proposed class under nonresponse are shown in Table 8.

The minimum MSE of $\hat{\bar{y}}'_{N\mathfrak{st}}$ is

$$\text{MSE}_{min}\left(\hat{\bar{y}}'_{N\mathfrak{st}}\right) = \left[\overline{Y}^2 L - \frac{A'_N}{B'_N}\right], \tag{44}$$

where $A'_N = \left[\phi_B\phi^2_D + \frac{\phi'_A\phi^2_E}{4} - \phi_C\phi_D\phi_E\right]$ and $B'_N = \left[\phi'_A\phi_B - \phi^2_C\right]$.

**Table 8** Some family members of proposed class under nonresponse.

| $\hat{\bar{y}}'_{N\mathfrak{st}}$ | $\gamma$ | g | $a_{\mathfrak{st}}$ | $b_{\mathfrak{st}}$ | $\beta$ |
|---|---|---|---|---|---|
| $\hat{\bar{y}}'_{N1\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | $\psi_b$ | $\psi_c$ | 1 |
| $\hat{\bar{y}}'_{N2\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | $\psi_c$ | $\psi_b$ | 1 |
| $\hat{\bar{y}}'_{N3\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_r$ | 1 |
| $\hat{\bar{y}}'_{N4\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_b$ | 1 |
| $\hat{\bar{y}}'_{N5\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_c$ | 1 |
| $\hat{\bar{y}}'_{N6\mathfrak{st}}$ | $\frac{1}{3}$ | 1 | $\psi_c$ | $\psi_r$ | 1 |
| $\hat{\bar{y}}'_{N7\mathfrak{st}}$ | $\frac{1}{2}$ | 1 | 1 | $\psi_b$ | 1 |
| $\hat{\bar{y}}'_{N8\mathfrak{st}}$ | 1 | 2 | $\psi_c$ | $\psi_r$ | 1 |
| $\hat{\bar{y}}'_{N9\mathfrak{st}}$ | 1 | 2 | $\psi_r$ | $\psi_c$ | 1 |

Further,

$$\phi'_{A1} = \overline{Y}^2 \left\{1 + \delta'_{2.0} + \left(a^2 + 2b + 1\right) \delta_{0.2} - 2a\delta_{11}\right\}.$$

## 7. EFFICIENCY COMPARISON

In current section, we will compare the proposed estimators with the reviewed estimators exhibited in this study in light of the MSE of every estimator.

- From Eqs. (25) and (3), MSE $\left(\hat{\bar{y}}_{R\mathfrak{st}}\right) - \text{MSE}_{min}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) > 0$ if

$$\frac{A_N}{B_N} - \overline{Y}^2 \left[L - (\delta_{2.0} + \delta_{0.2} - 2\delta_{1.1})\right] > 0.$$

- From Eqs. (25) and (5), MSE $\left(\hat{\bar{y}}_{st(lr)}\right) - \text{MSE}_{min}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) > 0$ if

$$\frac{A_N}{B_N} - \overline{Y}^2 \left[L - \delta_{2.0}\left(1 - \rho^2_{\mathfrak{st}}\right)\right] > 0.$$

- From Eqs. (25) and (8), $\text{MSE}_{min}\left(\hat{\bar{y}}_{kk(\mathfrak{st})}\right) - \text{MSE}_{min}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) > 0$ if

$$\left[\frac{A_N}{B_N} - \left\{\frac{B_{kk}D^2_{kk} + A_{kk}E^2_{kk} - 2C_{kk}D_{kk}E_{kk}}{A_{kk}B_{kk} - C^2_{kk}}\right\}\right] - \overline{Y}^2 (L - 1) > 0.$$

- From Eqs. (25) and (10), MSE $\left(\hat{\bar{y}}_{BT\mathfrak{st}}\right) - \text{MSE}_{min}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) > 0$ if

$$\frac{A_N}{B_N} - \overline{Y}^2 \left[L - \left(\delta_{2.0} + \frac{1}{4}\delta_{0.2} - \delta_{1.1}\right)\right] > 0.$$

