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# Bayes and Non-Bayes Estimation of Change Point in Nonstandard Mixture Inverse Weibull Distribution

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**ABSTRACT**

We consider a sequence of independent random variables $X_1, X_2, \dots, X_m, \dots, X_n$ $(n \geq 3)$ exhibiting a change in the probability distribution of the data generating mechanism. We suppose that the distribution changes at some point, called a change point, to a second distribution for the remaining observations. We propose Bayes estimators of change point under symmetric loss functions and asymmetric loss functions. The sensitivity analysis of Bayes estimators are carried out by simulation and numerical comparisons with R-programming.



## 1. INTRODUCTION

It is generally recognized that a physical entity experiences a structural change as it evolves over time. Such structural change problem are often used to describe abrupt changes in the mechanism underlying a sequence of random measurements. Further, in many real-life problems theoretical or empirical deliberations suggest models with occasionally changing one or more of its parameters. There is enormous frequentist and Bayesian literature on problems of detecting the change, inference concerning the change point, and related problems for various statistical models.

Control charts are one of the most important tools in statistical process control to monitor manufacturing processes and services. When a control chart shows an out-of-control condition, a search begins to identify and eliminate the root cause(s) of the process disturbance. The time when the disturbance has manifested itself to the process is referred to as change point. Identification of the change point is considered as an essential step in analyzing and eliminating the disturbance source(s) effectively.

Nonstandard mixture inverse Weibull (IW) distribution happens in many applied situations, for instance; life of a unit may have a IW distribution but some of the units fail instantaneously. In the study of tooth decay, the number of surfaces in a mouth which are filled, missing, or decayed are scored to produce a decay index. Healthy teeth are scored (0) for no evidence of decay. The distribution is a mixture of a mass point at (0) and a nontrivial continuous distribution of decay score. In the study of tumor characteristics, two variates can be recorded. A discrete variable to indicate the absence (0) or presence (1) of a tumor and a continuous variable measuring the tumor size.

A sequence of random variables $X_1, X_2, \dots, X_m, \dots, X_n$ has a change point at $m\ (1 \leq m \leq n-1)$, if $X_i\ (i = 1, \dots, m)$ has a probability distribution $F_1\left(x_i|\theta_1\right)$ and $X_i\ (i = m+1, \dots, n)$ has a probability distribution $F_2\left(x_i|\theta_2\right)$, where $F_1\left(x_i|\theta_1\right) \neq F_2\left(x_i|\theta_2\right)$ and $\theta_1 \neq \theta_2$. Change point inference has a long history. Many of statisticians like Ganji [1], Chernoff and Zacks [2], Kander and Zacks [3], Smith [4], Jani and Pandya [5], Pandya and Jani [6], Pandya and Jadav [7], and Ebrahimi and Ghosh [8] studied the change point models in Bayesian framework. The monograph of Broemeling and Tsurumi [9] is also useful reference.

## 2. CHANGE POINT MODEL

Let the sequence of observations $X_1, X_2, \dots, X_m$ come from mixture of IW and degenerate distribution. The probability density function of the sequences is as follows

$$f\left(x_i; \alpha_1, \beta, p_1\right) = \left(1 - p_1\right) I_{\{x_i=0\}}\left(x_i\right) + \left(p_1 \beta \alpha_1^{-\beta} x_i^{-\beta-1} e^{\left(-\alpha_1 x_i\right)^{-\beta}}\right) I_{\{x_i>0\}}\left(x_i\right);$$
$$\alpha_1 > 0, \beta > 0, 0 < P_1 < 1, i = 1, 2, \dots, m,$$

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and later $(n-m)$ observations $X_{m+1},\dots,X_n$ come from mixture of IW and degenerate distribution. The probability density function of the sequences is as follows:

$$f\left(x_i;\alpha_2,\beta,p_2\right)=\left(1-p_2\right)I_{\{x_i=0\}}\left(x_i\right)+\left(p_2\beta\alpha_2^{-\beta}x_i^{-\beta-1}e^{\left(-\alpha_2x_i\right)^{-\beta}}\right)I_{\{x_i>0\}}\left(x_i\right);$$
$$\alpha_2>0,\beta>0,0<P_2<1,i=m+1,m+2,\dots,n.$$

