

Rough Number–Based Three-Way Group Decisions and Application in Influenza Emergency Management

Fan Jia^{*}, Peide Liu

School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan, Shandong, PR China

ARTICLE INFO

Article History

Received 03 Sep 2018

Accepted 17 Apr 2019

Keywords

Three-way decisions
Group decision-making
Rough numbers
Risk preference

ABSTRACT

Group decision-making can effectively deal with complex decision problems in reality and takes important research status in the field of decision-making. In recent years, three-way decision has been a hot topic in the field of uncertain decision-making, so the models of three-way decision under group environment have become a new direction and problem in decision-making research. Based on the subjectivity and uncertainty of the group judgment, this paper adopts the rough number method to objectively and effectively integrate the group information so as to establish rough number–based three-way decision models. First of all, we use rough numbers to transfer loss functions of individuals into a rough number–based gathering loss function, and verify the rough number–based loss function meets the general characteristics of general loss functions. Then from the perspectives of optimism, pessimism, risk preference of decision-makers, and directly operation of rough numbers, we explore the calculation of thresholds and the acquisition method of three-way rules, and then establish rough number–based three-way group decision models. Finally, influenza emergency management is introduced to verify the effectiveness and advancement of the novel method.

© 2019 The Authors. Published by Atlantis Press SARL.

This is an open access article distributed under the CC BY-NC 4.0 license (<http://creativecommons.org/licenses/by-nc/4.0/>).

1. INTRODUCTION

Three-way decision-making is a decision-making theory developed on the basis of the decision-theoretic rough set (DTRS). Ever since its introduction by the Canadian scholar Yao in 2009, it has attracted wide attention in academia. To provide a reasonable semantic explanation for the three regions divided by the upper and lower approximate sets in the rough set theory, Yao corresponded the positive, boundary, and negative domains in the rough set model with three decision-making results: acceptance, noncommitment, and rejection [1,2]. The author also expanded the traditional two-way decision-making model with only two outputs (acceptance and rejection) into a three-way model, which more accurately describes the characteristics of human thinking and behavior. Three-way decision-making frequently occurs in real life. For example, a manuscript review by experts based on two-way decision-making leads to one of two results: acceptance or rejection. In the actual review process, however, most manuscripts cannot be directly categorized into the above two results. Instead, a third decision-making result, noncommitment, is selected, in which further revision required. This gives rise to a three-way decision-making problem. Three-way decision-making is proposed based on the idea of the DTRS. The DTRS starts with Bayes' theorem and uses loss functions to describe the risks in decision-making. It selects the decision-making behavior with the minimum risk in response to the expected risks of three-way decision-making results,

namely, acceptance, delayed decision-making, and rejection. Two important elements in the DTRS are the conditional probability and the threshold. The final decision-making behavior is determined by comparing their magnitude. Three-way decision-making has already been widely applied in many areas, such as investment decision-making [3,4,5], conflict analysis [6], face recognition [7,8], email filtering [9], medical diagnosis [10,11], and recommendation systems [12,13].

The greatest advantage of three-way decision-making over traditional decision-making methods is that it can reduce the loss caused by incorrect classification. There are many decision-making problems in real life. Once an incorrect decision is made, the resulting loss may be immeasurable, especially for emergencies. In complicated decision-making situations with short decision-making time, incorrect decision-making often results in a series of adverse consequences. The management of an influenza epidemic situation is a typical emergency management event. In recent years, a variety of influenza viruses have caused outbreaks one after another, for example, H1N1. There are both correct and incorrect judgments on influenza epidemic situations. A correct judgment can control the development of the epidemic situation, whereas an incorrect judgment can easily lead to a large-scale influenza outbreak or social disorder. Figure 1 shows four results caused by different decisions aiming at whether the influenza outbreak or not. Suppose there are several influenza cases in a certain city, and the health department needs to determine whether or not this is a harbinger of an influenza outbreak based on the current situation and whether or not preventive measures should be implemented, such as issuing influenza

^{*}Corresponding author. Email: fanta07@126.com

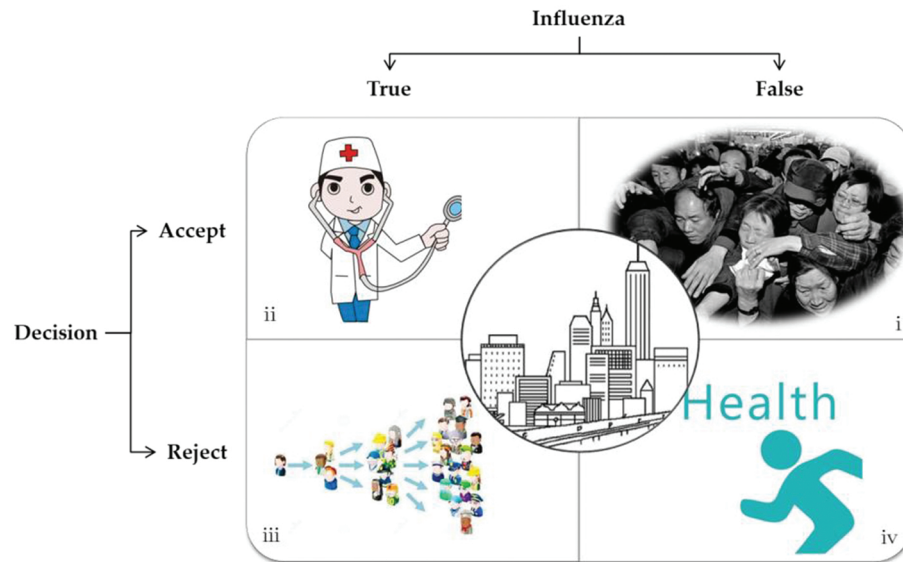


Figure 1 | Decision results for influenza emergency management.

warnings or deploying isolation procedures. The quadrant (ii) and (iv) depict the correct decisions, where quadrant (ii) represents that the city is suffering the influenza and the government decides to take protection actions to reduce the losses, and quadrant (iv) represents that no influenza outbreaks in this city and the government takes no action. While the quadrant (i) and (iii) demonstrate the mistakes in decision-making: the quadrant (iii) shows the type I error (rejection of a true null hypothesis), which means it is currently at the stage of an influenza outbreak, but the government has not taken any measures, which will result in spreading of influenza and more cases; the quadrant (i) shows the type II error (acceptance of a false null hypothesis), meaning that the current situation will not cause an influenza outbreak, but the government takes actions like issuing influenza warnings and deploying isolation procedures, which will cause the general public panic and disturb the normal functioning of society, and even social chaos. In particular, with the current development of the Internet, information spreads much faster, easily leading to the spread of rumors. For instance, a few years ago, a prevailing rumor that H7N9 would outbreak in China caused people to rush out and buy Banlangen (the root of *Isatis indigotica*) and white vinegar. This resulted in social unrest and consequences even worse than the influenza outbreak itself. Therefore, there is a new research approach in which three-way decision-making is used to cope with practical decision-making problems, especially emergency management problems.

As an essential element of three-way decisions, the determination of loss functions is a key procedure and has attracted most attention. It is almost impossible to obtain exact losses by instruments or technologies, so in most realistic problems, losses need to be determined by related experts. For example, the losses of medical diagnosis problems need to be determined by doctors according to their knowledge and experience. While it is undeniable that there exists uncertainty and subjectivity in experts' opinions, therefore, many researches considered uncertainty and constructed three-way decision-making models based on different expression forms of losses. Liu and Liang described the loss using the interval number and constructed DTRS models based on the interval

number [14,15], which has provided ideas for later research on three-way decision-making on uncertainty information. Liu introduced randomness into the loss function and constructed a random three-way decision-making model, the relevant characteristics of which were explored based on uniform distributions and normal distributions separately [5]. Subsequently, Liang *et al.* expanded the three-way decision-making model to fuzzy cases in which the loss function is expressed in the forms of triangular fuzzy numbers [16], intuitionistic fuzzy numbers [17], hesitant fuzzy numbers [18], and other data types. Corresponding three-way decision-making models were constructed, and the threshold calculation method was given to make the optimal decision. Liang *et al.* used linguistic terms to evaluate the selection and constructed a linguistic three-way decision-making method [19]. Three-way model was introduced into information table with order relation, and then the ordered three-way decisions were constructed as well as the algorithm [20].

As the complexity of decision environment and the limitation of human cognition, opinion of single decision-maker could not describe losses correctly. Therefore, to express losses in a comprehensive way, a few researches have introduced group decision-making approaches to three-way decisions. In traditional group decision-making methods, different aggregating approaches have been discussed according to different forms of evaluating values, some of which have been applied into three-way decision problems. By using the idea of granular computing, Liang aggregated and expressed the loss function as interval numbers and thus established a three-way group decision-making model [21]. After that, Liang expanded group decision-making to a loss function of linguistic value and constructed a three-way group decision-making model based on linguistic evaluation information [19]. Sun constructed a three-way group decision-making model based on a multigranularity fuzzy DTRS [22]. Liu constructed a three-way group decision-making model by taking the decision-makers' attitudes and preferences into consideration and integrating these with the prospect theory [23]. By aggregating group information, an aggregated loss function can be found, and then the thresholds. The conditional probability is always observed by historical experience, and

then decision rules can be obtained through comparing conditional probability and thresholds.