- From Eqs. (25) and (14), $\text{MSE}_{min}\left(\hat{\bar{y}}_{ki(\mathfrak{st})}\right) - \text{MSE}_{min}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) > 0$ if

$$\frac{A_N}{B_N} - \overline{Y}^2 \left[L - 1 + \frac{A^2_k}{B_k}\right] > 0.$$

- From Eqs. (25) and (17), $\mathrm{MSE}_{min}\left(\hat{\bar{y}}_{sg(\mathfrak{st})}\right) - \mathrm{MSE}_{min}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) > 0$ if

$$\left[\frac{A_N}{B_N} - \left\{\frac{B_{sg}D_{sg}^2 + \frac{A_{sg}E_{sg}^2}{4} - C_{sg}D_{sg}E_{sg}}{A_{sg}B_{sg} - C_{sg}^2}\right\}\right] - \overline{Y}^2 (L-1) > 0.$$

- From Eqs. (25) and (20), $\mathrm{MSE}_{min}\left(\hat{\bar{y}}_{nk(\mathfrak{st})}\right) - \mathrm{MSE}_{min}\left(\hat{\bar{y}}_{N\mathfrak{st}}\right) > 0$ if

$$\left[\frac{A_N}{B_N} - \left\{\frac{B_{nk}D_{nk}^2 + \frac{A_{nk}E_{nk}^2}{4} - C_{nk}D_{nk}E_{nk}}{A_{nk}B_{nk} - C_{nk}^2}\right\}\right] - \overline{Y}^2 (L-1) > 0.$$

From the abovementioned conditions, we can argue that proposed estimators are more efficient than the existing ones in absence of non-response. Further, one can also develop such type of efficiency conditions for the nonresponse case.

## 8. NUMERICAL ILLUSTRATION

Here we assess the merits of proposed estimator over existing ones on the premise of percentage relative efficiency (PRE).

### 8.1. Population

The data are taken from Koyuncu [7]; where number of teachers consider as $Y$ and for auxiliary attribute $\phi$ we use number of students classifying more (>) or less (<) than 750, in both primary and secondary schools as auxiliary variable for 923 districts at six regions in Turkey in 2007. For more detail see Koyuncu [7]. Some important descriptives are available in Table 9.

Further, we consider 10%, 20%, and 30% values for nonresponse. Some important calculations related to nonresponse are available in Table 10. The PRE of the proposed and existing estimators for absence and presence of nonresponse w.r.t $V\left(\bar{y}_{\mathfrak{st}}\right)$ and $V\left(\bar{y}'_{\mathfrak{st}}\right)$, respectively, available in Tables 11–14.

**Table 9** Descriptives of population for absence of nonresponse.

| | | | | |
|---|---|---|---|---|
| $N_{1st} = 127$ | $N_{2st} = 117$ | $N_{3st} = 103$ | $N_{4st} = 170$ | $N_{5st} = 205$ |
| $N_{6st} = 201$ | $n_{1st} = 31$ | $n_{2st} = 21$ | $n_{3st} = 29$ | $n_{4st} = 38$ |
| $n_{5st} = 22$ | $n_{6st} = 39$ | $\mathfrak{W}_{1st} = 0.11375$ | $\mathfrak{W}_{2st} = 0.1267$ | $\mathfrak{W}_{3st} = 0.1115$ |
| $\mathfrak{W}_{4st} = 0.1841$ | $\mathfrak{W}_{5st} = 0.2221$ | $\mathfrak{W}_{6st} = 0.2177$ | $P_{1st} = 0.952$ | $\underline{P}_{2st} = 0.974$ |
| $\underline{P}_{3st} = 0.932$ | $\underline{P}_{4st} = 0.888$ | $\underline{P}_{5st} = 0.912$ | $\underline{P}_{6st} = 0.950$ | $\overline{Y}_{1st} = 703.74$ |
| $\overline{Y}_{2st} = 413$ | $\overline{Y}_{3st} = 573.17$ | $\overline{Y}_{4st} = 424.66$ | $\overline{Y}_{5st} = 267.03$ | $\overline{Y}_{6st} = 393.84$ |
| $C_{\phi 1st} = 0.223$ | $C_{\phi 2st} = 0.163$ | $C_{\phi 3st} = 0.271$ | $C_{\phi 4st} = 0.355$ | $C_{\phi 5st} = 0.311$ |
| $C_{\phi 6st} = 0.229$ | $C_{y1st} = 1.256$ | $C_{y2st} = 1.562$ | $C_{y3st} = 1.803$ | $C_{y4st} = 1.909$ |
| $C_{y5st} = 1.512$ | $C_{y6st} = 1.807$ | $\rho_{1st} = 0.936$ | $\rho_{2st} = 0.066$ | $\rho_{3st} = 0.143$ |
| $\rho_{4st} = 0.174$ | $\rho_{5st} = 0.183$ | $\rho_{6st} = 0.120$ | $\rho_{\mathfrak{st}} = 0.141$ | $n = 180$ |