## 3. BAYES ESTIMATORS OF PARAMETERS

The likelihood function of the given sample information is

$$\begin{aligned}L\left(\alpha_1,\alpha_2,\beta,p_1,p_2,m|x\right)\ =&\ \left(1-p_1\right)^{N_m}p_1^{A_m}\left(1-p_2\right)^{N-N_m}p_2^{B_m}\beta^{n-N}\alpha_1^{-\beta A_m}\\&\times\alpha_2^{-\beta B_m}u_{A_m}^{-\beta-1}e^{-\alpha_1^{-\beta}v_{A_m}}u_{B_m}^{-\beta-1}e^{-\alpha_2^{-\beta}v_{B_m}},\end{aligned}$$

where

$$u_{A_m}=\prod_{i=1}^{A_m}y_i;\quad u_{B_m}=\prod_{j=1}^{B_m}z_j;\quad v_{A_m}=\sum_{i=1}^{A_m}y_i^{-\beta};\quad v_{B_m}=\sum_{j=1}^{B_m}z_j^{-\beta};$$

Let $N$ be a number of observations equal to zero, $N_m$ be a number of observations equal to zero before change point m, $A_m$ be a number of the nonzero observations before change point m, $B_m$ be a number of the nonzero observations after change point m. Denote by $y_1,y_2,\dots,y_{A_m}$ the nonzero observations before the change point m, and denote by $z_1,z_2\dots,z_{B_m}$ the nonzero observations after the change point m.

For Bayesian estimation, we need to specify a prior distribution for the parameters. As in Broemeling and Tsurumi [9], suppose that the marginal prior distribution for m is discrete uniform over the set $\{1,2,\dots,n-1\}$.

As in Calabria and Pulcini [10] and Erto and Guida [11], we assume that some prior information on the mechanism of failures in terms of reliability level at a prefixed time value are available. In addition, we assume that these prior technical information are given in terms of mean values $\mu_1$ and $\mu_2$. Following Pandya and Jadav [12] let a log inverse exponential density be represent this prior knowledge on $R_{1t}$ and $R_{1t}$ at a common prefixed time $t$ with respective means $\mu_1$ and $\mu_2$,

$$g\left(R_{1t}\right)=\frac{\left(Ln\left(\frac{1}{1-R_{1t}}\right)\right)^{a_1-1}}{\Gamma\left(a_1\right)},\quad 0\le R_{1t}\le 1,a_1>0$$
$$g\left(R_{2t}\right)=\frac{\left(Ln\left(\frac{1}{1-R_{2t}}\right)\right)^{a_2-1}}{\Gamma\left(a_2\right)},\quad 0\le R_{2t}\le 1,a_2>0.$$

If the prior information is given in terms of the prior means $\mu_1$ and $\mu_2$ then the parameters $a_i,i=1,2$ can be obtained as $a_i=\frac{Ln\left(\frac{1}{1-\mu_i}\right)}{Ln(2)}$, $i=1,2$.

Making change of variables $R_{it}=1-e^{\left(-\alpha_it\right)^{-\beta}}$, densities on $R_{it}$ can be converted into conditional prior densities on $\alpha_1$ and $\alpha_2$ as

$$g_i\left(\alpha_i|\beta\right)=\frac{t^{-\beta a_i}\beta\alpha_i^{-\beta a_i-1}\exp\left[\left(-\alpha_it\right)^{-\beta}\right]}{\Gamma\left(a_i\right)},\quad a_i>0,$$

Suppose the marginal prior distributions of $p_1$ and $p_2$ are Beta priors with respective means $\mu_3$, $\mu_4$ and common standard deviation $\sigma_1$,

$$g\left(p_1\right)=\frac{p_1^{a_3-1}\left(1-p_1\right)^{b_3-1}}{B\left(a_3,b_3\right)};\quad a_3,b_3>0;\quad 0\le p_1\le 1,$$

$$g\left(p_2\right)=\frac{p_2^{a_4-1}\left(1-p_2\right)^{b_4-1}}{B\left(a_4,b_4\right)};\quad a_4,b_4>0;\quad 0\le p_2\le 1,$$