Although there exist some studies about three-way group decision-making models, the research for group decision in three-way models is still in the exploratory stage. According to the discussion above, all the approaches for group aggregation in three-way decisions ignores the intrinsic links among evaluation information from decision-makers. Zhai proposed a novel approach, the rough number, which is proposed to quantify experts' cognition based on rough set theory, and can be used for group aggregation by considering the intrinsic relationship among decision-makers [24,25]. Therefore, this paper intends to discuss three-way group decision-making models in the novel perspective of rough number. A rough number is represented as an interval numbers formed by upper and lower limits. It does not require any prior information, but it is determined completely based on the original data. The rough number method can comprehensively take the evaluations from all experts and integrate decisions made by individual experts into a group decision to ensure the objectivity and accuracy of the data and results. The rough number method was originally applied in the development of new products initiated by quality function. Zhai used the rough number method to effectively compile subjective evaluations and judgments of consumers and integrated individual evaluation values into rough numbers, which were then employed for data mining and decision-making [25]. Subsequently, many scholars have integrated the rough number method with the multicriteria decision-making method and have obtained decision-making models, such as rough number AHP (analytic hierarchy process); [26], rough number BWM (best-worst method); [27], and rough number TOPSIS (technique for order preference by similarity to an ideal solution) [28]. Since its introduction, the rough number method has been widely used as an effective way of assembling group information.

Therefore, the aim of this research is to construct a novel three-way group decision-making model based on rough numbers, which can aggregate multiple experts' information in a reasonable way, obtain rough loss functions, and receive more effective three-way rules, and then be used in influenza emergency management problems. The remainder of this paper is organized as follows. Section 2 lists some basic concepts of three-way decisions with DTRSs and rough numbers. Section 3 proposes the construction process of group loss functions based on rough numbers. In section 4, according to group loss functions, we discuss the acquisition methods of three-way rules from four different perspectives. To illustrate the application in influenza emergency management, section 5 presents a practical case in China and discuss the effectiveness of the model. Section 6 concludes this paper and outlines the future work.

2. PRELIMINARIES

This section is composed of two subsections to review some preliminaries about three-way decisions with DTRSs and the method of rough numbers.

2.1. Three-Way Decisions Based on DTRSs

To illustrate the principle of rough sets, Pawlak defined the approximation space as $apr = (U, R)$, where U be a finite and nonempty set and R be an equivalence relation. Denote the partition of U induced by R as $[x]$. Then Yao proposed probabilistic rough sets (PRSs),

where for $\forall C \subseteq U$, the probabilistic lower and upper approximations of C can be defined as follows [1]:

$$\underline{apr}(C) = \{x \in U \mid \Pr(C \mid [x]) \geq \alpha\}, \quad (1)$$

$$\overline{apr}(C) = \{x \in U \mid \Pr(C \mid [x]) \geq \beta\}, \quad (2)$$

where $\Pr(C \mid [x])$ is the conditional probability of the classification expressed as

$$\Pr(C \mid [x]) = \frac{|[x] \cap C|}{|[x]|}, \quad (3)$$

and the thresholds α and β satisfy $0 \leq \beta < \alpha \leq 1$, where Equations (1) and (2) are degenerated into Pawlak approximations of classical rough sets when $\alpha = 1$ and $\beta = 0$ [2].

The PRSs divide the universe U into three regions: the positive region, the boundary region, and the negative region, which correspond to acceptance, the noncommitment, and the rejection, the actions of three-way decisions proposed by Yao [2], which can be expressed as

$$POS(C) = \{x \in U \mid \Pr(C \mid [x]) \geq \alpha\}, \quad (4)$$

$$BND(C) = \{x \in U \mid \beta < \Pr(C \mid [x]) < \alpha\}, \quad (5)$$

$$NEG(C) = \{x \in U \mid \Pr(C \mid [x]) \leq \beta\}, \quad (6)$$

In particular, the models can be degenerated to traditional two-way decisions when $\alpha = \beta$, and the positive region and negative region can be written as

$$POS_\gamma(C) = \{x \in U \mid \Pr(C \mid [x]) \geq \gamma\}, \quad (7)$$

$$NEG_\gamma(C) = \{x \in U \mid \Pr(C \mid [x]) < \gamma\}, \quad (8)$$

where $\gamma = \alpha = \beta$.

In order to give semantic interpretations of the thresholds and three regions in PRSs, Yao proposed DTRSs with the aid of Bayesian theory [29]. A DTRS model is composed of two states and three actions, which can be seen in Table 1. The set of states can be expressed as $\Omega = \{C, \neg C\}$, which presents that an object is either in C or not in C . The set of actions is denoted as $A = \{a_P, a_B, a_N\}$, in which a_P , a_B , and a_N denote $x \in POS(C)$, $x \in BND(C)$, and $x \in NEG(C)$, respectively. The loss function regarding the risks or losses of different actions is given by a 3×2 matrix exhibited in Table 1. λ_{PP} , λ_{BP} , and λ_{NP} indicate the losses of taking actions a_P , a_B , and a_N , respectively, when the object belongs to C , while λ_{PN} , λ_{BN} , and λ_{NN} denote the losses incurred for taking actions a_P , a_B , and a_N , respectively, when the object belongs to $\neg C$.

Table 1 | The loss functions.

	$C(P)$	$\neg C(N)$
a_P	λ_{PP}	λ_{PN}
a_B	λ_{BP}	λ_{BN}
a_N	λ_{NP}	λ_{NN}

For $\forall x \in U$, we denote $R(a_i \mid [x])$ as the expected loss when taking the action a_i ($i = P, B, N$), and it can be calculated as

$$R(a_P \mid [x]) = \lambda_{PP} \Pr(C \mid [x]) + \lambda_{PN} \Pr(\neg C \mid [x]), \quad (9)$$

$$R(a_B \mid [x]) = \lambda_{BP} \Pr(C \mid [x]) + \lambda_{BN} \Pr(\neg C \mid [x]), \quad (10)$$

$$R(a_N \mid [x]) = \lambda_{NP} \Pr(C \mid [x]) + \lambda_{NN} \Pr(\neg C \mid [x]), \quad (11)$$

where $\Pr(C|x)$ is interpreted as the conditional probability of an object x belonging to C given that the object is described by its equivalence class $[x]$, and $\Pr(\neg C|x)$ is interpreted as the conditional probability of an object x belonging to $\neg C$ given that the object is described by its equivalence class $[x]$.

According to Bayesian theory, the best decision is the one with minimum cost, so decision rules can be indicated as follows:

- (P) If $R(a_P|[x]) \leq R(a_B|[x])$ and $R(a_P|[x]) \leq R(a_N|[x])$; decide $x \in POS(C)$,
- (B) If $R(a_B|[x]) \leq R(a_P|[x])$ and $R(a_B|[x]) \leq R(a_N|[x])$; decide $x \in BND(C)$,
- (N) If $R(a_N|[x]) \leq R(a_P|[x])$ and $R(a_N|[x]) \leq R(a_B|[x])$; decide $x \in NEG(C)$.

Since $\Pr(C|x) + \Pr(\neg C|x) = 1$, we can use $1 - \Pr(C|x)$ instead of $\Pr(\neg C|x)$ in Equations (9–11). As the losses satisfy the conditions that $\lambda_{PP} \leq \lambda_{BP} < \lambda_{NP}$ and $\lambda_{NN} \leq \lambda_{BN} < \lambda_{PN}$ by a reasonable semantic interpretation, we can obtain the simplified rules (P')-(N') as [2]

- (P') If $\Pr(C|x) \geq \alpha$ and $\Pr(C|x) \geq \gamma$; decide $x \in POS(C)$,
- (B') If $\Pr(C|x) \leq \alpha$ and $\Pr(C|x) \geq \beta$; decide $x \in BND(C)$,
- (N') If $\Pr(C|x) \leq \beta$ and $\Pr(C|x) \leq \gamma$; decide $x \in NEG(C)$,

where the thresholds α, β, γ are given as

$$\alpha = \frac{(\lambda_{PN} - \lambda_{BN})}{(\lambda_{PN} - \lambda_{BN}) + (\lambda_{BP} - \lambda_{PP})}, \quad (12)$$

$$\beta = \frac{(\lambda_{BN} - \lambda_{NN})}{(\lambda_{BN} - \lambda_{NN}) + (\lambda_{NP} - \lambda_{BP})}, \quad (13)$$

$$\gamma = \frac{(\lambda_{PN} - \lambda_{NN})}{(\lambda_{PN} - \lambda_{NN}) + (\lambda_{NP} - \lambda_{PP})}, \quad (14)$$

Suppose there exists an assumption

$$(\lambda_{PN} - \lambda_{BN})(\lambda_{NP} - \lambda_{BP}) > (\lambda_{BP} - \lambda_{PP})(\lambda_{BN} - \lambda_{NN}), \quad (15)$$

then the thresholds satisfy $0 \leq \beta < \gamma < \alpha \leq 1$, so the rules (P')-(N') can be further simplified

- (P'') If $\Pr(C|x) \geq \alpha$, decide $x \in POS(C)$;
- (B'') If $\beta < \Pr(C|x) < \alpha$, decide $x \in BND(C)$;
- (N'') If $\Pr(C|x) \leq \beta$, decide $x \in NEG(C)$.

