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Multi-adjoint based group decision-making under an intuitionistic fuzzy information system

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Abstract

The construction of belief intervals is crucial for decision-making in multi-attribute group information integration. Based on multi-adjoint and evidence theory, an approach to multi-criteria group decision-making(MCGDM) in intuitionistic fuzzy information system is proposed. First, the upper and lower approximations of alternatives are calculated by multi-adjoint operators under the correlation matrices which were given by different experts. After that the belief and plausibility functions are gained by intuitionistic fuzzy probability formulas. Second, the belief intervals of alternatives are acquired by combining all experts' evidence. Then the alternatives are ranked by comparing the belief intervals. Finally, the effectiveness of the method is verified by an application of business transaction. Compared with the existing model, the method introduced in this paper is more effective and accurate.

Keywords: Multi-criteria Group Decision Making; Group Decision Making; Intuitionistic Fuzzy Sets; Evidence Theory;

1. Introduction

Decision-making(DM), which is an important form of human thinking and cognitive activity, widely exists in every field of human social life. Multi-criteria group decision making (MCGDM) [1,2] is to find the optimal object by ranking a given set of alternatives according to a group of decision makers. During the process of MCGDM, firstly, preference of alternatives is provided by each decision maker according to the values of multi-criteria; secondly, all preference relations are comprehensively considered and a satisfactory solution is presented for the decision makers. Fuzzy decision is the most common research field of MCGDM.

Fuzzy set theory was initiated by Zadeh [3] in 1965. After that, Atanassov [4] proposed intuitionistic fuzzy set (IFS) theory. The membership and the non-membership are used to describe the relationship between the objects and attributes, which makes the hesitant degree to be clearly described. IFSs depict more exquisite, and the membership degree, the non-membership degree and the hesitation degree are implied in the decision-making behavior to accept, reject, and hesitation. Following this, many scholars applied the theory to the area of decision making field [5,6], and propose many different intuitionistic fuzzy decision-making models [7,8,9].

A large body of pioneering works has focused on the problems of DM based on fuzzy set theory, which has been mentioned in the literature [10,11]. These studies are mainly focused on two aspects. The first is the construction of aggregation operators which combined the information carried by attributes together. Considering the relation among attributes, Yager [12] proposed a power average operator. Zhou et al [13] extended the power average operator. After that, Zhou and Chen [14], Xu and Wang [15] applied power average operator to deal with linguistic information. Other related works on the construction of aggregation operators have been reported in [16,17]. The second is the weights of decision makers, since different decision maker play different role in the decision group. Xu and Cai [18] determined the weights of experts according to nonlinear optimization models. Wan [19] established a model to calculate weighting vector on account of the similarity degree matrix of decision makers' judgement. Liang and Wei [20] introduced a new similarity measure by the relationship between entropy and similarity measure to derive the relative importance weight of experts. More recently, Ren and Wei [21] developed a correctional score function of dual hesitant fuzzy elements to solve multi-attribute decision-making problems in which the attributes were in different priority levels. Considering the attribute values are hybrid types, Jin [22] introduced certitude structure to tackle with such MAGDM problems. Zhu [23] defined t-norms and t-conorms products to construct $AND-fpfs$ decision making and $OR-fpfs$ decision making methods. Zhang and Guo [24] introduced some formulaes to compute priority weights which helped decision makings to express their preference flexibly. Khalid and Beg [25] mainly focused on incomplete hesitant fuzzy preference relations in group decision making. By defining a hesitant upper bound condition, this model promises to estimate missing information that is expressible and without voiding the originality of information provided by the expert. Based on the $I2LGA$ operator, Liu and Chen [26] proposed a new method for multi-attribute group decision making.

These methods mentioned above are important for the study of MCGDM, and have been used in many organizational decisions. Despite these achievement, there are still some certain decision problems which are not taken into consideration yet. In many real decision making of business transactions, for example, a man want to buy a house or an CEO want to choose an optimal project for his(her) company. There is only one decision maker for this MD problem. In fact, it is difficult for decision maker to rank all objects and make a clear decision after the attributes value of the decision objects are known. There are two reasons. Firstly, the decision maker may have a poor understanding of attributes and is not clear about the intrinsic relationship between them. Secondly, it is difficult to get any reasonable and reliable combination results by all attributes. Thus, the man may seek help from the property consultants and the CEO may consult the board of directors. We denote the property consultants and the board of directors as consulting experts. The problem can be described as following:

Suppose that there are several objects $O = \{o_1, o_2, \cdots, o_n\}$. Let $AT = \{a_1, a_2, \cdots, a_m\}$ is a set of attributes or decision parameters. The IFV is applied to describe the value of an object with an attribute as shown in Table 1. The decision maker wants to rank the objects completely and finds the optimal object by AT.

Table 1. The IF information system.

$Objects$	a_1	a_2	$\cdots$	a_m
o_1	$\langle\mu_{11}, \nu_{11}\rangle$	$\langle\mu_{12}, \nu_{12}\rangle$	$\cdots$	$\langle\mu_{1m}, \nu_{1m}\rangle$
o_2	$\langle\mu_{21}, \nu_{21}\rangle$	$\langle\mu_{22}, \nu_{22}\rangle$	$\cdots$	$\langle\mu_{2m}, \nu_{2m}\rangle$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
o_n	$\langle\mu_{m1}, \nu_{m1}\rangle$	$\langle\mu_{m2}, \nu_{m2}\rangle$	$\cdots$	$\langle\mu_{mm}, \nu_{mm}\rangle$

Zhao [27] proposed an approach of multi-criteria ranking based on intuitionistic fuzzy soft set to deal with decision making problems above. In this paper, several principles were proposed for solving the M-CDM completely ranking problem and also getting the only one optimal selection. However, the result-s obtained by this method may be contrary to our intuitive judgment.

To overcome this shortcoming, A group of consulting experts is introduced to provide advice for decision maker. Because of the work experience, the professional field are different, each expert may provide different viewpoint in multi-criteria which is described by the intuitionistic fuzzy relation matrix D^k. And we propose an approach to group decision-making in intuitionistic fuzzy information system. By consulting some experts, and using adjoint triple and evidence theory, decision maker can rank all objects and make a clear decision based on individual preferences. The contribution of this paper are listed as follows:

(1) The multi-adjoint theory and D-S theory are applied to intuitionistic fuzzy group decision, which can effectively enhance the ability of the model to deal with intuitionistic fuzzy decision-making problem.

(2) From the intuitionistic fuzzy relation matrix D^k, the weight of each attribute is obtained according to the kth expert, which effectively avoid the error caused by a given weight.