Mean and standard deviation of $p_1$ and $p_2$ are

$$\mu_i=\frac{a_i}{a_i+b_i};\quad \sigma_1=\frac{a_ib_i}{\left(a_i+b_i\right)^2\left(a_i+b_i+1\right)};\quad i=3,4,$$

then,

$$a_i = \frac{(1-\mu_i)\,\mu_i^2 - \mu_i\sigma_1}{\sigma_1}; \quad b_i = \frac{(1-\mu_i)\,a_i}{\mu_i}; \quad i = 3, 4. \tag{1}$$

For $\beta$, consider the uniform density on $[\beta_1, \beta_2]$, that is,

$$g(\beta) = \frac{1}{\beta_2 - \beta_1}, \quad \beta_1 \le \beta \le \beta_2.$$

Then, the joint prior distribution of $\alpha_1, \alpha_2, \beta, p_1, p_2$, and m is given by

$$\begin{aligned} g\left(\alpha_1, a_2, \beta, m, p_1, p_2\right) \;=\; & kt^{-\beta(a_1, a_2)}\beta^2\alpha_1^{-\beta a_1 - 1}\exp\left[-(t\alpha_1)^{-\beta}\right] \\ & \times \alpha_2^{-\beta a_2 - 1}\exp\left[-(t\alpha_2)^{-\beta}\right] \\ & \times \alpha_2^{-\beta a_2 - 1}\exp\left[-(t\alpha_2)^{-\beta}\right] \\ & \times p_1^{a_3 - 1}\left(1-p_1\right)^{b_3 - 1} p_2^{a_4 - 1}\left(1-p_2\right)^{b_4 - 1}, \end{aligned}$$

where

$$k = \frac{1}{\Gamma(a_1)\,\Gamma(a_2)\,(\beta_2 - \beta_1)\,(n-1)\,B(a_3, b_3)\,B(a_4, b_4)}. \tag{2}$$

Also, the joint posterior distribution of $\alpha_1, \alpha_2, \beta, p_1, p_2$, and m is given by

$$\begin{aligned} \mathrm{g}\left(\alpha_1, \alpha_2, \beta, m, p_1, p_2 | x\right) \;=\; & kt^{-\beta(a_1 + a_2)} p_1^{A_m + a_3 - 1}\left(1-p_1\right)^{N_m + b_3 - 1} \\ & \times \left(1-p_2\right)^{N - N_m + b_4 - 1} p_2^{B_m + a_4 - 1}\beta^{n-N+2} \\ & \times \alpha_1^{-\beta(A_m + \mathrm{a}_1) - 1}\alpha_2^{-\beta(B_m + \mathrm{a}_2) - 1} u_{A_m}^{-\beta - 1} \\ & \times e^{-\alpha_1^{-\beta}\left(v_{A_m} + t^{-\beta}\right)} u_{B_m}^{-\beta - 1} e^{-\alpha_2^{-\beta}\left(v_{B_m} + t^{-\beta}\right)}\left[h(x)\right]^{-1}, \end{aligned}$$

where

$$h(x) = k\sum_{m=1}^{n-1} I_1(m),$$

and

$$\begin{aligned} I_1(m) \;=\; & \int_{\beta_1}^{\beta_2} u_{A_m}^{-\beta-1} u_{B_m}^{-\beta-1}\beta^{n-N}\left(v_{A_m} + t^{-\beta}\right)^{-(A_m + \mathrm{a}_1)}\left(v_{B_m} + t^{-\beta}\right)^{-(B_m + \mathrm{a}_2)} \\ & \times B(A_m + a_3, N_m + b_3)\,\Gamma(A_m + \mathrm{a}_1)\,B(B_m + a_4, N - N_m + b_4)\,\Gamma(B_m + \mathrm{a}_2)\,d\beta. \end{aligned}$$