On the contrary, if the assumption is

$$(\lambda_{PN} - \lambda_{BN})(\lambda_{NP} - \lambda_{BP}) \leq (\lambda_{BP} - \lambda_{PP})(\lambda_{BN} - \lambda_{NN}), \quad (16)$$

then the thresholds satisfy $0 \leq \alpha \leq \gamma \leq \beta \leq 1$, and (B') is not working in this case because there is no $\Pr(C|x)$ satisfying $\Pr(C|x) \leq \alpha$ and $\Pr(C|x) \leq \beta$ simultaneously. So the rules can be rewritten as a two-way decision rules

- (P''') If $\Pr(C|x) \geq \gamma$, decide $x \in POS(C)$,
- (N''') If $\Pr(C|x) < \gamma$, decide $x \in NEG(C)$.

The rules (P'')-(N'') and (P''')-(N''') indicate that three-way decisions must satisfy the condition Equation (16) or $0 \leq \beta < \gamma < \alpha \leq 1$, while two-way decisions satisfy the condition Equation (16) or $0 \leq \alpha \leq \gamma \leq \beta \leq 1$.

2.2. Rough Numbers

Inspired by rough set theory, rough number is firstly proposed by Zhai [24] in order to handle subjective preferences of customers in quality function deployment. Similar to the notion of approximates in rough sets, a rough number is constructed by lower and upper limits, which determine a rough boundary interval to characterize imprecise information. Rough numbers merely depend on original data without any prior knowledge, so they can capture the experts' real perception effectively, and aggregate every individual's preference into an objective and consistent group judgement. In this subsection, we review some basic concepts of rough numbers.

Suppose U is the universe containing all the objects, and there are n classes expressed as $R = \{C_1, C_2, \dots, C_n\}$. If they are ordered as $C_1 < C_2 < \dots < C_n$, then for $\forall Y \in U, \forall C_i \in R$, the lower approximation $\underline{Apr}(C_i)$ and the upper approximation $\overline{Apr}(C_i)$ of C_i can be defined as

$$\underline{Apr}(C_i) = \cup \{Y \in U/R(Y) \leq C_i\}, \quad (17)$$

$$\overline{Apr}(C_i) = \cup \{Y \in U/R(Y) \geq C_i\}, \quad (18)$$

Then C_i can be transformed to a rough number $RN(C_i)$, which is expressed as

$$RN(C_i) = [C_i] = [\underline{RN}(C_i), \overline{RN}(C_i)], \quad (19)$$

where $\underline{RN}(C_i)$ and $\overline{RN}(C_i)$ are the lower limit and upper limit of C_i , and denoted as

$$\underline{RN}(C_i) = \frac{1}{M_L} \sum R(Y) | Y \in \underline{Apr}(C_i), \quad (20)$$

$$\overline{RN}(C_i) = \frac{1}{M_U} \sum R(Y) | Y \in \overline{Apr}(C_i), \quad (21)$$

where M_L and M_U are the numbers of objects contained in $\underline{Apr}(C_i)$ and $\overline{Apr}(C_i)$, respectively.

To compare the values of rough numbers, Zhai proposed the ranking rules [25]. For any two rough numbers $RN_1 = [\underline{RN}_1, \overline{RN}_1]$ and $RN_2 = [\underline{RN}_2, \overline{RN}_2]$, there are five possible cases, which are shown in Figure 2, where M_1 and M_2 are the medians of RN_1 and RN_2 , respectively.

For an easy-to-operate purpose, we illustrate the ranking rules in an accurate mathematical linguistics rather than a graphical representation. Denote $S_a = \underline{RN}_a + \overline{RN}_a$ as the sum of upper limit and lower limit for a rough number $RN_a = [\underline{RN}_a, \overline{RN}_a]$, and $D_a = \overline{RN}_a - \underline{RN}_a$ as the difference of upper limit and lower limit. For any two rough numbers $RN_1 = [\underline{RN}_1, \overline{RN}_1]$ and $RN_2 = [\underline{RN}_2, \overline{RN}_2]$, we can obtain the ranking rules as follows:

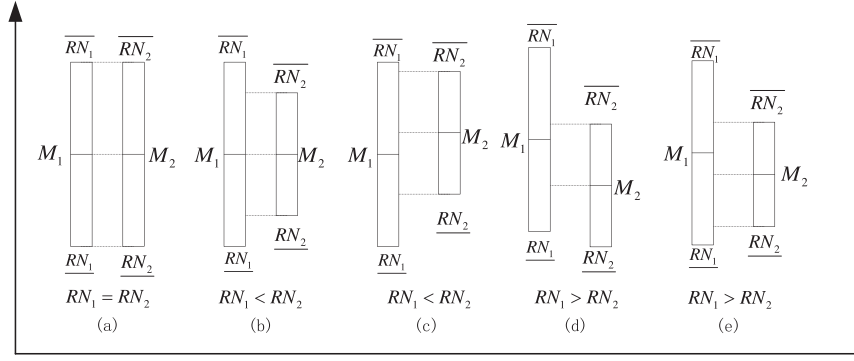


Figure 2 | Ranking rules for rough numbers.

- (a) If $S_1 > S_2$; then $RN_1 > RN_2$,
 (b) If $S_1 < S_2$; then $RN_1 < RN_2$,
 (c) If $S_1 = S_2$; then:
 i. if $D_1 < D_2$; then $RN_1 > RN_2$,
 ii. if $D_1 > D_2$; then $RN_1 < RN_2$,
 iii. if $D_1 = D_2$; then $RN_1 = RN_2$.

Where $S_a = \underline{RN}_a + \overline{RN}_a$ and $D_a = \overline{RN}_a - \underline{RN}_a$.

Based on the construction of S and D , we convert the graphical ranking rules to mathematical linguistics, which could facilitate the further research, and it is easy to find out that the two expression forms are equivalent.

For a convenient expression, $\underline{RN}(C_i)$, $\overline{RN}(C_i)$, and $RN(C_i)$ can be denoted as $\underline{C}_i$, $\overline{C}_i$, and $[C_i]$ for short, respectively.

Rough numbers are similar to interval numbers in form, so with the aid of arithmetic operations of interval analysis, Zhai proposed the operations of rough numbers [24]. Suppose $[a] = [\underline{a}, \overline{a}]$ and $[b] = [\underline{b}, \overline{b}]$ are two rough numbers and satisfy $\underline{a}, \overline{a}, \underline{b}, \overline{b} > 0$, and $\alpha > 0$ is a real number. Then the arithmetic operations of rough numbers can be expressed as follows:

$$[a] \times \alpha = \alpha \times [a] = [\alpha \times \underline{a}, \alpha \times \overline{a}], \quad (22)$$

$$[a] + [b] = [\underline{a} + \underline{b}, \overline{a} + \overline{b}], \quad (23)$$

$$[a] \times [b] = [\underline{a} \times \underline{b}, \overline{a} \times \overline{b}], \quad (24)$$

$$\frac{[a]}{[b]} = \left[\frac{\underline{a}}{\overline{b}}, \frac{\overline{a}}{\underline{b}} \right]. \quad (25)$$

3. THE AGGREGATION OF LOSS FUNCTIONS BASED ON ROUGH NUMBERS

3.1. Description of Three-Way Group Decision-Making

The most important issue of the three-way decision-making model is the determination of the loss function. For losses caused by incorrect classification, researchers usually use subjective experiences

and evaluation by relevant experts to determine the loss values. While due to the lack of knowledge and experience, and the limitation of individual subjective knowledge, individuals find it difficult to have systematic and comprehensive understanding of decision-making problems. Therefore, group decision-making has become an important approach in the field. In a group decision-making perspective, h experts can be selected to form a decision-making group, which is represented as $E = \{e_1, e_2, \dots, e_h\}$, among which e_k denotes the k th expert. Each expert makes a subjective assessment on various incorrect classifications, and h loss functions can be obtained. The loss function obtained by expert e_k through evaluation and judgement is shown in Table 2. The process of three-way group decision-making is to determine the aggregated group loss function based on the individual loss functions given by h individual experts and to describe the opinions of the decision-making group. The aggregated loss function is used to explore the threshold calculation method in three-way decision-making and the acquisition of three-way decision-making rules.

Table 2 | The loss function of e_k .

	$C(P)$	$\neg C(N)$
a_P	λ_{PP}^k	λ_{PN}^k
a_B	λ_{BP}^k	λ_{BN}^k
a_N	λ_{NP}^k	λ_{NN}^k

3.2. Rough Number-Based Loss Functions

By integrating the evaluation information from h experts, an evaluation summary table for the decision-making group can be obtained, as shown in Table 3.

Table 3 | The results of experts' evaluation.

E	λ_{PP}	λ_{BP}	λ_{NP}	λ_{PN}	λ_{BN}	λ_{NN}
e_1	λ_{PP}^1	λ_{BP}^1	λ_{NP}^1	λ_{PN}^1	λ_{BN}^1	λ_{NN}^1
e_2	λ_{PP}^2	λ_{BP}^2	λ_{NP}^2	λ_{PN}^2	λ_{BN}^2	λ_{NN}^2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
e_k	λ_{PP}^k	λ_{BP}^k	λ_{NP}^k	λ_{PN}^k	λ_{BN}^k	λ_{NN}^k
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
e_h	λ_{PP}^h	λ_{BP}^h	λ_{NP}^h	λ_{PN}^h	λ_{BN}^h	λ_{NN}^h

For any λ_{**} , there exist h evaluation results, meaning that the group evaluation result on the loss λ_{**} can be denoted as $\{\lambda_{**}^1, \lambda_{**}^2, \dots, \lambda_{**}^h\}$. With the help of the construction method of

rough number, the evaluating values can be aggregated to rough numbers, and then the loss function can be expressed by rough number-based values. By using Equations (17–19), each evaluation value λ_{**}^k can be converted and expressed as a rough number and represented as $[\lambda_{**}^k] = [\underline{\lambda}_{**}^k, \overline{\lambda}_{**}^k]$, where the values of $\underline{\lambda}_{**}^k$ and $\overline{\lambda}_{**}^k$ are related to the evaluation values of all decision-makers. For example, suppose five experts have evaluated the loss λ_{pp} as $\{2, 3, 3, 4, 2\}$, then we can convert each value to a rough number-based expression as

$$\begin{aligned} [\lambda_{pp}^1] &= [\lambda_{pp}^5] = [2] = [2, 2.8], \\ [\lambda_{pp}^2] &= [\lambda_{pp}^3] = [3] = [2.5, 3.33], \\ [\lambda_{pp}^4] &= [4] = [2.8, 4]. \end{aligned}$$

Therefore, the rough number-based evaluation values of λ_{pp} is $\{[2, 2.8], [2.5, 3.33], [2.5, 3.33], [2, 2.8], [2.8, 4]\}$.