**Table 10** Some important results for nonresponse.

| | | | | |
|---|---|---|---|---|
| 10% | | | | |
| | $N_{(2)1st} = 13$ | $S^2_{(2)y_{1st}} = 260681.7436$ | $N_{(2)2st} = 12$ | $S^2_{(2)y_{2st}} = 149590.9697$ |
| | $N_{(2)3st} = 11$ | $S^2_{(2)y_{3st}} = 3210544.9636$ | $N_{(2)4st} = 17$ | $S^2_{(2)y_{4st}} = 1736239.066$ |
| | $N_{(2)5st} = 21$ | $S^2_{(2)y_{5st}} = 76837.2619$ | $N_{(2)6st} = 20$ | $S^2_{(2)y_{6st}} = 71170.42105$ |
| 20% | | | | |
| | $N_{(2)1st} = 25$ | $S^2_{(2)y_{1st}} = 157424.9167$ | $N_{(2)2st} = 23$ | $S^2_{(2)y_{2st}} = 164954.0198$ |
| | $N_{(2)3st} = 21$ | $S^2_{(2)y_{3st}} = 2737122.4000$ | $N_{(2)4st} = 34$ | $S^2_{(2)y_{4st}} = 1773172.939$ |
| | $N_{(2)5st} = 41$ | $S^2_{(2)y_{5st}} = 117155.4524$ | $N_{(2)6st} = 40$ | $S^2_{(2)y_{6st}} = 224079.7276$ |
| 30% | | | | |
| | $N_{(2)1st} = 38$ | $S^2_{(2)y_{1st}} = 250264.1031$ | $N_{(2)2st} = 35$ | $S^2_{(2)y_{2st}} = 127413.0672$ |
| | $N_{(2)3st} = 31$ | $S^2_{(2)y_{3st}} = 1914712.2796$ | $N_{(2)4st} = 51$ | $S^2_{(2)y_{4st}} = 1451756.296$ |
| | $N_{(2)5st} = 62$ | $S^2_{(2)y_{5st}} = 90337.04733$ | $N_{(2)6st} = 60$ | $S^2_{(2)y_{6st}} = 271760.609$ |