The remainder of this paper is structured as follows. In Section 2, we recall preliminaries from intuitionistic fuzzy sets, and D-S theory. In Section 3, we introduce the multi-adjoint intuitionistic fuzzy rough sets, after that the believe function and plausibility function based on intuitionistic fuzzy probability are calculated. Next, in Section 4, we present intuitionistic fuzzy MCGDM method. An application of business transaction on housing purchase for using the proposed method is demonstrated, and we also compare this model with other methods. Finally, we conclude the paper with a summary and give an outlook for further research.

2. Preliminaries

As a generalization of fuzzy set, since considering the support, oppose and hesitation of the three aspects of information, IFS is more flexible and practical in dealing with ambiguity and uncertainty information systems. In this section, we briefly introduce the basic notions and definitions of IFSs theory and D-S theory.

2.1. *IFSs and IF approximation space*

Definition 1. (Atanassov 1986) [4] Let U be a nonempty objects set. An IFS $\mathbf{A}$ has the form $\mathbf{A} = \{\langle\mu_{\mathbf{A}}(x), \nu_{\mathbf{A}}(x)\rangle : x \in U\}$, where the mappings $\mu_{\mathbf{A}} : U \to [0,1]$ and $\nu_{\mathbf{A}} : U \to [0,1]$ denote the degree of membership and nonmembership of each element $x \in U$ to the set $\mathbf{A}$ (namely $\mu_{\mathbf{A}}(x)$ and $\nu_{\mathbf{A}}(x)$) respectively, and $0 \leqslant \mu_{\mathbf{A}}(x) + \nu_{\mathbf{A}}(x) \leqslant 1$ for each $x \in U$. $1 - \mu_{\mathbf{A}}(x) - \nu_{\mathbf{A}}(x)$ can be interpreted as the hesitancy degree of x to $\mathbf{A}$.

The family of all IFS subsets of U is denoted by $IFS(U)$. We call $\mathbf{A}(x) = \langle\mu_{\mathbf{A}}(x), \nu_{\mathbf{A}}(x)\rangle$ an intuitionistic fuzzy value(IFV). Especially, for any $\mathbf{A} \in P(U)$, if $x \in \mathbf{A}$, then $\mathbf{A}(x) = \langle 1, 0\rangle$, if $x \notin \mathbf{A}$, then $\mathbf{A}(x) = \langle 0, 1\rangle$. Let $Bel_{\mathbf{A}}(x) = \mu_{\mathbf{A}}(x)$, $Pl_{\mathbf{A}}(x) = 1 - \nu_{\mathbf{A}}(x)$, then the intuitionistic fuzzy set $\mathbf{A}$ can also be written as $\mathbf{A} = \{BI_{\mathbf{A}}(x) : x \in U\}$, in which $BI_{\mathbf{A}}(x) = [Bel_{\mathbf{A}}(x), Pl_{\mathbf{A}}(x)]$ expresses the belief interval of x to $\mathbf{A}$.

Distance measure is an important uncertainty measurements in IFS theory.

Definition 2. Let U be a finite and nonempty set, $\mathbf{A}, \mathbf{B} \in IFS(U)$, the distance between $\mathbf{A}$ and $\mathbf{B}$ is defined as $D(\mathbf{A},\mathbf{B}) = \frac{1}{|U|}\sum_{x\in U}\left[\frac{|\mu_{\mathbf{A}}(x)-\mu_{\mathbf{B}}(x)|+|\nu_{\mathbf{A}}(x)-\nu_{\mathbf{B}}(x)|}{4} + \frac{\max\{|\mu_{\mathbf{A}}(x)-\mu_{\mathbf{B}}(x)|,|\nu_{\mathbf{A}}(x)-\nu_{\mathbf{B}}(x)|\}}{2}\right]$.

The intuitionistic fuzzy binary relation is an intuitionistic subset on $U\times U$. IR is defined as $IR = \{\langle \mu_{IR}(x,y), \nu_{IR}(x,y)\rangle : (x,y)\in U\times U\}$, where $\mu_{IR} : U\times U \to [0,1]$, $\nu_{IR} : U\times U\to[0,1]$, and satisfied the condition $0\leqslant \mu_{IR}(x,y)+\nu_{IR}(x,y)\leqslant 1, \forall(x,y)\in U\times U$. We denote all intuitionistic fuzzy binary relation sets on U as $IFR(U\times U)$. For $IR\in IFR(U\times U)$, if $IR(x,x) = \langle 1,0\rangle$ is satisfied for all $x\in U$, then IR is reflexive; if $IR(x,y) = IR(x,y)$ is satisfied for all $(x,y)\in U\times U$, then IR is symmetric.

Definition 3. (Wu & Zhou 2008) [28] Let U be a finite and nonempty set, IR is an intuitionistic fuzzy binary relation on $U\times U$. The ordered pair (U,IR) is called intuitionistic fuzzy approximation space.

2.2. D-S theory

Question assumptions are all possible outcomes which can be discerned by people on a subject. If these assumptions are mutually exclusive and complete with all possible descriptions of the problem, then these assumption collections Θ are called the recognition framework. The power set of Θ is denoted by 2^{Θ}.

Definition 4. (Shafer 1976) [29] A set function $m : 2^{\Theta}\to[0,1]$ is referred to as a basic probability assignment or mass distribution, if it satisfies the following axioms:

(1) $m(\emptyset) = 0$,

(2) $\sum_{A\subseteq\Theta} m(A) = 1$.

If $m(A)\neq 0$, then subset A is called the focal element of m. The value of $m(A)$ represents the degree of belief that a specific element of Θ belongs to A. All focal elements of m is denoted as M, the pair (M,m) is called a belief structure on Θ.

Definition 5. (Shafer 1976) [29] Let $m : 2^{\Theta}\to[0,1]$ be a basic belief assignment function. $\forall X\subseteq\Theta$, the belief function ($Bel : m\to[0,1]$) and plausibility function ($Pl : m\to[0,1]$) of X are respectively defined as:

$$Bel(X) = \sum_{Y\subseteq X} m(Y);$$

$$Pl(X) = \sum_{Y\cap X\neq\emptyset} m(Y).$$

The belief interval of X is defined as $Bl(X) = [Bel(X),Pl(X)]$.