By using Equations (20) and (21), aggregated loss function can be obtained and represented as $[\lambda_{**}] = [\underline{\lambda}_{**}, \overline{\lambda}_{**}]$. Table 4 shows the aggregated rough loss function obtained through the rough number method. For the above example, the aggregated rough number can be calculated as

$$\begin{aligned} \underline{\lambda}_{pp} &= \frac{1}{5} \times (2 + 2.5 + 2.5 + 2.8 + 2) = 2.36, \\ \overline{\lambda}_{pp} &= \frac{1}{5} \times (2.8 + 3.33 + 3.33 + 4 + 2.8) = 3.25. \end{aligned}$$

Table 4 | Rough loss function.

	$C(P)$	$\neg C(N)$
a_P	$[\lambda_{pp}] = [\underline{\lambda}_{pp}, \overline{\lambda}_{pp}]$	$[\lambda_{PN}] = [\underline{\lambda}_{PN}, \overline{\lambda}_{PN}]$
a_B	$[\lambda_{BP}] = [\underline{\lambda}_{BP}, \overline{\lambda}_{BP}]$	$[\lambda_{BN}] = [\underline{\lambda}_{BN}, \overline{\lambda}_{BN}]$
a_N	$[\lambda_{NP}] = [\underline{\lambda}_{NP}, \overline{\lambda}_{NP}]$	$[\lambda_{pp}] = [\underline{\lambda}_{pp}, \overline{\lambda}_{pp}]$

Therefore, the aggregated rough loss $[\lambda_{pp}]$ can be denoted as

$$[\lambda_{pp}] = [2.36, 3.25].$$

According to the discussion in section 2, three-way decisions must satisfy the conditions $\lambda_{pp} \leq \lambda_{BP} < \lambda_{NP}$ and $\lambda_{NN} \leq \lambda_{BN} < \lambda_{PN}$. In order to guarantee the effectiveness of the following research, we have to verify that rough loss functions should also satisfy the conditions.

Theorem 1. *If for any $k = 1, 2, \dots, h$ there exist $\lambda_{pp}^k \leq \lambda_{BP}^k < \lambda_{NP}^k$ and $\lambda_{NN}^k \leq \lambda_{BN}^k < \lambda_{PN}^k$, then we can obtain*

1. $\underline{\lambda}_{pp} \leq \underline{\lambda}_{BP} < \underline{\lambda}_{NP}, \underline{\lambda}_{NN} \leq \underline{\lambda}_{BN} < \underline{\lambda}_{PN},$
2. $\overline{\lambda}_{pp} \leq \overline{\lambda}_{BP} < \overline{\lambda}_{NP}, \overline{\lambda}_{NN} \leq \overline{\lambda}_{BN} < \overline{\lambda}_{PN},$
3. $[\lambda_{pp}] \leq [\lambda_{BP}] < [\lambda_{NP}], [\lambda_{NN}] \leq [\lambda_{BN}] < [\lambda_{PN}].$

Proof. Firstly, we prove $\underline{\lambda}_{pp} \leq \underline{\lambda}_{BP} < \underline{\lambda}_{NP}, \overline{\lambda}_{pp} \leq \overline{\lambda}_{BP} < \overline{\lambda}_{NP}$ and $[\lambda_{pp}] \leq [\lambda_{BP}] < [\lambda_{NP}]$ based on $\lambda_{pp}^k \leq \lambda_{BP}^k < \lambda_{NP}^k$.

It is obvious that there are $\underline{\lambda}_{pp} = \underline{\lambda}_{BP}$ and $\overline{\lambda}_{pp} = \overline{\lambda}_{BP}$ when $\lambda_{pp}^k = \lambda_{BP}^k$ for any $k = 1, 2, \dots, h$. Suppose there exists at least one decision-maker e_s such that $\lambda_{pp}^s < \lambda_{BP}^s$, then there must be

$[\lambda_{pp}^s] < [\lambda_{BP}^s]$ according to the construction process of rough numbers, and $[\lambda_{pp}^k] = [\lambda_{BP}^k]$ or $[\lambda_{pp}^k] < [\lambda_{BP}^k]$ for $k \neq s$. By Equations (20) and (21) there exist $\underline{\lambda}_{pp} < \underline{\lambda}_{BP}$ and $\overline{\lambda}_{pp} < \overline{\lambda}_{BP}$. So $\underline{\lambda}_{pp} \leq \underline{\lambda}_{BP}$ and $\overline{\lambda}_{pp} \leq \overline{\lambda}_{BP}$ are proved when $\lambda_{pp}^k \leq \lambda_{BP}^k$, and the equations are satisfied if and only if $\lambda_{pp}^k = \lambda_{BP}^k$ for any $k = 1, 2, \dots, h$. Similarly, if $\lambda_{BP}^k < \lambda_{NP}^k$ for any $k = 1, 2, \dots, h$, there are $\underline{\lambda}_{BP} < \underline{\lambda}_{NP}$ and $\overline{\lambda}_{BP} < \overline{\lambda}_{NP}$. Therefore, we can obtain that if $\lambda_{pp}^k \leq \lambda_{BP}^k < \lambda_{NP}^k$, there are $\underline{\lambda}_{pp} \leq \underline{\lambda}_{BP} < \underline{\lambda}_{NP}$ and $\overline{\lambda}_{pp} \leq \overline{\lambda}_{BP} < \overline{\lambda}_{NP}$. In the same way, we can prove the corresponding conclusion if $\lambda_{NN}^k \leq \lambda_{BN}^k < \lambda_{PN}^k$.

For $[\lambda_{pp}] \leq [\lambda_{BP}] < [\lambda_{NP}]$, we can prove $[\lambda_{pp}] \leq [\lambda_{BP}]$ firstly. According to the ranking rules of rough numbers proposed in Section 2, we should explore the relationship between $\underline{\lambda}_{pp} + \overline{\lambda}_{pp}$ and $\underline{\lambda}_{BP} + \overline{\lambda}_{BP}$. The discussion above has proved that if $\lambda_{pp}^k \leq \lambda_{BP}^k$ there are $\underline{\lambda}_{pp} \leq \underline{\lambda}_{BP}$ and $\overline{\lambda}_{pp} \leq \overline{\lambda}_{BP}$, so there must be $\underline{\lambda}_{pp} + \overline{\lambda}_{pp} \leq \underline{\lambda}_{BP} + \overline{\lambda}_{BP}$. As we know, there is $\underline{\lambda}_{pp} + \overline{\lambda}_{pp} = \underline{\lambda}_{BP} + \overline{\lambda}_{BP}$ if and only if $\lambda_{pp}^k = \lambda_{BP}^k$, so it is easy to obtain that if $\lambda_{pp}^k \leq \lambda_{BP}^k$ then $[\lambda_{pp}] \leq [\lambda_{BP}]$. Similarly, according to $\lambda_{BP}^k < \lambda_{NP}^k$ we can obtain $[\lambda_{BP}] < [\lambda_{NP}]$. So $[\lambda_{pp}] \leq [\lambda_{BP}] < [\lambda_{NP}]$ is proved.

According to the discussion, we can prove $\underline{\lambda}_{pp} \leq \underline{\lambda}_{BP} < \underline{\lambda}_{NP}, \overline{\lambda}_{pp} \leq \overline{\lambda}_{BP} < \overline{\lambda}_{NP}$, and $[\lambda_{pp}] \leq [\lambda_{BP}] < [\lambda_{NP}]$ based on $\lambda_{pp}^k \leq \lambda_{BP}^k < \lambda_{NP}^k$. And in the same way, we can also prove $\underline{\lambda}_{NN} \leq \underline{\lambda}_{BN} < \underline{\lambda}_{PN}, \overline{\lambda}_{NN} \leq \overline{\lambda}_{BN} < \overline{\lambda}_{PN}$, and $[\lambda_{NN}] \leq [\lambda_{BN}] < [\lambda_{PN}]$ based on $\lambda_{NN}^k \leq \lambda_{BN}^k < \lambda_{PN}^k$.

So the theorem has been proved.

Theorem 1 illustrates that the aggregated loss functions based on rough numbers still satisfy the conditions in classical three-way decisions, which guarantee the validity and reasonability of the further research.