**Table 11** | PRE of proposed and existing estimators in absence of nonresponse.

| Estimator | PRE | Estimator | PRE | Estimator | PRE |
|---|---|---|---|---|---|
| $\bar{y}_{st}$ | 100 | $\hat{\bar{y}}_{kk1(st)}$ | 103.0878 | $\hat{\bar{y}}_{nk6(st)}$ | 1356.459 |
| $\bar{y}_{Rst}$ | 101.6047 | $\hat{\bar{y}}_{kk2(st)}$ | 103.0877 | $\hat{\bar{y}}_{N1st}$ | 19242.52 |
| $\bar{y}_{BTst}$ | 101.5375 | $\hat{\bar{y}}_{kk3(st)}$ | 103.0875 | $\hat{\bar{y}}_{N2st}$ | 19020.75 |
| $\hat{\bar{y}}_{st(lr)}$ | 101.7598 | $\hat{\bar{y}}_{kk4(st)}$ | 103.0876 | $\hat{\bar{y}}_{N3st}$ | 32694.96 |
| $\hat{\bar{y}}_{k1(st)}$ | 103.0875 | $\hat{\bar{y}}_{sg(st)}$ | 103.0874 | $\hat{\bar{y}}_{N4st}$ | 23249.10 |
| $\hat{\bar{y}}_{k2(st)}$ | 103.08750 | $\hat{\bar{y}}_{nk1(st)}$ | 17514.15 | $\hat{\bar{y}}_{N5st}$ | 57329.18 |
| $\hat{\bar{y}}_{k3(st)}$ | 103.0667 | $\hat{\bar{y}}_{nk2(st)}$ | 1961.927 | $\hat{\bar{y}}_{N6st}$ | 48330.94 |
| $\hat{\bar{y}}_{k4(st)}$ | 102.496 | $\hat{\bar{y}}_{nk3(st)}$ | 4500.757 | $\hat{\bar{y}}_{N7st}$ | 61076.41 |
| $\hat{\bar{y}}_{k5(st)}$ | 103.0249 | $\hat{\bar{y}}_{nk4(st)}$ | 1154.523 | $\hat{\bar{y}}_{N8st}$ | 59545.86 |
| $\hat{\bar{y}}_{k6(st)}$ | 102.4960 | $\hat{\bar{y}}_{nk5(st)}$ | 6346.253 | $\hat{\bar{y}}_{N9st}$ | 38961.67 |

PRE, percentage relative efficiency.

**Table 12** | PRE of proposed and existing estimators for 10% nonresponse.

| Estimator | PRE | Estimator | PRE | Estimator | PRE |
|---|---|---|---|---|---|
| $\bar{y}'_{st}$ | 100 | $\hat{\bar{y}}'_{kk1(st)}$ | 103.0542 | $\hat{\bar{y}}'_{nk6(st)}$ | 1214.55 |
| $\bar{y}'_{Rst}$ | 101.4037 | $\hat{\bar{y}}'_{kk2(st)}$ | 103.0541 | $\hat{\bar{y}}'_{N1st}$ | 17167.73 |
| $\bar{y}'_{BTst}$ | 101.3449 | $\hat{\bar{y}}'_{kk3(st)}$ | 103.0539 | $\hat{\bar{y}}'_{N2st}$ | 16970.11 |
| $\hat{\bar{y}}'_{st(lr)}$ | 101.53 | $\hat{\bar{y}}'_{kk4(st)}$ | 103.054 | $\hat{\bar{y}}'_{N3st}$ | 29169.73 |
| $\hat{\bar{y}}'_{k1(st)}$ | 103.054 | $\hat{\bar{y}}'_{sg(st)}$ | 103.0538 | $\hat{\bar{y}}'_{N4st}$ | 20742.58 |
| $\hat{\bar{y}}'_{k2(st)}$ | 103.053 | $\hat{\bar{y}}'_{nk1(st)}$ | 15686.5 | $\hat{\bar{y}}'_{N5st}$ | 51204.11 |
| $\hat{\bar{y}}'_{k3(st)}$ | 103.0365 | $\hat{\bar{y}}'_{nk2(st)}$ | 1752.667 | $\hat{\bar{y}}'_{N6st}$ | 43133.63 |
| $\hat{\bar{y}}'_{k4(st)}$ | 102.5359 | $\hat{\bar{y}}'_{nk3(st)}$ | 4015.272 | $\hat{\bar{y}}'_{N7st}$ | 54509.73 |
| $\hat{\bar{y}}'_{k5(st)}$ | 102.9984 | $\hat{\bar{y}}'_{nk4(st)}$ | 1034.06 | $\hat{\bar{y}}'_{N8st}$ | 53122.13 |
| $\hat{\bar{y}}'_{k6(st)}$ | 102.5359 | $\hat{\bar{y}}'_{nk5(st)}$ | 5661.139 | $\hat{\bar{y}}'_{N9st}$ | 34758.74 |