$\forall X,Y\subseteq\Theta$, The belief intervals of X,Y are $Bl(X) = [Bel(X),Pl(X)]$ and $Bl(Y) = [Bel(Y),Pl(Y)]$ respectively. If $Bel(X) > Bel(Y)$, then $Bl(X) > Bl(Y)$, denoted as $X > Y$. If $Bel(X) = Bel(Y)$ and $Pl(X) > Pl(Y)$, then $Bl(X) > Bl(Y)$, denoted as $X > Y$. If $Bel(X) = Bel(Y)$ and $Pl(X) = Pl(Y)$, then $Bl(X) = Bl(Y)$, denoted as $X = Y$.

Definition 6. (Shafer 1976) [29] Let $Bel_1, Bel_2,\cdots,Bel_n$ be belief functions of Θ. Then combined evidence can be calculated by orthogonal sum $m = m_1\oplus m_2\oplus\cdots\oplus m_n$ for fusing independent information sources m_i. The orthogonal sum is associative and commutative; it is defined in Dempster's rule of combination:

$$m(A) = \frac{\sum\limits_{A_1\cap A_2\cap\cdots\cap A_n = A} m_1(A_1)m_2(A_2)\cdots m_n(A_n)}{1-N},$$

in which

$$N = \sum_{A_1\cap A_2\cap\cdots\cap A_n=\emptyset} m_1(A_1)m_2(A_2)\cdots m_n(A_n).$$

3. Multi-adjoint IFRSs and the belief interval

In this section, construction of the belief interval has been proposed in intuitionistic fuzzy information system. First, multi-adjoint theory has been introduced in intuitionistic fuzzy information system, the lower and the upper approximations of each object according to expert are obtained. Second, the belief function and plausibility function are calculated by intuitionistic fuzzy probability for each object.

3.1. Multi-adjoint IFRSs

Now we mainly introduce the definition of multi-adjoint intuitionistic fuzzy rough sets (short for Multi-adjoint IFRSs) in this subsection.

Definition 7. (Eugenia, Medina & Ramirez 2013)[30] Let $(P_1,\leqslant_1),(P_2,\leqslant_2),(P_3,\leqslant_3)$ be three posets. The pair $(\&,\swarrow,\nwarrow)$ is called an adjoint triple with respect to P_1,P_2,P_3, if the three mappings $\& : P_1\times P_2\to P_3$, $\swarrow : P_3\times P_2\to P_1$, $\nwarrow : P_3\times P_1\to P_2$ satisfy,

- $x \leqslant_1 z \swarrow y$ iff $x \& y \leqslant_3 z$ iff $y \leqslant_2 z \nwarrow x$, for any $x \in P_1, y \in P_2, z \in P_3$;
- & is order-preserving on both arguments;
- $\swarrow, \nwarrow$ are order-preserving on the first argumentand order-reserving on the second argument.

There are some special examples of adjoint triples:

Gödel adjoint triple:

$x \&_G y = min\{x,y\}$, $z \nwarrow_G x = \begin{cases} 1, \text{if } x \leqslant z \\ z, otherwise; \end{cases}$

Product adjoint triple:

$x \&_P y = x \cdot y$, $z \nwarrow_G x = min\{1, z/x\}$;

Łukasiewicz adjoint triple:

$x \&_L y = max\{0, x+y-1\}$, $z \nwarrow_L x = min\{1, 1-x+z\}$.

The above adjoint pairs formed by a t-norm and its residuated implications, which can be seen as degenerate examples of general adjoint triples, and in this three adjoint pairs $\nwarrow_G = \swarrow_G$, $\nwarrow_P = \swarrow_P$, $\nwarrow_L = \swarrow_L$, since $\&_G$, $\&_P$, $\&_L$ are commutative.

Definition 8. Given an intuitionistic fuzzy approximation space (U, IR), $\forall \mathbf{A} \in IFS(U)$, the lower and the upper approximations of $\mathbf{A}$ are defined respectively as

$$\underline{IR}(\mathbf{A}) = \{\langle \mu_{\underline{IR}(\mathbf{A})}(x), \nu_{\underline{IR}(\mathbf{A})}(x) \rangle : x \in U\}; \quad (1)$$

$$\overline{IR}(\mathbf{A}) = \{\langle \mu_{\overline{IR}(\mathbf{A})}(x), \nu_{\overline{IR}(\mathbf{A})}(x) \rangle : x \in U\}, \quad (2)$$

where

$\mu_{\underline{IR}(\mathbf{A})}(x) = inf\{\mu_{\mathbf{A}}(y) \nwarrow_{xy} \mu_{IR}(x,y) : y \in U\}$,
$\nu_{\underline{IR}(\mathbf{A})}(x) = sup\{(1 - \nu_{IR}(x,y)) \&_{xy} \nu_{\mathbf{A}}(y) : y \in U\}$;
$\mu_{\overline{IR}(\mathbf{A})}(x) = sup\{\mu_{IR}(x,y) \&_{xy} \mu_{\mathbf{A}}(y) : y \in U\}$,
$\nu_{\overline{IR}(\mathbf{A})}(x) = inf\{\nu_{\mathbf{A}}(y) \nwarrow_{xy} (1 - \nu_{IR}(x,y)) : y \in U\}$.

Definition 9. For all $\mathbf{A} \in IFS(U)$, the pair $(\underline{IR}(\mathbf{A}), \overline{IR}(\mathbf{A}))$ is called a multi-adjoint intuitionistic fuzzy rough set of $\mathbf{A}$ with respect to IR.

An important feature of the presented framework is that the user may represent explicit preferences among the objects and acquire different upper and lower approximations in an intuitionistic fuzzy decision system, by associating different family adjoint triples. In next section, the probability of approximations will be calculated to construct the belief interval which will be used in the decision system.

3.2. *The belief function and plausibility function based on IF probability*

According to literature (Feng, Zhang & Mi) [31], a novel probability of an A-IF set is defined by using the (α, β) level set. And now we just show the conclusion as the following definition will be used later.

Definition 10. (Feng, Zhang & Mi) [31] Let U be a nonempty finite set, $(U, P(U), P)$ is a probability space. $\forall \mathbf{A} \in IFS(U)$ the probability measure P^* of $\mathbf{A}$ is defined as

$$P^*(\mathbf{A}) = \sum_{x \in U} ((1 - \nu_{\mathbf{A}}(x))^2 - (1 - \mu_{\mathbf{A}}(x) - \nu_{\mathbf{A}}(x))^2) P(x). \quad (3)$$

It is obviously that $P^*(\emptyset) = 0 P^*(U) = 1$.

If $(U, P(U), P)$ is a probability space, $IR \in IFR(U \times U)$ is a reflexive and symmetric intuitionistic fuzzy binary relation, (U, IR) is an intuitionistic fuzzy approximate space mentioned in Definition 3. Let $IR(x,y) = IR_S(x)(y)$, $IR_S(x)$ is an intuitionistic fuzzy set on U.