4. THREE-WAY GROUP DECISION MODELS BASED ON ROUGH LOSS FUNCTIONS

By using rough numbers to aggregate the information of decision-makers, the rough loss function, as shown in Table 4, has been obtained, wherein the losses are expressed as rough numbers. In this section, we intend to explore how thresholds, rules, and classifications are obtained in group three-way decisions based on rough loss functions. In consideration of the risk preferences of decision-makers, this section explores decision-making strategies from three perspectives, namely optimism, pessimism, and risk preference coefficient. Then a direct method to acquire the threshold and three-way rule is given based on the algorithm of rough numbers without taking into account the decision-maker's risk preference.

4.1. Three-Way Group Decision Models with Risk Preference

4.1.1. Model 1: An optimistic perspective

The optimistic perspective refers to a situation in which decision-makers believe that the loss caused by incorrect classification is the smallest. Therefore, only the lower limits of rough losses $\underline{\lambda}_{**}$ are taken as the final decision-making losses. The optimistic loss function can be shown in Table 5.

Table 5 Optimistic loss function.

	$C(P)$	$\neg C(N)$
a_P	$\underline{\lambda}_{PP}$	$\underline{\lambda}_{PN}$
a_B	$\underline{\lambda}_{BP}$	$\underline{\lambda}_{BN}$
a_N	$\underline{\lambda}_{NP}$	$\underline{\lambda}_{NN}$

According to the acquisition procedure of three-way decision rules, the expected losses of different actions a_i under optimistic perspective can be denoted as

$$R_o(a_P|x) = \underline{\lambda}_{PP} \Pr(C|x) + \underline{\lambda}_{PN} \Pr(\neg C|x), \quad (26)$$

$$R_o(a_B|x) = \underline{\lambda}_{BP} \Pr(C|x) + \underline{\lambda}_{BN} \Pr(\neg C|x), \quad (27)$$

$$R_o(a_N|x) = \underline{\lambda}_{NP} \Pr(C|x) + \underline{\lambda}_{NN} \Pr(\neg C|x). \quad (28)$$

According to Bayesian theory, the optimistic decision rules can be indicated as follows:

- (P_o) If $R_o(a_P|x) \leq R_o(a_B|x)$ and $R_o(a_P|x) \leq R_o(a_N|x)$, decide $x \in POS(C)$,
- (B_o) If $R_o(a_B|x) \leq R_o(a_P|x)$ and $R_o(a_B|x) \leq R_o(a_N|x)$, decide $x \in BND(C)$,
- (N_o) If $R_o(a_N|x) \leq R_o(a_P|x)$ and $R_o(a_N|x) \leq R_o(a_B|x)$, decide $x \in NEG(C)$.

Based on the properties shown in Theorem 1, we can obtain that $\underline{\lambda}_{PP} \leq \underline{\lambda}_{BP} < \underline{\lambda}_{NP}$ and $\underline{\lambda}_{NN} \leq \underline{\lambda}_{BN} < \underline{\lambda}_{PN}$. Then because of $\Pr(C|x) + \Pr(\neg C|x) = 1$, simplified decision rules can be obtained as follows:

- (P'_o) If $\Pr(C|x) \geq \alpha_o$ and $\Pr(C|x) \geq \gamma_o$, decide $x \in POS(C)$,
- (B'_o) If $\Pr(C|x) \leq \alpha_o$ and $\Pr(C|x) \geq \beta_o$, decide $x \in BND(C)$,
- (N'_o) If $\Pr(C|x) \leq \beta_o$ and $\Pr(C|x) \leq \gamma_o$, decide $x \in NEG(C)$,

where

$$\alpha_o = \frac{(\underline{\lambda}_{PN} - \underline{\lambda}_{BN})}{(\underline{\lambda}_{PN} - \underline{\lambda}_{BN}) + (\underline{\lambda}_{BP} - \underline{\lambda}_{PP})}, \quad (29)$$

$$\beta_o = \frac{(\underline{\lambda}_{BN} - \underline{\lambda}_{NN})}{(\underline{\lambda}_{BN} - \underline{\lambda}_{NN}) + (\underline{\lambda}_{NP} - \underline{\lambda}_{BP})}, \quad (30)$$

$$\gamma_o = \frac{(\underline{\lambda}_{PN} - \underline{\lambda}_{NN})}{(\underline{\lambda}_{PN} - \underline{\lambda}_{NN}) + (\underline{\lambda}_{NP} - \underline{\lambda}_{PP})}. \quad (31)$$

Example 1. Suppose in a group decision-making problem, we have obtained the aggregated rough loss function, and the losses of misclassifications are expressed as $[\lambda_{PP}] = [0.2, 0.6]$, $[\lambda_{BP}] = [0.7, 1]$, $[\lambda_{NP}] = [1.5, 2.1]$, $[\lambda_{PN}] = [2.8, 3.4]$, $[\lambda_{BN}] = [1.7, 2.5]$, and $[\lambda_{NN}] = [0.3, 0.8]$. Based on the optimistic perspective, we choose

the lower limits as the final losses, so the losses can be written as $\lambda_{PP} = 0.2$, $\lambda_{BP} = 0.7$, $\lambda_{NP} = 1.5$, $\lambda_{PN} = 2.8$, $\lambda_{BN} = 1.7$, and $\lambda_{NN} = 0.3$. According to Equations (29–31), we can calculate the thresholds as $\alpha = 0.687$, $\beta = 0.636$, and $\gamma = 0.658$.

4.1.2. Model 2: A pessimistic perspective

The pessimistic perspective means that the decision-maker believes that the loss caused by incorrect classification is the greatest. Hence, for the rough number loss function, only the upper limits of the rough losses $\overline{\lambda}_{**}$ are taken as the final losses in decision-making. The pessimistic loss function is shown in Table 6.

Table 6 Pessimistic loss function.

	$C(P)$	$\neg C(N)$
a_P	$\overline{\lambda}_{PP}$	$\overline{\lambda}_{PN}$
a_B	$\overline{\lambda}_{BP}$	$\overline{\lambda}_{BN}$
a_N	$\overline{\lambda}_{NP}$	$\overline{\lambda}_{NN}$

Based on the acquisition procedure of three-way decision rules, the expected losses when choosing different actions a_i under pessimistic perspective can be expressed as

$$R_p(a_P|x) = \overline{\lambda}_{PP} \Pr(C|x) + \overline{\lambda}_{PN} \Pr(\neg C|x), \quad (32)$$

$$R_p(a_B|x) = \overline{\lambda}_{BP} \Pr(C|x) + \overline{\lambda}_{BN} \Pr(\neg C|x), \quad (33)$$

$$R_p(a_N|x) = \overline{\lambda}_{NP} \Pr(C|x) + \overline{\lambda}_{NN} \Pr(\neg C|x). \quad (34)$$

According to Bayesian theory, the pessimistic decision rules can be denoted as follows:

- (P_p) If $R_p(a_P|x) \leq R_p(a_B|x)$ and $R_p(a_P|x) \leq R_p(a_N|x)$, decide $x \in POS(C)$,
- (B_p) If $R_p(a_B|x) \leq R_p(a_P|x)$ and $R_p(a_B|x) \leq R_p(a_N|x)$, decide $x \in BND(C)$,
- (N_p) If $R_p(a_N|x) \leq R_p(a_P|x)$ and $R_p(a_N|x) \leq R_p(a_B|x)$, decide $x \in NEG(C)$.

From Theorem 1, we can obtain that there exist $\overline{\lambda}_{PP} \leq \overline{\lambda}_{BP} < \overline{\lambda}_{NP}$ and $\overline{\lambda}_{NN} \leq \overline{\lambda}_{BN} < \overline{\lambda}_{PN}$. Then according to $\Pr(C|x) + \Pr(\neg C|x) = 1$ we can get the simplified decision rules as

- (P'_p) If $\Pr(C|x) \geq \alpha_p$ and $\Pr(C|x) \geq \gamma_p$, decide $x \in POS(C)$,
- (B'_p) If $\Pr(C|x) \leq \alpha_p$ and $\Pr(C|x) \geq \beta_p$, decide $x \in BND(C)$,
- (N'_p) If $\Pr(C|x) \leq \beta_p$ and $\Pr(C|x) \leq \gamma_p$, decide $x \in NEG(C)$,

where

$$\alpha_p = \frac{(\overline{\lambda}_{PN} - \overline{\lambda}_{BN})}{(\overline{\lambda}_{PN} - \overline{\lambda}_{BN}) + (\overline{\lambda}_{BP} - \overline{\lambda}_{PP})}, \quad (35)$$

$$\beta_p = \frac{(\overline{\lambda_{BN}} - \overline{\lambda_{NN}})}{(\overline{\lambda_{BN}} - \overline{\lambda_{NN}}) + (\overline{\lambda_{NP}} - \overline{\lambda_{BP}})}, \quad (36)$$

$$\gamma_p = \frac{(\overline{\lambda_{PN}} - \overline{\lambda_{NN}})}{(\overline{\lambda_{PN}} - \overline{\lambda_{NN}}) + (\overline{\lambda_{NP}} - \overline{\lambda_{PP}})}. \quad (37)$$

Example 2. Continue the Example 1. Based on the pessimistic viewpoint, we determine the upper limits of rough numbers as the final losses, which can be written as $\lambda_{PP} = 0.6$, $\lambda_{BP} = 1$, $\lambda_{NP} = 2.1$, $\lambda_{PN} = 3.4$, $\lambda_{BN} = 2.5$, and $\lambda_{NN} = 0.8$. And according to Equations (35–37), the thresholds under pessimistic perspective can be calculated as $\alpha = 0.692$, $\beta = 0.607$, and $\gamma = 0.634$.