**Table 13** | PRE of proposed and existing estimators for 20% nonresponse.

| Estimator | PRE | Estimator | PRE | Estimator | PRE |
|---|---|---|---|---|---|
| $\bar{y}'_{st}$ | 100 | $\hat{\bar{y}}'_{kk1(st)}$ | 103.0692 | $\hat{\bar{y}}'_{nk6(st)}$ | 1356.459 |
| $\bar{y}'_{Rst}$ | 101.2502 | $\hat{\bar{y}}'_{kk2(st)}$ | 103.0691 | $\hat{\bar{y}}'_{N1st}$ | 19242.52 |
| $\bar{y}'_{BTst}$ | 101.198 | $\hat{\bar{y}}'_{kk3(st)}$ | 103.0688 | $\hat{\bar{y}}'_{N2st}$ | 19020.75 |
| $\hat{\bar{y}}'_{st(lr)}$ | 101.3705 | $\hat{\bar{y}}'_{kk4(st)}$ | 103.069 | $\hat{\bar{y}}'_{N3st}$ | 32694.96 |
| $\hat{\bar{y}}'_{k1(st)}$ | 103.069 | $\hat{\bar{y}}'_{sg(st)}$ | 103.0687 | $\hat{\bar{y}}'_{N4st}$ | 23249.10 |
| $\hat{\bar{y}}'_{k2(st)}$ | 103.0689 | $\hat{\bar{y}}'_{nk1(st)}$ | 17514.15 | $\hat{\bar{y}}'_{N5st}$ | 57392.18 |
| $\hat{\bar{y}}'_{k3(st)}$ | 103.054 | $\hat{\bar{y}}'_{nk2(st)}$ | 1961.927 | $\hat{\bar{y}}'_{N6st}$ | 48330.94 |
| $\hat{\bar{y}}'_{k4(st)}$ | 102.6068 | $\hat{\bar{y}}'_{nk3(st)}$ | 4500.757 | $\hat{\bar{y}}'_{N7st}$ | 61076.41 |
| $\hat{\bar{y}}'_{k5(st)}$ | 103.0187 | $\hat{\bar{y}}'_{nk4(st)}$ | 1154.523 | $\hat{\bar{y}}'_{N8st}$ | 59545.86 |
| $\hat{\bar{y}}'_{k6(st)}$ | 102.6068 | $\hat{\bar{y}}'_{nk5(st)}$ | 6346.253 | $\hat{\bar{y}}'_{N9st}$ | 38961.67 |

PRE, percentage relative efficiency.

**Table 14** PRE of proposed and existing estimators for 30% nonresponse.

| Estimator | PRE | Estimator | PRE | Estimator | PRE |
|---|---|---|---|---|---|
| $\bar{y}'_{st}$ | 100 | $\hat{\bar{y}}'_{kk1(st)}$ | 103.1002 | $\hat{\bar{y}}'_{nk6(st)}$ | 1456.213 |
| $\bar{y}'_{Rst}$ | 101.1609 | $\hat{\bar{y}}'_{kk2(st)}$ | 103.1001 | $\hat{\bar{y}}'_{N1st}$ | 20700.97 |
| $\bar{y}'_{BTst}$ | 101.1125 | $\hat{\bar{y}}'_{kk3(st)}$ | 103.0998 | $\hat{\bar{y}}'_{N2st}$ | 20462.23 |
| $\hat{\bar{y}}'_{st(lr)}$ | 101.2726 | $\hat{\bar{y}}'_{kk4(st)}$ | 103.1 | $\hat{\bar{y}}'_{N3st}$ | 35172.98 |
| $\hat{\bar{y}}'_{k1(st)}$ | 103.1001 | $\hat{\bar{y}}'_{sg(st)}$ | 103.0997 | $\hat{\bar{y}}'_{N4st}$ | 25011.03 |
| $\hat{\bar{y}}'_{k2(st)}$ | 103.0999 | $\hat{\bar{y}}'_{nk1(st)}$ | 18799.27 | $\hat{\bar{y}}'_{N5st}$ | 61742.00 |
| $\hat{\bar{y}}'_{k3(st)}$ | 103.0865 | $\hat{\bar{y}}'_{nk2(st)}$ | 2109.021 | $\hat{\bar{y}}'_{N6st}$ | 51984.30 |
| $\hat{\bar{y}}'_{k4(st)}$ | 102.6703 | $\hat{\bar{y}}'_{nk3(st)}$ | 4842.024 | $\hat{\bar{y}}'_{N7st}$ | 65692.43 |
| $\hat{\bar{y}}'_{k5(st)}$ | 103.0528 | $\hat{\bar{y}}'_{nk4(st)}$ | 1239.21 | $\hat{\bar{y}}'_{N8st}$ | 64061.32 |
| $\hat{\bar{y}}'_{k6(st)}$ | 102.6703 | $\hat{\bar{y}}'_{nk5(st)}$ | 6827.846 | $\hat{\bar{y}}'_{N9st}$ | 41916.07 |