Definition 11. (Feng, Zhang & Mi) [31] Let U be a nonempty finite set, a set function $m : IFS(U) \to [0,1]$ is referred to as a basic probability assignment (also called mass function) if it satisfies axioms (M1) and (M2)

(M1) $m(\emptyset) = 0$;

(M2) $\sum_{\mathbf{A} \in IFS(U)} m(\mathbf{A}) = 1$.

For $\mathbf{A} \in IFS(U)$ with $m(\mathbf{A}) \neq 0$ is referred to as a focal element of m. We denote all focal elements of m by M. The pair (M, m) is called an IF belief structure.

Theorem 1. *(Feng, Zhang & Mi)* [31] *Let $U = \{x_1, x_2, \cdots, x_n\}$ be a nonempty and finite universe of discourse, (U, IR) a reflective and symmetric intuitionistic fuzzy approximation space. $\forall \mathbf{A} \in IFS(U)$ we define*

$$m_{IR}(\mathbf{A}) = \begin{cases} \sum_{x: \mathbf{A} = IR_S(x)} P^*(1_x), & if \ \mathbf{A} \in G; \\ 0, & otherwise. \end{cases}$$

Then m_{IR} is a mass function.

The probability measures of $\underline{IR}(\mathbf{A})$ and $\overline{IR}(\mathbf{A})$ have the following expressions.

Definition 12. Suppose U be a nonempty and finite universe of discourse, (U, IR) a reflective and symmetric intuitionistic fuzzy approximation space. P is a probability measure on U, $\forall \mathbf{A} \in IFS(U)$,

$$Bel_{IR}(\mathbf{A}) = P^*(\underline{IR}(\mathbf{A})); \tag{4}$$

$$Pl_{IR}(\mathbf{A}) = P^*(\overline{IR}(\mathbf{A})). \tag{5}$$

Therefore, we have the following results.

Theorem 2. *Suppose U be a nonempty and finite universe of discourse, (U, IR) a reflective and symmetric intuitionistic fuzzy approximation space. P is a probability measure on U, $\forall \mathbf{A} \in IFS(U)$,*

$Bel_{IR}(\mathbf{A}) = \sum_{X\in G} m_{IR}(X)((1-\nu_{\underline{IR}(X\subseteq\mathbf{A})}(x))^2 - (1-\mu_{\underline{IR}(X\subseteq\mathbf{A})}(x) - \nu_{\underline{IR}(X\subseteq\mathbf{A})}(x))^2)$

$Pl_{IR}(\mathbf{A}) = \sum_{X\in G} m_{IR}(X)((1-\nu_{\overline{IR}(X\subseteq\mathbf{A})}(x))^2 - (1-\mu_{\overline{IR}(X\subseteq\mathbf{A})}(x) - \nu_{\overline{IR}(X\subseteq\mathbf{A})}(x))^2)$.

where

$\mu_{\underline{IR}(X\subseteq\mathbf{A})}(x) = inf\{\mu_{\mathbf{A}}(y) \nwarrow_{xy} \mu_X(y) : y \in U\}$,

$\nu_{\underline{IR}(X\subseteq\mathbf{A})}(x) = sup\{(1-\nu_X(y))\&_{xy}\nu_{\mathbf{A}}(y) : y \in U\}$;

$\mu_{\overline{IR}(X\cap\mathbf{A})}(x) = sup\{\mu_X(y)\&_{xy}\mu_{\mathbf{A}}(y) : y \in U\}$,

$\nu_{\overline{IR}(X\cap\mathbf{A})}(x) = inf\{\nu_{\mathbf{A}}(y) \nwarrow_{xy} (1-\nu_X(y)) : y \in U\}$.

Proof. $Bel_{IR}(\mathbf{A}) = P^*(\underline{IR}(\mathbf{A})) = \sum_{x\in U} P(x)((1-\nu_{\underline{IR}(\mathbf{A})}(x))^2 - (1-\mu_{\underline{IR}(\mathbf{A})}(x) - \nu_{\underline{IR}(\mathbf{A})}(x))^2) = \sum_{R_S(x)\in G}(\sum_{y:R_S(y)=R_S(x)} P(1_y))((1-\nu_{\underline{IR}(\mathbf{A})}(x))^2 - (1-\mu_{\underline{IR}(\mathbf{A})}(x) - \nu_{\underline{IR}(\mathbf{A})}(x))^2) = \sum_{X\in G} m_{IR}(X)((1-sup\{(1-\nu_X(y))\&_{xy}\nu_{\mathbf{A}}(y) : y \in U\})^2 - (1-inf\{\mu_{\mathbf{A}}(y) \nwarrow_{xy} \mu_X(y) : y \in U\} - sup\{(1-\nu_X(y))\&_{xy}\nu_{\mathbf{A}}(y) : y \in U\})^2) = \sum_{X\in G} m_{IR}(X)((1-\nu_{\underline{IR}(X\subseteq\mathbf{A})}(x))^2 - (1-\mu_{\underline{IR}(X\subseteq\mathbf{A})}(x) - \nu_{\underline{IR}(X\subseteq\mathbf{A})}(x))^2)$.

Similarly, we get

$Pl_{IR}(\mathbf{A}) = P^*(\overline{IR}(\mathbf{A})) = \sum_{x\in U} P(x)((1-\nu_{\overline{IR}(\mathbf{A})}(x))^2 - (1-\mu_{\overline{IR}(\mathbf{A})}(x) - \nu_{\overline{IR}(\mathbf{A})}(x))^2) = \sum_{R_S(x)\in G}(\sum_{y:R_S(y)=R_S(x)} P(1_y))((1-\nu_{\overline{IR}(\mathbf{A})}(x))^2 - (1-\mu_{\overline{IR}(\mathbf{A})}(x) - \nu_{\overline{IR}(\mathbf{A})}(x))^2) = \sum_{X\in G} m_{IR}(X)((1-inf\{\nu_{\mathbf{A}}(y) \nwarrow_{xy} (1-\nu_X(y)) : y \in U\})^2 - (1-sup\{\mu_X(y)\&_{xy}\mu_{\mathbf{A}}(y) : y \in U\} - inf\{\nu_{\mathbf{A}}(y) \nwarrow_{xy} (1-\nu_X(y)) : y \in U\})^2) = \sum_{X\in G} m_{IR}(X)((1-\nu_{\overline{IR}(X\cap\mathbf{A})}(x))^2 - (1-\mu_{\overline{IR}(X\cap\mathbf{A})}(x) - \nu_{\overline{IR}(X\cap\mathbf{A})}(x))^2)$. □