4.1.3. Model 3: Risk preference coefficient

For many practical problems, the risk preferences of decision-makers are neither pessimistic nor optimistic, and they may be between the two instead. In such a case, a coefficient can be employed to describe the risk preference so that the losses in three-way decisions can be determined.

Definition 1. For any rough number $[a] = [\underline{a}, \overline{a}]$, let $\theta \in [0, 1]$, and the rough number can be converted into

$$\theta([a]) = (1 - \theta)\underline{a} + \theta\overline{a}, \quad (38)$$

where θ is the risk preference coefficient of decision-makers and $\theta([a])$ is a conversion from a rough number $[a]$ to a real number.

From Definition 1, we know that $\theta([a])$ is a real number. Therefore, it is possible to use this conversion method to convert rough numbers into real numbers. The greater the risk preference coefficient θ is, the more the decision-maker prefers a larger value. On the contrary, the smaller the coefficient θ is, the more the decision-maker prefers a smaller value. The cases $\theta = 0$ and $\theta = 1$, respectively, represent two special situations corresponding to the previously mentioned optimistic and pessimistic perspectives. Thus, the model based on risk preference coefficient is an extension of the optimistic and pessimistic models.

With the risk preference coefficient, it is possible to determine the loss function expressed using real numbers, which are shown in Table 7.

Table 7 | Loss function based on θ .

	$C(P)$	$\neg C(N)$
a_P	$\theta([\lambda_{PP}])$	$\theta([\lambda_{PN}])$
a_B	$\theta([\lambda_{BP}])$	$\theta([\lambda_{BN}])$
a_N	$\theta([\lambda_{NP}])$	$\theta([\lambda_{NN}])$

Where $\theta([\lambda_{**}]) = (1 - \theta)\underline{\lambda_{**}} + \theta\overline{\lambda_{**}}$ represents the loss value based on the risk preference coefficient θ .

Based on the loss function derived from risk preference coefficient, the expected losses of different actions a_i can be denoted as

$$R_\theta(a_P|x) = \theta([\lambda_{PP}]) \Pr(C|x) + \theta([\lambda_{PN}]) \Pr(\neg C|x), \quad (39)$$

$$R_\theta(a_B|x) = \theta([\lambda_{BP}]) \Pr(C|x) + \theta([\lambda_{BN}]) \Pr(\neg C|x), \quad (40)$$

$$R_\theta(a_N|x) = \theta([\lambda_{NP}]) \Pr(C|x) + \theta([\lambda_{NN}]) \Pr(\neg C|x). \quad (41)$$

According to Bayesian theory, the decision rules under risk preference coefficient model can be expressed as follows:

- (P_θ) If $R_\theta(a_P|x) \leq R_\theta(a_B|x)$ and $R_\theta(a_P|x) \leq R_\theta(a_N|x)$, decide $x \in POS(C)$,
- (B_θ) If $R_\theta(a_B|x) \leq R_\theta(a_P|x)$ and $R_\theta(a_B|x) \leq R_\theta(a_N|x)$, decide $x \in BND(C)$,
- (N_θ) If $R_\theta(a_N|x) \leq R_\theta(a_P|x)$ and $R_\theta(a_N|x) \leq R_\theta(a_B|x)$, decide $x \in NEG(C)$.

Theorem 1 illustrates that $[\lambda_{PP}] \leq [\lambda_{BP}] < [\lambda_{NP}]$ and $[\lambda_{NN}] \leq [\lambda_{BN}] < [\lambda_{PN}]$, and based on the form of risk preference coefficient shown in Equation (38), there are

$$\theta([\lambda_{PP}]) \leq \theta([\lambda_{BP}]) < \theta([\lambda_{NP}]), \quad (42)$$

$$\theta([\lambda_{NN}]) \leq \theta([\lambda_{BN}]) < \theta([\lambda_{PN}]), \quad (43)$$

so according to $\Pr(C|x) + \Pr(\neg C|x) = 1$, we can obtain the simplified decision rules as follows:

- (P'_θ) If $\Pr(C|x) \geq \alpha_\theta$ and $\Pr(C|x) \geq \gamma_\theta$, decide $x \in POS(C)$,
- (B'_θ) If $\Pr(C|x) \leq \alpha_\theta$ and $\Pr(C|x) \geq \beta_\theta$, decide $x \in BND(C)$,
- (N'_θ) If $\Pr(C|x) \leq \beta_\theta$ and $\Pr(C|x) \leq \gamma_\theta$, decide $x \in NEG(C)$.

Where

$$\alpha_\theta = \frac{(\theta([\lambda_{PN}]) - \theta([\lambda_{BN}]))}{(\theta([\lambda_{PN}]) - \theta([\lambda_{BN}])) + (\theta([\lambda_{BP}]) - \theta([\lambda_{PP}]))}, \quad (44)$$

$$\beta_\theta = \frac{(\theta([\lambda_{BN}]) - \theta([\lambda_{NN}]))}{(\theta([\lambda_{BN}]) - \theta([\lambda_{NN}])) + (\theta([\lambda_{NP}]) - \theta([\lambda_{BP}]))}, \quad (45)$$

$$\gamma_\theta = \frac{(\theta([\lambda_{PN}]) - \theta([\lambda_{NN}]))}{(\theta([\lambda_{PN}]) - \theta([\lambda_{NN}])) + (\theta([\lambda_{NP}]) - \theta([\lambda_{PP}]))}. \quad (46)$$

Example 3. We continue to use the data in Example 1. Denote the risk preference coefficient as $\theta = 0.7$, meaning that the decision-makers intend to relatively pessimistic individuals with the degree of 0.7. So based on Equation (38) we can calculate the losses as $\lambda_{PP} = 0.48$, $\lambda_{BP} = 0.91$, $\lambda_{NP} = 1.92$, $\lambda_{PN} = 3.22$, $\lambda_{BN} = 2.26$, and $\lambda_{NN} = 0.65$. And according to Equations (44–46), the thresholds of risk preference coefficient model can be calculated as $\alpha = 0.691$, $\beta = 0.615$, and $\gamma = 0.641$.

4.2. Three-Way Group Decision Model Based on Rough Numbers

The models discussed above are all based on the viewpoint that converting rough numbers into real numbers. And in this section we intend to solve rough-number based three-way decisions in a direct way. According to the rough loss function shown in Table 4, we can obtain three-way decision rules by utilizing the arithmetic operations and ranking rules of rough numbers. The specific procedure is shown as follows.

Based on rough loss function in Table 4, the expected losses of different actions a_i can be denoted as

$$[R(a_p|x)] = [\lambda_{pp}] \Pr(C|x) + [\lambda_{pn}] \Pr(\neg C|x),$$

$$[R(a_b|x)] = [\lambda_{bp}] \Pr(C|x) + [\lambda_{bn}] \Pr(\neg C|x),$$

$$[R(a_n|x)] = [\lambda_{np}] \Pr(C|x) + [\lambda_{nn}] \Pr(\neg C|x).$$

According to the arithmetic operations of rough numbers, we can obtain the expected rough losses as

$$[R(a_p|x)] = [\lambda_{pp} \Pr(C|x) + \lambda_{pn} \Pr(\neg C|x), \lambda_{pp} \Pr(C|x) + \lambda_{pn} \Pr(\neg C|x)], \quad (47)$$

$$[R(a_b|x)] = [\lambda_{bp} \Pr(C|x) + \lambda_{bn} \Pr(\neg C|x), \lambda_{bp} \Pr(C|x) + \lambda_{bn} \Pr(\neg C|x)], \quad (48)$$

$$[R(a_n|x)] = [\lambda_{np} \Pr(C|x) + \lambda_{nn} \Pr(\neg C|x), \lambda_{np} \Pr(C|x) + \lambda_{nn} \Pr(\neg C|x)]. \quad (49)$$

Simplify the decision rules based on Bayesian theory and we can get

- (P_R) If $[R(a_p|x)] \leq [R(a_b|x)]$ and $[R(a_p|x)] \leq [R(a_n|x)]$, decide $x \in POS(C)$,
- (B_R) If $[R(a_b|x)] \leq [R(a_p|x)]$ and $[R(a_b|x)] \leq [R(a_n|x)]$, decide $x \in BND(C)$,
- (N_R) If $[R(a_n|x)] \leq [R(a_p|x)]$ and $[R(a_n|x)] \leq [R(a_b|x)]$, decide $x \in NEG(C)$.