So, the marginal posterior distribution of $p_1, p_2$ and m is given by

$$g\left(m|x\right) = \frac{I_1(m)}{\sum_{m=1}^{n-1} I_1(m)}, \tag{3}$$

$$g\left(p_1|x\right) = k\sum_{m=1}^{n-1} p_1^{A_m + a_3 - 1}\left(1-p_1\right)^{N_m + b_3 - 1} k_2(m)\,J(m)\,[h(x)]^{-1}, \tag{4}$$

$$g\left(p_2|x\right) = k\sum_{m=1}^{n-1}\left(1-p_2\right)^{N - N_m + b_4 - 1} p_2^{B_m + a_4 - 1} k_3(m)\,J(m)\,[h(x)]^{-1}, \tag{5}$$

where

$$k_2(m) = \Gamma(A_m + a_1)\Gamma(B_m + a_2) B(B_m + a_4, N - N_m + b_4), \tag{6}$$

$$k_3(m) = \Gamma(A_m + \mathrm{a}_1)\Gamma(B_m + \mathrm{a}_2) B(A_m + a_3, N_m + b_3), \tag{7}$$

and

$$J(m) = \int_{\beta_l}^{\beta_2} u_{A_m}^{-\beta-1} u_{B_m}^{-\beta-1} \beta^{n-N} \left(v_{A_m} + t^{-\beta}\right)^{-(A_m+\mathrm{a}_1)} \left(v_{B_m} + t^{-\beta}\right)^{-(B_m+\mathrm{a}_2)} d\beta. \tag{8}$$

## 3.1. Point Estimation under Symmetric Loss Functions

In Bayesian framework a loss function is used to minimize the expected loss an estimator generates. The Bayes estimator of a generic parameter (or function thereof) $\theta$ based on symmetric loss function (SEL) function

$$L_1(\theta, d) \propto (\theta - d)^2, \theta, d \epsilon \mathfrak{R},$$

is the posterior mean, where d is the decision rule to estimate $\theta$. For estimation of change point m, which has a nonnegative integer value, the loss function $L_1(m, \nu)$ is defined only for integer value m and $\nu$. Hence, Bayes estimator of change point under SEL function, $m^*$, is the posterior mean. The posterior mean is

$$Posterior\ mean = \frac{\sum_{m=1}^{n-1} m.I_1(m)}{\sum_{m=1}^{n-1} I_1(m)} \tag{9}$$

The Bayes estimators of $p_1$ and $p_2$ under SEL function are as follows:

$$p_1^* = k \sum_{m=1}^{n-1} B(A_m + a_3 + 1, N_m + b_3) k_2(m) J(m) [h(x)]^{-1}, \tag{10}$$

and

$$p_2^* = k \sum_{m=1}^{n-1} B(B_m + a_4 + 1, N - N_m + b_4) k_3(m) J(m) [h(x)]^{-1}. \tag{11}$$

Other Bayes estimators of change point under loss functions,

$$L_2(m, d) = |m - d|,$$

and

$$L_3(m, d) = \begin{cases} 0, & if \quad |m - d| < \varepsilon, \varepsilon > 0, \\ 1, & \quad otherwise \end{cases}$$

are the posterior median and the posterior mode, respectively.

## 3.2. Point Estimation under Asymmetric Loss Functions

In this section, we obtain Bayes estimator of change point under Linex loss function. The Linex loss function, proposed by Varian [13] and discussed its behavior by Zellner [14], is defined as

$$L_4(\theta, d) \propto \exp[q_1(d - \theta)] - q_1(d - \theta) - 1, \ q_1 > 0, \theta, d \epsilon \mathfrak{R},$$

where d is the decision role to estimate parameter $\theta$. It was found to be appropriate in the situation where over estimation is considered more heavily penalized than underestimation and vice versa. The Bayes estimate of change point, m, under Linex loss function is $m_L^*$ as

$$m_L^* = -\left(\frac{1}{q_1}\right) Ln\left[\sum_{m=1}^{n-1}\left(e^{-q_1 m} I_1(m)\right) \Big/ \sum_{m=1}^{n-1} I_1(m)\right]. \tag{12}$$