Theorem 3. *Suppose U be a nonempty and finite universe of discourse, $IR \in IFR(U\times U)$ a reflective and symmetric intuitionistic fuzzy binary relation, the following properties hold:*

1. *$Bel_{IR}(\emptyset) = Pl_{IR}(\emptyset) = 0$, $Bel_{IR}(U) = Pl_{IR}(U) = 1$;*
2. *$Bel_{IR}(\mathbf{A}) \leqslant P^*(\mathbf{A}) \leqslant Pl_{IR}(\mathbf{A}), \forall \mathbf{A} \in IFS(U)$;*
3. *$Bel_{IR}(\mathbf{A}) \leqslant Bel_{IR}(\mathbf{B}), Pl_{IR}(\mathbf{A}) \leqslant Pl_{IR}(\mathbf{B})$, $\forall \mathbf{A}, \mathbf{B} \in IFS(U), \mathbf{A} \subseteq \mathbf{B}$.*

Proof. (1) According to Definition 8 and 10, we have $Bel_{IR}(\emptyset) = Pl_{IR}(\emptyset) = P^*(\emptyset) = 0$, $Bel_{IR}(U) = Pl_{IR}(U) = P^*(U) = 0$;

(2) $Bel_{IR}(\mathbf{A}) = P^*(\underline{IR}(\mathbf{A})) \leqslant P^*(\mathbf{A}) \leqslant P^*(\overline{IR}(\mathbf{A})) = Pl_{IR}(\mathbf{A})$

(3) $\forall \mathbf{A}, \mathbf{B} \in IFS(U)$, if $\mathbf{A} \subseteq \mathbf{B}$, then $Bel_{IR}(\mathbf{A}) = P^*(\underline{IR}(\mathbf{A})) \leqslant P^*(\underline{IR}(\mathbf{B})) = Bel_{IR}(\mathbf{B})$. Similarly, we have $Pl_{IR}(\mathbf{A}) \leqslant Pl_{IR}(\mathbf{B})$. □

It is easy to verify that Bel_{IR} and Pl_{IR} degenerate into the crisp belief and plausibility functions when the belief structure (M, m) and $\mathbf{A}$ are crisp. Thus Bel_{IR} and Pl_{IR} are measures of belief and plausibility functions on $IFS(U)$. Convert belief function and plausibility function into Mass functions, we have:

$m(yes) = Bel_{IR}$,

$m(no) = 1 - Pl_{IR}$,

$m(yes, no) = Pl_{IR} - Bel_{IR}$.

4. The construction of multi-adjoint IF group decision-making method

To make our model easier to understand and realize, we suppose that $O = \{o_1, o_2, \cdots, o_n\}$ is a set of decision objects. A man wants to choose the best one from O. There are m attributes to take into consideration, the attribute set $AT = \{a_1, a_2, \cdots, a_m\}$. For each object $o_l, l \leqslant n$, there is an intuitionistic fuzzy set of o_i on AT denoted as $\mathbf{A}_l = \{\langle \mu_j(o_l), \nu_j(o_l)\rangle : j = 1, 2, \cdots, m\}$. There is an advisory group consisting of K experts $M_k (k = 1, 2, \cdots, K)$ each expert gets the intuitionistic fuzzy relation matrix D^k according to the attributes.

$$\begin{pmatrix} \langle \mu^k_{11}, \nu^k_{11}\rangle & \langle \mu^k_{12}, \nu^k_{12}\rangle & \cdots & \langle \mu^k_{1m}, \nu^k_{1m}\rangle \\ \langle \mu^k_{21}, \nu^k_{21}\rangle & \langle \mu^k_{22}, \nu^k_{22}\rangle & \cdots & \langle \mu^k_{2m}, \nu^k_{2m}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \mu^k_{m1}, \nu^k_{m1}\rangle & \langle \mu^k_{m2}, \nu^k_{m2}\rangle & \cdots & \langle \mu^k_{mm}, \nu^k_{mm}\rangle \end{pmatrix}$$

Where μ_{ij}^k, ν_{ij}^k denote the degree to which the expert k considers the attribute i to be relevant and irrelevant with the attribute $j$and $0 \leqslant \mu_{ij}^k + \nu_{ij}^k \leqslant 1$, for $\forall i, j \in \{1, 2, \cdots m\}$, $k \in \{1, 2, \cdots K\}$. satisfies the properties

$\langle \mu_{ii}^k, \nu_{ii}^k \rangle = \langle 1, 0 \rangle$, for $\forall i \leqslant m$;

$\langle \mu_{ij}^k, \nu_{ij}^k \rangle = \langle \mu_{ji}^k, \nu_{ji}^k \rangle$, for $\forall i, j \leqslant m$.

It is clear that (AT, D^k) is an intuitionistic fuzzy approximation space. Before group decision-making information integration, a hypothesis must be proposed firstly.

4.1. *Group decision-making information integration*

Before group decision-making information integration, a hypothesis must be proposed firstly.

Assume The sources of information in decision-making system are mostly reliable. That means the intuitionistic fuzzy relation matrices given by most experts are reliable. Otherwise, the system has lapsed and the group decision-making information integration is meaningless.

Let yes express supportive of the expert M_k to a decision object; No indicate appositive of expert M_k to a decision objects; (yes, no) mean cannot judge (or hesitate).

Step 1 From Definition 8, the lower and upper approximations according to expert are obtained. They are $\underline{D^k}(o_l)$ and $\overline{D^k}(o_l)$, which indicate the degrees of certainty and probability of the expert M_k with regard to decision object o_l on each attribute. The calculation is as follows

$\underline{\mu}_i^k(o_l) = inf\{\mu_j(o_l) \nwarrow_{ij} \mu_{ij}^k : j \leqslant m\}$;

$\underline{\nu}_i^k(o_l) = sup\{(1 - \nu_{ij}^k) \&_{ij} \nu_j(o_l) : j \leqslant m\}$;

$\overline{\mu}_i^k(o_l) = sup\{\mu_{ij}^k \&_{ij} \mu_j(o_l) : j \leqslant m\}$;

$\overline{\nu}_i^k(o_l) = inf\{\nu_j(o_l) \nwarrow_{ij} (1 - \nu_{ij}^k) : j \leqslant m\}$.