Where $[R(a_p|x)]$, $[R(a_b|x)]$ and $[R(a_n|x)]$ are all rough numbers. According to the ranking rules of rough numbers shown in Section 2.2, we can do some further research about decision rules. Taking $[R(a_p|x)] \leq [R(a_b|x)]$ as example, we can obtain the following equations:

$$S_p = (\lambda_{pp} + \lambda_{pp}) \Pr(C|x) + (\lambda_{pn} + \lambda_{pn}) (1 - \Pr(C|x)), \quad (50)$$

$$S_b = (\lambda_{bp} + \lambda_{bp}) \Pr(C|x) + (\lambda_{bn} + \lambda_{bn}) (1 - \Pr(C|x)), \quad (51)$$

$$D_p = (\lambda_{pp} - \lambda_{pp}) \Pr(C|x) + (\lambda_{pn} - \lambda_{pn}) (1 - \Pr(C|x)), \quad (52)$$

$$D_b = (\lambda_{bp} - \lambda_{bp}) \Pr(C|x) + (\lambda_{bn} - \lambda_{bn}) (1 - \Pr(C|x)). \quad (53)$$

If $[R(a_p|x)] \leq [R(a_b|x)]$ is expected, $S_p < S_b$ or $S_p = S_b$ and $D_p \geq D_b$ must be satisfied, meaning that

$$[R(a_p|x)] \leq [R(a_b|x)]$$

$$\Leftrightarrow (S_p < S_b) \text{ or } ((S_p = S_b) \wedge (D_p \geq D_b)).$$

For the two cases: (1) if $S_p < S_b$, the Equation 54 can be obtained; (2) if $S_p = S_b$ and $D_p \geq D_b$, the Equation 55 can be obtained.

$$\Pr(C|x) >$$

$$\frac{(\lambda_{pn} + \lambda_{pn}) - (\lambda_{bn} + \lambda_{bn})}{(\lambda_{pn} + \lambda_{pn}) - (\lambda_{bn} + \lambda_{bn}) + (\lambda_{bp} + \lambda_{bp}) - (\lambda_{pp} + \lambda_{pp})}, \quad (54)$$

$$\left[\frac{(\lambda_{pn} - \lambda_{pn}) - (\lambda_{bn} - \lambda_{bn})}{(\lambda_{pp} - \lambda_{pp})} \Pr(C|x) < \frac{(\lambda_{pn} - \lambda_{pn}) - (\lambda_{bn} - \lambda_{bn})}{(\lambda_{pp} - \lambda_{pp})} \right]. \quad (55)$$

It is worth noting that, in Equation (55), whether $\left[\frac{(\lambda_{pn} - \lambda_{pn}) - (\lambda_{bn} - \lambda_{bn})}{(\lambda_{pp} - \lambda_{pp})} \right]$ is positive or negative cannot be determined at this moment. So it is impossible to simplify Equation (55) and obtain an expression of $\Pr(C|x)$ as Equation (54). Therefore, when comparing the relationship between $[R(a_p|x)]$ and $[R(a_b|x)]$, it is convenient if there is $S_p < S_b$ or $S_p > S_b$, and in this case we can get the result by the threshold α_s directly. Similarly, if we can compare other expected losses only by S , the thresholds β_s and γ_s can also be obtained, which are expressed as Equations (57) and (58). While if there is $S_p = S_b$, deducing that $\Pr(C|x) = \alpha_s$, then we need to compare D_p and D_b . If there is $D_p \geq D_b$, then $[R(a_p|x)] \leq [R(a_b|x)]$; while if there is $D_p < D_b$, then $[R(a_p|x)] > [R(a_b|x)]$. For the comparison of other expected losses, the process is consistent. Therefore, according to the discussion above, the acquisition process of decision rules under different cases can be illustrated as follows, and the final results are tabulated in Table 8.

In Table 8, three-way decision rules can be classified to three cases: (1) the normal case, where there is $\Pr(C|x) \neq \alpha_s$, $\Pr(C|x) \neq \beta_s$, and $\Pr(C|x) \neq \gamma_s$, corresponding to No. (a-c); the extreme case I, where $\Pr(C|x) = \alpha_s$ or $\Pr(C|x) = \beta_s$ or $\Pr(C|x) = \gamma_s$ with $\alpha_s \neq \beta_s \neq \gamma_s$, corresponding to No. (d-o); (3) the extreme case II, where $\Pr(C|x) = \alpha_s = \beta_s = \gamma_s$, corresponding to No. (p-r). The process of decision rules of each case in detail can be explained as follows:

1. Rules in normal case

In this case, $\Pr(C|x) \neq \alpha_s$, $\Pr(C|x) \neq \beta_s$, and $\Pr(C|x) \neq \gamma_s$ are satisfied simultaneously, then there is no extreme situation. Theorem 1 has illustrated that $[\lambda_{pp}] \leq [\lambda_{bp}] < [\lambda_{np}]$ and $[\lambda_{nn}] \leq [\lambda_{bn}] < [\lambda_{pn}]$, so threshold values α_s , β_s , and γ_s can be used directly to determine the three-way decision rules as follows, which are corresponding to the rule (a)-(c), respectively.

Table 8 | Rough three-way decision rules.

Conditions	Subconditions	Sub-subconditions	Results	No.
$\Pr(C x) > \alpha_S \wedge \Pr(C x) > \gamma_S$	—	—	$x \in POS(C)$	(a)
$\Pr(C x) < \alpha_S \wedge \Pr(C x) > \beta_S$	—	—	$x \in BND(C)$	(b)
$\Pr(C x) < \beta_S \wedge \Pr(C x) < \gamma_S$	—	—	$x \in NEG(C)$	(c)
$\Pr(C x) = \alpha_S$	$D_P \geq D_B$	$\alpha_S > \gamma_S$	$x \in POS(C)$	(d)
		$\alpha_S < \gamma_S$	$x \in NEG(C)$	(e)
	$D_P < D_B$	$\alpha_S > \beta_S$	$x \in BND(C)$	(f)
$\Pr(C x) = \beta_S$		$\alpha_S < \beta_S$	$x \in NEG(C)$	(g)
	$D_B \geq D_N$	$\beta_S > \alpha_S$	$x \in POS(C)$	(h)
		$\beta_S < \alpha_S$	$x \in BND(C)$	(i)
$\Pr(C x) = \gamma_S$	$D_B < D_N$	$\beta_S > \gamma_S$	$x \in POS(C)$	(j)
		$\beta_S < \gamma_S$	$x \in NEG(C)$	(k)
	$D_P \geq D_N$	$\gamma_S > \alpha_S$	$x \in POS(C)$	(l)
$\Pr(C x) = \alpha_S = \beta_S = \gamma_S$		$\gamma_S < \alpha_S$	$x \in BND(C)$	(m)
	$D_P < D_N$	$\gamma_S > \beta_S$	$x \in BND(C)$	(n)
		$\gamma_S < \beta_S$	$x \in NEG(C)$	(o)
	$(D_P \geq D_B) \wedge (D_P \geq D_N)$	—	$x \in POS(C)$	(p)
	$(D_B \geq D_P) \wedge (D_B \geq D_N)$	—	$x \in BND(C)$	(q)
	$(D_N \geq D_P) \wedge (D_N \geq D_B)$	—	$x \in NEG(C)$	(r)

- (P'_R) If $\Pr(C|x) > \alpha_S$ and $\Pr(C|x) > \gamma_S$, decide $x \in POS(C)$,
- (B'_R) If $\Pr(C|x) < \alpha_S$ and $\Pr(C|x) > \beta_S$, decide $x \in BND(C)$,
- (N'_R) If $\Pr(C|x) < \gamma_S$ and $\Pr(C|x) < \beta_S$, decide $x \in NEG(C)$.

Where

$$\alpha_S = \frac{(\lambda_{PN} + \overline{\lambda_{PN}}) - (\lambda_{BN} + \overline{\lambda_{BN}})}{(\lambda_{PN} + \overline{\lambda_{PN}}) - (\lambda_{BN} + \overline{\lambda_{BN}}) + (\lambda_{BP} + \overline{\lambda_{BP}}) - (\lambda_{PP} + \overline{\lambda_{PP}})}, \quad (56)$$

$$\beta_S = \frac{(\lambda_{BN} + \overline{\lambda_{BN}}) - (\lambda_{NN} + \overline{\lambda_{NN}})}{(\lambda_{BN} + \overline{\lambda_{BN}}) - (\lambda_{NN} + \overline{\lambda_{NN}}) + (\lambda_{NP} + \overline{\lambda_{NP}}) - (\lambda_{BP} + \overline{\lambda_{BP}})}, \quad (57)$$

$$\gamma_S = \frac{(\lambda_{PN} + \overline{\lambda_{PN}}) - (\lambda_{NN} + \overline{\lambda_{NN}})}{(\lambda_{PN} + \overline{\lambda_{PN}}) - (\lambda_{NN} + \overline{\lambda_{NN}}) + (\lambda_{NP} + \overline{\lambda_{NP}}) - (\lambda_{PP} + \overline{\lambda_{PP}})}. \quad (58)$$

2. Rules in extreme case I

In this case there exists $\Pr(C|x) = \alpha_S$, $\Pr(C|x) = \beta_S$ or $\Pr(C|x) = \gamma_S$, and $\alpha_S \neq \beta_S \neq \gamma_S$. $\alpha_S \neq \beta_S \neq \gamma_S$ denotes that $\Pr(C|x)$ equals to only one of the threshold. Here it is necessary to explore the relations among D_P , D_B , D_N .

Taking the comparison of $[R(a_P|x)]$ and $[R(a_B|x)]$, for example.

Suppose there are $\Pr(C|x) = \alpha_S$ and $\alpha_S \neq \beta_S \neq \gamma_S$, so we can obtain $\Pr(C|x) \neq \beta_S$ and $\Pr(C|x) \neq \gamma_S$, then

- if there is $D_P \geq D_B$, we can obtain $[R(a_P|x)] \leq [R(a_B|x)]$, and then
for $\alpha_S > \gamma_S$ we have $\Pr(C|x) > \gamma_S$, so there is $[R(a_P|x)] < [R(a_N|x)]$, and $x \in POS(C)$, (Rule (d))
for $\alpha_S < \gamma_S$ we have $\Pr(C|x) < \gamma_S$, so there is $[R(a_P|x)] > [R(a_N|x)]$, and $x \in NEG(C)$, (Rule (e))
- if there is $D_P < D_B$, we can obtain $[R(a_P|x)] > [R(a_B|x)]$, and then
for $\alpha_S > \beta_S$ we have $\Pr(C|x) > \beta_S$, so there is $[R(a_B|x)] < [R(a_N|x)]$, and $x \in BND(C)$, (Rule (f))
for $\alpha_S < \beta_S$ we have $\Pr(C|x) < \beta_S$, so there is $[R(a_B|x)] > [R(a_N|x)]$, and $x \in NEG(C)$. (Rule (g))
Similarly, the cases when $\Pr(C|x) = \beta_S$ or $\Pr(C|x) = \gamma_S$ can be discussed in the same process, and obtain the rules (h)-(o).