Calabria and Pulcini [13] introduced the following asymmetric loss function

$$L_5(\theta, d) = \left[\left(\frac{d}{\theta}\right)^{q_2}\right] - q_2\left[Ln\left(\frac{d}{\theta}\right)\right] - 1.$$

This loss function is known as general entropy loss function (GEL). The Bayes estimate of change point, m, under GEL is $m_E^*$

$$m_E^* = \left[\sum_{m=1}^{n-1} I_1(m) \Big/ \sum_{m=1}^{n-1}\left(m^{-q_2} I_1(m)\right)\right]^{\frac{1}{q_2}}. \tag{13}$$

Also, the Bayes estimates of $p_1$ and $p_2$ are given by

$$p_{1E}^* = \left[k\sum_{m=1}^{n-1} B\left(A_m + a_3 - q_2, N_m + b_3\right) k_2(m) J(m)\left[h(x)\right]^{-1}\right]^{-\frac{1}{q_2}}, \tag{14}$$

$$p_{2E}^* = \left[k\sum_{m=1}^{n-1} B\left(B_m + a_4 - q_2, N - N_m + b_4\right) k_3(m) J(m)\left[h(x)\right]^{-1}\right]^{-\frac{1}{q_2}}. \tag{15}$$

## 4. MAXIMUM LIKELIHOOD ESTIMATORS

In this section, we obtain the maximum likelihood estimate of change point. We suppose $\beta, \alpha_1$, and $\alpha_2$ are known. Logarithm of the likelihood function is

$$\begin{aligned} Ln\left(L\left(\alpha_1, \alpha_2, \beta, p_1, p_2, m|x\right)\right) = {} & N_m Ln\left(1-p_1\right) + A_m Ln\left(p_1\right) + \left(N - N_m\right) Ln\left(1 - p_2\right) \\ & + B_m Ln\left(p_2\right) + (n - N) Ln\beta - \beta A_m Ln\left(\alpha_1\right) \\ & - \beta B_m Ln\left(\alpha_2\right) + (-\beta - 1) Ln\left(u_{A_m}\right) - \alpha_1^{-\beta} v_{A_m} \\ & + (-\beta - 1) Ln\left(u_{B_m}\right) - \alpha_2^{-\beta} v_{B_m}. \end{aligned}$$

Then, the maximum likelihood estimates of $p_1$ and $p_2$ are given by

$$\hat{p}_1 = \frac{A_m}{N_m + A_m}, \quad \hat{p}_2 = \frac{B_m}{N - N_m + B_m}. \tag{16}$$

So, the maximum likelihood estimate of change point is the value of m which maximize the likelihood function

$$\begin{aligned} L\left(\alpha_1, \alpha_2, \beta, \hat{p}_1, \hat{p}_2, m|x\right) = {} & \left(1 - \hat{p}_1\right)^{N_m} \hat{p}_1^{A_m} \left(1 - \hat{p}_2\right)^{N - N_m} \hat{p}_2^{B_m} \beta^{n-N} \alpha_1^{-\beta A_m} \\ & \times \alpha_2^{-\beta B_m} u_{A_m}^{-\beta - 1} e^{-\alpha_1^{-\beta} v_{A_m}} u_{B_m}^{-\beta - 1} e^{-\alpha_2^{-\beta} v_{B_m}}. \end{aligned} \tag{17}$$

## 5. NUMERICAL STUDY, SENSITIVITY ANALYSIS OF BAYES ESTIMATES

The data given in Table 1 is a random sample of size n = 20 which is generated by using R-programming from the introduced change point model. We considered m = 10, its mean that, the change point in sequence is occurred after $10^{th}$ observation. The first 10 observations from mixture of IW and degenerate distribution with $\beta = 1,\ \alpha_1 = 0.06,\ p_1 = 0.8,\ R_{1t} = 0.96$ at $t = 5$ and next 10 observations from mixture of IW and degenerate distribution with $\beta = 1,\ \alpha_2 = 9.5,\ p_2 = 0.6,\ R_{2t} = 0.02$ at $t = 5$. The posterior median and the posterior mode of change point, m, under informative prior are also calculated. The results are shown in Table 2. We calculated Bayes estimators proportions $p_1$ and $p_2$ under squared error loss function and GEL by making programs in R-Programming which is a statistical software. The results are shown in Tables 3 and 4.