Step 2 From Definition 12, the belief function $Bel_{D^k}(o_l) = P^*(\underline{D^k}(o_l))$ and plausibility function $Pl_{D^k}(o_l) = P^*(\underline{D^k}(o_l))$ of the expert M_k with regard to decision object o_l are calculated. The weight of a_i in D^k can be used as the probability of the attribute a_i under expert M_k. The calculation is as follows

Let D^k be the intuitionistic fuzzy relation matrix given by expert M_k. μ_{ij}^k, ν_{ij}^k denote the degree to which the expert k considers the attribute i to be relevant and irrelevant with the attribute j. Then the function $s_{ij}^k = \mu_{ij}^k - \nu_{ij}^k$ is denoted as the relevant score of the attribute i to the attribute j by expert k.

The weight of a_i under expert M_k is

$$P^k(a_i) = \frac{\sum\limits_{j=1}^{m} s_{ij}}{\sum\limits_{i=1}^{m} (\sum\limits_{j=1}^{m} s_{ij})}. \tag{6}$$

$P^k(a_i)$ reflects the degree to which attribute a_i is recognized by expert M_k. The higher the correlation with other attributes, the more important of attribute a_i, and the bigger weight of a_i. Therefore, the weights of attributes based on correlation are reasonable.

Step 3 Convert the belief function and the plausibility function to the Mass function. Let

$m_l^k(yes) = Bel_{D^k}(o_l)$,

$m_l^k(no) = 1 - Pl_{D^k}(o_l)$,

$m_l^k(yes, no) = Pl_{D^k}(o_l) - Bel_{D^k}(o_l)$.

Step 4 Due to the interference of different factors, some experts give evidence that may be larger in conflict with other experts, and the rule of evidence combination does not recognize strong evidence nor the weak evidence. To get a reasonable and reliable evidence combination results, it is necessary to analyze the conflict of evidence before combination. We use distance measure to modify the evidence, the bigger the distance, the smaller the degree of support and the bigger the discount factor. According to definition 2, the distance between any two evidences can be calculated. Let $\Delta = (\vartheta_{ij})_{K \times K}$ be the distance matrix of objects, discount factor of evidences is defined as:

$$\omega_k = \frac{1 - \sum\limits_{k=1}^{K} \vartheta_{ij}}{1 - min\{\sum\limits_{k=1}^{K} \vartheta_{ij} : i = 1, 2 \cdots K\}}. \tag{7}$$

The revised evidence indicates that:

$m_l^{*k}(yes) = \omega_k m_l^k(yes)$,

$m_l^{*k}(no) = \omega_k m_l^k(no)$,

$m_l^{*k}(yes, no) = 1 - m_l^{*k}(yes) - m_l^{*k}(no)$.

Step 5 Combine all experts evidence for each decision object. The calculation is as follow

$$m_l(A)=\frac{\sum\limits_{A_1\bigcap A_2\bigcap\cdots\bigcap A_K\neq\emptyset} m_l^{*1}(A_1)m_l^{*2}(A_2)\cdots m_l^{*K}(A_K)}{1-N}. \tag{8}$$

in which
$N = \sum\limits_{A_1\bigcap A_2\bigcap\cdots\bigcap A_K=\emptyset} m_l^{*1}(A_1)m_l^{*2}(A_2)\cdots m_l^{*K}(A_K)$,
and $A,A_1,A_2,\cdots,A_K\in\{yes,no,(yes,no)\}$.

Step 6 Compares the belief interval of each decision object and makes the optimal decision.

4.2. *Instance analyses on business transaction*

Many business transactions [32] are made through evaluation a set of potential trading alternatives. In such a decision making problem, decision maker needs to rank the potential alternatives or select an optimal one based on the evaluation information associated a set of criteria. Before decision making, a group of consultant experts is asked to provide their advise.

As one of the basic condition of living and an important component part of household property, housing has been highly valued by Chinese people. Many families even take out two or three generations of savings to buy a house. Therefore, how to buy a satisfactory house is very important to an individual or a family. In this process, the decision maker first collects all kinds of information about the purchase of houses such as the quality of the property service, location etc. If the advantages and disadvantages of the alternative houses are obvious, the decision maker can make a direct choice. Otherwise, he needs to consult property consultants, and then make decisions based on his own needs. Now we verify the validity of the model with the examples of housing transaction.

Example 1 Suppose $O=\{o_1,o_2,o_3,o_4,o_5,o_6\}$ represent six alternative houses for sale, $AT=\{a_1,a_2,a_3,a_4\}$ be a set of attributes such as the location of the house, the quality of the property service, the building structure and the average price. The intuitionistic fuzzy sets of the six houses according to AT are shown in Table 2.

Table 2. The IFSs of the houses according to AT.

Objects	a_1	a_2	a_3	a_m
o_1	$\langle 0.80,0.20\rangle$	$\langle 0.78,0.18\rangle$	$\langle 0.69,0.28\rangle$	$\langle 0.78,0.12\rangle$
o_2	$\langle 0.76,0.23\rangle$	$\langle 0.78,0.20\rangle$	$\langle 0.70,0.28\rangle$	$\langle 0.75,0.18\rangle$
o_3	$\langle 0.80,0.10\rangle$	$\langle 0.60,0.24\rangle$	$\langle 0.75,0.20\rangle$	$\langle 0.70,0.25\rangle$
o_4	$\langle 0.75,0.20\rangle$	$\langle 0.82,0.15\rangle$	$\langle 0.55,0.28\rangle$	$\langle 0.80,0.18\rangle$
o_5	$\langle 0.65,0.28\rangle$	$\langle 0.74,0.20\rangle$	$\langle 0.70,0.15\rangle$	$\langle 0.75,0.20\rangle$
o_6	$\langle 0.80,0.10\rangle$	$\langle 0.60,0.24\rangle$	$\langle 0.75,0.15\rangle$	$\langle 0.70,0.25\rangle$

$P=\{M_1,M_2,M_3,M_4\}$ is a advisory group consists of four property consultants. The four members evaluate the attributes with their personal experience and information. Their evaluate intuitionistic fuzzy relation matrix about attribute set AT are shown in Table 3. Decision maker should rank houses based on individual preferences and specialists' opinions. And the MD process by means of the proposed method is described as follows.