PRE, percentage relative efficiency.

## 9. CONCLUSION

In this paper, we propose an estimator for the estimation of population mean $Y$ under stratified random sampling scheme. The execution (performance) of the proposed estimator is assessed theoretically and numerically using natural population. We also consider 10%, 20%, and 30% values for nonrespondents. The results of PRE showed that the proposed estimator is more efficient as compare to the customary ratio, regression, Bahl and Tuteja [1], Diana [2], Kadilar and Cingi [14], Koyuncu and Kadilar [3], Koyuncu and Kadilar [4], Koyuncu and Kadilar [10], Shabbir and Gupta [5], and Koyuncu [6] for both situations. Hence, it is advisable to utilize the proposed class of estimators for the estimation of population mean under stratified random sampling scheme when the information of auxiliary attribute is known.

## REFERENCES


1. S. Bahl, R.K. Tuteja, J. Inf. Optim. Sci. 12(1) (1991), 159–164.
2. G. Diana, Statistica. 53(1) (1993), 59–66.
3. N. Koyuncu, C. Kadilar, Pak. J. Statist. 26(2) (2010), 427–443.
4. N. Koyuncu, C. Kadilar, J. Stat. Plan. Inference. 139(8) (2009), 2552–2558.
5. J. Shabbir, S. Gupta, Commun. Stat. Theory Methods. 40(2) (2011), 199–212.
6. N. Koyuncu, Pak. J. Stat. Oper. Res. 12(3) (2016), 429–441.
7. N. Koyuncu, in Proceeding of 59-th ISI world Statistics Congress, Hong Kong, China, August 2013, pp. 25–30.
8. M.H. Hansen, W.N. Hurwitz, M. Gurney, J. Am. Stat. Assoc. 41(234) (1946), 173–189.
9. P. Kaur, Biom. J. 27(1) (1985), 101–105.
10. N. Koyuncu, C. Kadilar, J. Appl. Stat. 37(6) (2010), 999–1013.
11. B.V.S. Sisodia, V.K. Dwivedi, J. Indian Soc. Agric. Stat. 33 (1981), 13–18.
12. H.P. Singh, M.S. Kakran, Unpublished Paper. 1993.
13. L.N. Upadhyaya, H.P. Singh, Biom. J. 41(5) (1999), 627–636.
14. C. Kadilar, H. Cingi, Biom. J. 45(2) (2003), 218–225.
15. C. Kadilar, H. Cingi, Commun. Stat. Theory Methods. 34(3) (2005), 597–602.
16. B. Prasad, Commun. Stat. Theory Methods. 18(1) (1989), 379–392.
17. M.K. Chaudhary, R. Singh, R.K. Shukla, M. Kumar, F. Smarandache, Pak. J. Stat. Oper. Res. 5(1) (2009), 47–54.
18. M.H. Hansen, W.N. Hurwitz, J. Am. Stat. Assoc. 41(236) (1946), 517–529.