**Table 1** Generated observations from mixture of IW and degenerate distribution.

| $i$ | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
|---|---|---|---|---|---|---|---|---|---|---|
| $X_i$ | 2.72 | 4.40 | 11.94 | 6.91 | 25.15 | 4.53 | 261.35 | 0 | 46.18 | **0** |
| $i$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | **20** |
| $X_i$ | 0 | 0.10 | 0.12 | 0.07 | 0.16 | 0.06 | 0 | 0 | 0.17 | **0** |

IW, inverse Weibull.

**Table 2** The values of Bayes estimators of change point.

| **Prior** | **Bayes estimates of change point** | | |
|---|---|---|---|
| | $m^*$ | **Posterior median** | **Posterior mode** |
| Informative | 10 | 10 | 10 |

**Table 3** The values of Bayes estimators of proportions $p_1$ and $p_2$.

| **Prior** | **Bayes estimates of proportions** | |
|---|---|---|
| | **Posterior mean** $p_1$ | **Posterior mean** $p_2$ |
| Informative | 0.83 | 0.62 |

**Table 4** The Bayes estimates using general entropy loss.

| **Prior** | **Bayes estimates of proportions $p_1$ and $p_2$** | | |
|---|---|---|---|
| | $q_2$ | $p^*_{1E}(t_0)$ | $p^*_{2E}(t_0)$ |
| | −2 | 0.84 | 0.63 |
| | −1 | 0.83 | 0.62 |
| Informative | 0.09 | 0.82 | 0.60 |
| | 0.5 | 0.81 | 0.59 |
| | 0.9 | 0.84 | 0.58 |

**Table 5** The Bayes estimates using asymmetric loss functions.

| **Prior** | **Shape parameter** | | **Bayes estimates of change point** | |
|---|---|---|---|---|
| | $q_1$ | $q_2$ | $m^*_L$ | $m^*_E$ |
| | 2 | −2 | 11 | 10 |
| | −1 | −1 | 10 | 10 |
| Informative | 0.09 | 0.09 | 9 | 10 |
| | 0.5 | 0.5 | 9 | 9 |
| | 0.9 | 0.9 | 8 | 9 |

The results of Bayes estimates of change point, m, under Linex Loss function and GEL function by considering the different values of the shape parameters $q_1$ and $q_2$, which are shown in Table 5. Also, the sensitivity of the Bayes estimators of change point and proportions $p_1$ and $p_2$ with respect to the parameters of prior distribution have been studied. In Tables 6 and 7, we computed Bayes estimator of change point under SEL function considering different set of values of $(\mu_1, \mu_2)$ and $(\mu_3, \mu_4)$. In addition, Table 8 contains Bayes estimates of proportions under GEL function by considering different set of the values of $(\mu_3, \mu_4)$. The mean square error (MSE) of the estimators are given in Table 9. The results of Tables 6–8 lead to the conclusion that, Bayes estimates of change point and proportions are robust with appropriate choice of parameters of the prior distribution. From Fig. 1, by repeated the experiment 1000 times, we see that the Bayes estimator is better than MLE.

**Table 6** The Bayes estimates of m under SEL function for different values of $\mu_1$ and $\mu_2$.

| $\mu_1$ | $\mu_2$ | $m^*$ |
|---|---|---|
| 0.95 | 0.05 | 10 |
| 0.85 | 0.05 | 9 |
| 0.75 | 0.05 | 9 |
| 0.95 | 0.04 | 10 |
| 0.95 | 0.03 | 10 |
| 0.95 | 0.02 | 9 |
| 0.85 | 0.01 | 9 |
| 0.75 | 0.01 | 9 |

SEL, symmetric loss function.