Table 3. The IF relation matrixes.

		a_1	a_2	a_3	a_4
	a_1	$\langle 1.00,0.00\rangle$	$\langle 0.73,0.25\rangle$	$\langle 0.58,0.35\rangle$	$\langle 0.53,0.44\rangle$
D^1	a_2	$\langle 0.73,0.25\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.55,0.40\rangle$	$\langle 0.25,0.61\rangle$
	a_3	$\langle 0.58,0.35\rangle$	$\langle 0.55,0.40\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.55,0.39\rangle$
	a_4	$\langle 0.53,0.44\rangle$	$\langle 0.25,0.61\rangle$	$\langle 0.55,0.39\rangle$	$\langle 1.00,0.00\rangle$
	a_1	$\langle 1.00,0.00\rangle$	$\langle 0.93,0.06\rangle$	$\langle 0.69,0.27\rangle$	$\langle 0.71,0.26\rangle$
D^2	a_2	$\langle 0.93,0.06\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.76,0.20\rangle$	$\langle 0.64,0.31\rangle$
	a_3	$\langle 0.69,0.27\rangle$	$\langle 0.76,0.20\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.86,0.13\rangle$
	a_4	$\langle 0.71,0.26\rangle$	$\langle 0.64,0.31\rangle$	$\langle 0.86,0.13\rangle$	$\langle 1.00,0.00\rangle$
	a_1	$\langle 1.00,0.00\rangle$	$\langle 0.75,0.25\rangle$	$\langle 0.59,0.39\rangle$	$\langle 0.65,0.30\rangle$
D^3	a_2	$\langle 0.75,0.25\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.74,0.24\rangle$	$\langle 0.80,0.19\rangle$
	a_3	$\langle 0.59,0.39\rangle$	$\langle 0.74,0.24\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.89,0.11\rangle$
	a_4	$\langle 0.65,0.30\rangle$	$\langle 0.80,0.19\rangle$	$\langle 0.89,0.11\rangle$	$\langle 1.00,0.00\rangle$
	a_1	$\langle 1.00,0.00\rangle$	$\langle 0.93,0.06\rangle$	$\langle 0.80,0.10\rangle$	$\langle 0.66,0.27\rangle$
D^4	a_2	$\langle 0.93,0.06\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.73,0.23\rangle$	$\langle 0.44,0.43\rangle$
	a_3	$\langle 0.80,0.10\rangle$	$\langle 0.73,0.23\rangle$	$\langle 1.00,0.00\rangle$	$\langle 0.63,0.33\rangle$
	a_4	$\langle 0.66,0.27\rangle$	$\langle 0.44,0.43\rangle$	$\langle 0.63,0.33\rangle$	$\langle 1.00,0.00\rangle$

Step 1 Let $\tau : a_i\times a_j \longrightarrow product$. According to Definition 8, the the lower and upper approximations of alternatives w.r.t experts are obtained.

Step 2 According to formulas (6)we can easily get the weight of a_i under expert M^k, as is shown in Table 4. According to the formulas (3) to (5) the belief function and plausibility function of each house in the intuitionistic fuzzy relation matrix given by the specialists are calculated.

Table 4. The weight of attributes.

Experts	$P^k(a_1)$	$P^k(a_2)$	$P^k(a_3)$	$P^k(a_4)$
M_1	0.33	0.23	0.28	0.16
M_2	0.26	0.26	0.25	0.23
M_3	0.21	0.27	0.25	0.28
M_4	0.28	0.29	0.26	0.17

Step 3 Translate them into mass functionsas shown in Table 5.

Table 5. The mass function of each house in Example 1.

Experts	o_1	o_2	o_3	o_4	o_5	o_6
M_1	$\langle 0.65,0.33,0.02\rangle$	$\langle 0.59,0.41,0.00\rangle$	$\langle 0.62,0.28,0.11\rangle$	$\langle 0.61,0.37,0.02\rangle$	$\langle 0.60,0.36,0.04\rangle$	$\langle 0.64,0.27,0.09\rangle$
M_2	$\langle 0.61,0.29,0.11\rangle$	$\langle 0.57,0.36,0.07\rangle$	$\langle 0.58,0.24,0.18\rangle$	$\langle 0.57,0.31,0.11\rangle$	$\langle 0.57,0.34,0.09\rangle$	$\langle 0.58,0.24,0.18\rangle$
M_3	$\langle 0.61,0.28,0.10\rangle$	$\langle 0.57,0.37,0.06\rangle$	$\langle 0.59,0.28,0.12\rangle$	$\langle 0.57,0.33,0.10\rangle$	$\langle 0.61,0.35,0.04\rangle$	$\langle 0.59,0.28,0.13\rangle$
M_4	$\langle 0.63,0.32,0.05\rangle$	$\langle 0.59,0.39,0.02\rangle$	$\langle 0.58,0.25,0.17\rangle$	$\langle 0.59,0.33,0.08\rangle$	$\langle 0.57,0.35,0.08\rangle$	$\langle 0.59,0.25,0.16\rangle$

Table 6. The mass function of each house in Example 2.

Experts	o_1	o_2	o_3	o_4	o_5	o_6
M_1	$\langle 0.64,0.36,0.00\rangle$	$\langle 0.59,0.41,0.00\rangle$	$\langle 0.62,0.31,0.07\rangle$	$\langle 0.61,0.38,0.01\rangle$	$\langle 0.61,0.36,0.02\rangle$	$\langle 0.64,0.29,0.07\rangle$
M_2	$\langle 0.65,0.32,0.03\rangle$	$\langle 0.59,0.39,0.02\rangle$	$\langle 0.58,0.30,0.11\rangle$	$\langle 0.59,0.35,0.06\rangle$	$\langle 0.60,0.35,0.06\rangle$	$\langle 0.50,0.28,0.11\rangle$
M_3	$\langle 0.60,0.31,0.04\rangle$	$\langle 0.60,0.39,0.01\rangle$	$\langle 0.60,0.34,0.06\rangle$	$\langle 0.60,0.36,0.04\rangle$	$\langle 0.62,0.36,0.01\rangle$	$\langle 0.62,0.32,0.06\rangle$
M_4	$\langle 0.57,0.35,0.01\rangle$	$\langle 0.59,0.39,0.02\rangle$	$\langle 0.58,0.29,0.13\rangle$	$\langle 0.60,0.35,0.05\rangle$	$\langle 0.59,0.35,0.06\rangle$	$\langle 0.60,0.27,0.16\rangle$

Step 4-5 Revise the evidence above. Combine all the experts' evidence, we get the belief interval for each house:

$BI(o_1) = [0.947, 0.948]$, $BI(o_2) = [0.846, 0.847]$,
$BI(o_3) = [0.958, 0.963]$, $BI(o_4) = [0.904, 0.905]$,
$BI(o_5) = [0.889, 0.889]$, $BI(o_6) = [0.961, 0.956]$.

Step 6 Compare the belief interval of each house, we know that $o_6 > o_3 > o_1 > o_4 > o_5 > o_2$. Therefore, the consumer should buy the house o_6.

Example 2 In the example above, we only used Product adjoint pair to calculate the intuitionistic fuzzy upper and lower approximations. Assumes that the main consideration of the consumer is good education resources and community environment when buying a house. The consumer insists that the location of the house, the quality of the property service are more important than other factors. Thus, let

$$\tau_{a_i a_j} = \begin{cases} \text{Łukasiewicze}, & \text{if } i \in \{1,2\}; \\ \text{Product triple}, & otherwise. \end{cases}$$

Because the Product adjoint triple implication results in lower values and has more influence on the infimum in the lower approximation. According to the formulas (1) to (6), the belief function and plausibility function of each house in the intuitionistic fuzzy relation matrix given by the specialists are calculated. Translate them into mass functions, as shown in Table 6.

Calculate the discount factor of each specialist and revise the evidence above. Then using the same method to combine all the experts' evidence, we get the belief interval for each house:

$BI(o_1) = [0.934, 0.934]$, $BI(o_2) = [0.837, 0.837]$,
$BI(o_3) = [0.931, 0.932]$, $BI(o_4) = [0.885, 0.885]$,
$BI(o_5) = [0.893, 0.893]$, $BI(o_6) = [0.952, 0.953]$.

Comparing the belief interval of each house, we know that $o_6 > o_1 > o_3 > o_5 > o_4 > o_2$. Therefore, the consumer should buy the house o_6.

If the consumer puts the dwelling Environment quality in the first place, the quality of the property service and the building structure will be considered more than other factors. Thus, let

$$\tau_{a_i a_j} = \begin{cases} \text{Łukasiewicze}, & \text{if } i \in \{2,3\}; \\ \text{Product triple}, & otherwise. \end{cases}$$

Similarly, we get the following results $o_6 > o_3 > o_1 > o_4 > o_2 > o_5$. Therefore, the consumer should buy the house o_6.

4.3. Comparisons with other methods

In this subsection, we will compare the model with other literatures. In literature [27], the main MCDM ranking process is Max{choice-value} - Min{hesitation} - Max{score}. Now let us take a simulation.

Example 3 We use five different pairs of level value to rank the objects. For example, the $L(0.70, 0.30)$-level, the $L(mid, mid)$-level, the $L(top, bot)$-level, the $L(top, top)$-level, the $L(bot, bot)$-level. Compute the level soft set $L(s,t)$, as is shown in Table 7.

Table 7. The choice-value of different level soft set L(s,t).

O	L(0.70,0.30)	L(mid,mid)	L(top,bot)	L(top,top)	L(bot,bot)
o_1	4	2	0	1	1
o_2	3	2	0	0	0
o_3	3	2	1	2	1
o_4	3	2	1	2	1
o_5	3	3	0	0	1
o_6	2	2	2	2	2

Rank the objects according to the choice-value from the largest to the smallest. The result is as following.

$L(0.70, 0.30) : o_2 > o_1 = o_3 = o_4 = o_5 > o_6$;

$L(mid,mid) : o_5 > o_1 = o_2 = o_3 = o_4 = o_6$;
$L(top,bot) : o_6 > o_3 = o_4 > o_1 = o_2 = o_5$;
$L(top,top) : o_3 = o_4 = o_6 > o_1 > o_2 = o_5$;
$L(bot,bot) : o_6 > o_1 = o_3 = o_4 = o_5 > o_2$.

The alternatives cannot be sorted by the strict order from the choice-value. Thus, we have to continue to calculate the degree-hesitation of the objects, as is shown in Table 8.

Table 8. The degree-hesitation of objects.

O	o_1	o_2	o_3	o_4	o_5	o_6
degree-hesitation	0.17	0.12	0.36	0.27	0.33	0.41

Using the Min{hesitation} principle based on the first principle, the result is as follows.

$L(0.70,0.30) : o_2 > o_1 > o_4 > o_5 > o_3 > o_6$;
$L(mid,mid) : o_5 > o_2 > o_1 > o_4 > o_3 > o_6$;
$L(top,bot) : o_6 > o_4 > o_3 > o_2 > o_1 > o_5$;
$L(top,top) : o_4 > o_3 > o_6 > o_1 > o_2 > o_5$;
$L(bot,bot) : o_6 > o_1 > o_4 > o_5 > o_3 > o_2$.

Now, we rank the objects in a strict order completely in any level value. From the form of the results, both models can get a complete ordering of candidates, and also get the optimal object.

It is easy to see from the Table 3 that $\mathbf{A}_3 \subseteq \mathbf{A}_6$, we should get $o_6 > o_3$ in the whole order. However, we get $o_3 > o_6$ in complete ranking order on the level value of $L(0.70,0.30)$, $L(mid,mid)$, and $L(top,top)$. And the intuitionistic fuzzy numbers of o_1 is lager than o_2 according to the first, second and forth attributes, and the third nearly the same. we should get $o_1 > o_2$ in the whole order. However, we get $o_2 > o_1$ in complete ranking order on the level value of $L(0.70,0.30)$, $L(mid,mid)$, and $L(top,bot)$. Thus, the results obtained by the method in literature[27] are not very accurate.

The reasons for the above problems are the personal limitations of the decision-makers. Therefore, it is necessary to consult relevant experts when making decisions. By consulting relevant experts, the decision-makers can understand the decision alternatives more deeply. From the intuitionistic fuzzy relation matrix D^k, we can get the weight of each attribute according to the kth expert. This method can effectively avoid the error caused by a given weight. Moreover, by using the different adjoint operators, the decision-maker can express different preferences among attributes or decision parameters. In other words, the research work in this paper is more accurate and generalized.

5. Conclusions and further study

With respect to the problem of multiple attribute group decision-making in intuitionistic fuzzy information system, multi-adjoint theory and D-S theory have been first used in this paper. An approach to group decision-making in intuitionistic fuzzy information system based on multi-adjoint and evidence theory is proposed. Firstly, using multi-adjoint intuitionistic fuzzy approximation operators and probability formula, we obtain the belief function and plausibility function of objects. Then, the D-S theory is used to combine all experts' evidence to obtain the belief intervals. Ranking the objects by comparing the belief interval, the optimal scheme is acquired. Finally, the feasibility and effectiveness of this method is verified by an example. The multi-adjoint theory and D-S theory are applied to intuitionistic fuzzy group decision, which can effectively enhance the ability of the model to deal with intuitionistic fuzzy decision-making problem. At the same time, different use of adjoint triples increases the flexibility to deal with fuzzy decision-making problems.

In the future work, we shall continue to study the effects of general adjoint triples on solving the decision-making problem.

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