**Table 7** The Bayes estimates of m under SEL function for different values of $\mu_3$ and $\mu_4$.

| $\mu_3$ | $\mu_4$ | $m^*$ |
|---|---|---|
| 0.8 | 0.6 | 10 |
| 0.7 | 0.6 | 10 |
| 0.6 | 0.6 | 10 |
| 0.8 | 0.5 | 10 |
| 0.8 | 0.4 | 10 |
| 0.8 | 0.3 | 10 |
| 0.7 | 0.5 | 10 |

SEL, symmetric loss function.

**Table 8** Bayes estimates of proportions.

| | | $q_2 = 1.2$ | | $q_2 = 0.5$ | | $q_2 = -2$ | |
|---|---|---|---|---|---|---|---|
| $\mu_3$ | $\mu_4$ | $p^*_{1E}$ | $p^*_{2E}$ | $p^*_{1E}$ | $p^*_{2E}$ | $p^*_{1E}$ | $p^*_{2E}$ |
| 0.8 | 0.6 | 0.83 | 0.62 | 0.81 | 0.59 | 0.84 | 0.63 |
| 0.8 | 0.5 | 0.84 | 0.59 | 0.85 | 0.57 | 0.84 | 0.61 |
| 0.7 | 0.6 | 0.80 | 0.60 | 0.79 | 0.59 | 0.82 | 0.63 |
| 0.7 | 0.5 | 0.80 | 0.60 | 0.79 | 0.56 | 0.82 | 0.61 |
| 0.6 | 0.6 | 0.78 | 0.61 | 0.76 | 0.60 | 0.79 | 0.63 |
| 0.6 | 0.5 | 0.77 | 0.60 | 0.77 | 0.56 | 0.79 | 0.62 |

**Table 9** The values of MSE estimates of change point.

| $m$ | $m^{MLE}$ | $MSE\left(m^{MLE}\right)$ | $m^B$ | $MSE\left(m^B\right)$ |
|---|---|---|---|---|
| 1 | 1 | 0.15 | 2 | 2.3 |
| 2 | 2 | 0.17 | 2 | 0.90 |
| 3 | 3 | 0.20 | 3 | 0.5 |
| 4 | 4 | 0.25 | 4 | 0.30 |
| 5 | 5 | 0.26 | 5 | 0.25 |
| 6 | 6 | 0.29 | 6 | 0.24 |
| 7 | 7 | 0.31 | 7 | 0.27 |
| 8 | 8 | 0.35 | 8 | 0.22 |
| 9 | 9 | 0.37 | 9 | 0.21 |
| 10 | 10 | 0.38 | 10 | 0.16 |
| 11 | 11 | 0.39 | 11 | 0.15 |
| 12 | 12 | 0.40 | 12 | 0.18 |
| 13 | 13 | 0.40 | 13 | 0.23 |
| 14 | 14 | 0.42 | 14 | 0.24 |
| 15 | 15 | 0.45 | 15 | 0.22 |
| 16 | 16 | 0.48 | 16 | 0.30 |
| 17 | 17 | 0.48 | 17 | 0.33 |
| 18 | 18 | 0.35 | 18 | 0.34 |
| 19 | 19 | 0.10 | 18 | 2.9 |

MSE, mean square error.

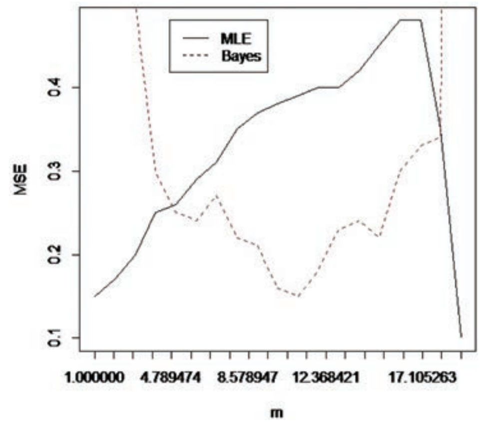


**Figure 1** | Comparison of Bayes and Maximum Likelihood Estimators.

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