3. Rules in extreme case II

In this case, there is $\Pr(C|x) = \alpha_S = \beta_S = \gamma_S$. According to classical three-way decisions, thresholds only satisfy three situations: $\alpha < \gamma < \beta$, $\beta < \gamma < \alpha$, and $\alpha = \gamma = \beta$, as discussed by Equations (15) and (16). In decision rules (d)-(o), we have not considered the case $\alpha = \gamma = \beta$, so the rule (p)-(r) state the case of $\Pr(C|x) = \alpha_S = \beta_S = \gamma_S$ separately. When $\Pr(C|x) = \alpha_S = \beta_S = \gamma_S$, there is $S_P = S_B = S_N$, and to compare the relationship among expected losses, D_P , D_B , and D_N need to be considered. In this situation, the largest D_* corresponds to the smallest value of $[R(a_*(x))]$, so we can make the corresponding decision, and yield rule (p)-(r).

In conclusion, 18 three-way decision rules derived from rough loss functions can be obtained, as shown in Table 8.

Example 4. We discuss the problem in Example 1 by using the direct model proposed in this subsection. The expected rough losses can be calculated as

$$[R(a_P|x)] = [2.8 - 2.6 \Pr(C|x), 3.4 - 2.8 \Pr(C|x)],$$

$$[R(a_B|x)] = [1.7 - \Pr(C|x), 2.5 - 1.5 \Pr(C|x)],$$

$$[R(a_N|x)] = [0.3 + 1.2 \Pr(C|x), 0.8 + 1.3 \Pr(C|x)].$$

According to Equations (56–58), we can obtain the values of three thresholds, which are denoted as $\alpha_S = 0.690$, $\beta_S = 0.620$, and $\gamma_S = 0.680$. If there is $\Pr(C|x) \neq \alpha_S, \beta_S, \gamma_S$, then the three-way decision rules can be determined by thresholds directly. For example, suppose $\Pr(C|x) = 0.670$, then we can decide that $x \in BND(C)$ according to rule (b). While if there is $\Pr(C|x) = \alpha_S$, $\Pr(C|x) = \beta_S$, or $\Pr(C|x) = \gamma_S$, then it needs further discussion. For example, suppose $\Pr(C|x) = \alpha_S = 0.690$, then we need to compare the relationship between $D[R(a_P|x)]$ and $D[R(a_B|x)]$. It is easy to find that $[R(a_P|x)] = [1.007, 1.469]$ and $[R(a_B|x)] = [1.010, 1.466]$, so there is $S([R(a_P|x)]) = S([R(a_B|x)])$, and then $D([R(a_P|x)]) = 0.462$ and $D([R(a_B|x)]) = 0.455$, which illustrate that $D([R(a_P|x)]) > D([R(a_B|x)])$. As there is $\alpha_S \geq \gamma_S$, we can determine that the conditions satisfy rule (d), so when $\Pr(C|x) = \alpha_S$ we can make the decision as $x \in POS(C)$.

5. AN APPLICATION IN INFLUENZA EMERGENCY MANAGEMENT

5.1. Problem Description

As an important practical issue in human society, influenza emergency management is discussed in this section by using rough number-based three-way group decisions. A city in China is suffering from a suspected influenza outbreak. The health department of the city plans to isolate regions where influenza cases have been diagnosed, and takes no preventive measures in regions where there is no influenza outbreak. Based on the judgment on the influenza epidemic, two states $\Omega = \{C, \neg C\}$ are obtained, which represent that an outbreak has occurred and has not yet occurred in this region, respectively. The set of actions taken by the health department and the local government is expressed as $A = \{a_P, a_B, a_N\}$, which denote isolation, continuous monitoring, and nonisolation, respectively. Here, $\Pr(C|x)$ represents the probability of an influenza outbreak in region x . There are six administrative regions in this city, denoted as $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$. Based on past experience as well as data collection and analysis for each administrative region, the conditional probabilities of an influenza outbreak in the six administrative regions are $\Pr(C|x_1) = 0.54$, $\Pr(C|x_2) = 0.21$, $\Pr(C|x_3) = 0.32$, $\Pr(C|x_4) = 0.65$, $\Pr(C|x_5) = 0.58$, $\Pr(C|x_6) = 0.26$, $\Pr(C|x_7) = 0.25$, and $\Pr(C|x_8) = 0.57$, respectively.

It is difficult to accurately calculate losses caused by incorrect classification of influenza epidemic situations. Therefore, five experts in influenza emergency management are selected to form a decision-making group $E = \{e_1, e_2, e_3, e_4, e_5\}$ to estimate losses caused by different classifications. The evaluating results of each expert are depicted in Table 9.

Table 9 Results of experts' evaluation.

E	λ_{PP}	λ_{BP}	λ_{NP}	λ_{PN}	λ_{BN}	λ_{NN}
e_1	2	13	20	14	7	2
e_2	3	10	21	13	6	1
e_3	3	8	20	15	5	2
e_4	4	7	19	13	7	3
e_5	2	6	28	17	8	2

Based on the constructing process of rough numbers shown in Equations (17–21), the evaluation values of decision group can be aggregated to rough losses, as shown in Table 10.

Table 10 Rough loss function.

	$C(P)$	$\neg C(N)$
a_P	[2.36, 3.25]	[13.50, 15.43]
a_B	[7.21, 10.63]	[5.92, 7.25]
a_N	[20.15, 23.88]	[1.65, 2.35]

5.2. Three-Way Decisions of Influenza Emergency Management

According to the four models proposed in Section 4, that is to say, the optimistic model, the pessimistic model, the risk preference coefficient model, and the rough model, we can obtain different decision results under various environments.

1. According to model 1 based on the optimistic perspective, the optimistic loss function can be denoted as Table 11. And by using Equations (29–31), we can calculate the values of thresholds as $\alpha_o = 0.610$, $\beta_o = 0.248$, and $\gamma_o = 0.400$.

Table 11 Optimistic loss function.

	$C(P)$	$\neg C(N)$
a_P	2.36	13.50
a_B	7.21	5.92
a_N	20.15	1.65

2. According to model 2 based on the pessimistic perspective, the pessimistic loss function can be denoted as Table 12. By using Equations (35–37), we can obtain the values of thresholds as $\alpha_p = 0.526$, $\beta_p = 0.270$, and $\gamma_p = 0.388$.

Table 12 Pessimistic loss function.

	$C(P)$	$\neg C(N)$
a_P	3.25	15.43
a_B	10.63	7.25
a_N	23.88	2.35

3. According to model 3 based on the risk preference coefficient perspective, we can obtain the loss function based on θ . In this case we suppose $\theta = 0.7$, so the loss function can be denoted as Table 13. Calculating the values of thresholds based on Equations (44–46), we can obtain $\alpha_\theta = 0.547$, $\beta_\theta = 0.264$, and $\gamma_\theta = 0.391$. To illustrate the sensitivity of thresholds to the value of θ , the curves of α , β , and γ with different values of θ are elaborated in Figure 3. It is clear that when $\theta = 0$ the

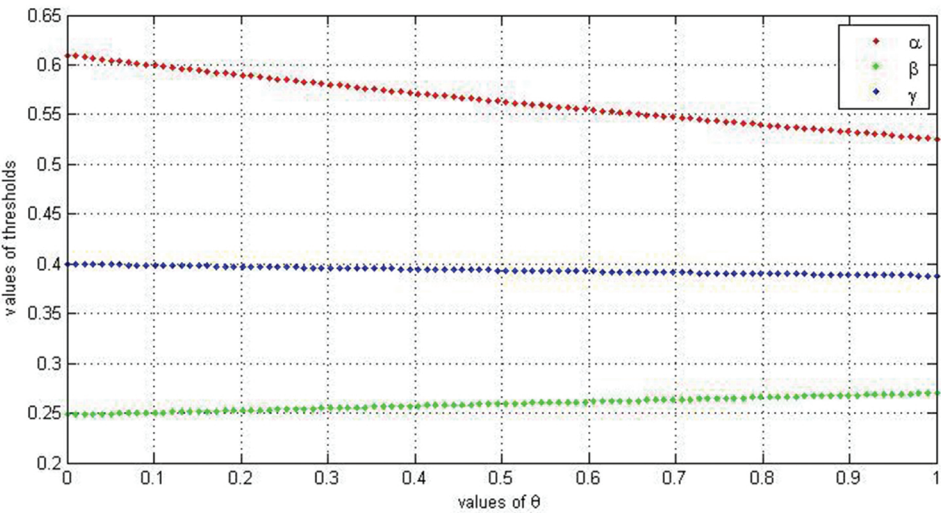


Figure 3 | The thresholds values with different values of θ .

values of thresholds correspond to the optimistic model, and when $\theta = 1$ they correspond to the pessimistic model.

Table 13 | Loss function based on $\theta = 0.7$.

	$C(P)$	$\neg C(N)$
a_P	2.98	14.85
a_B	9.60	6.85
a_N	22.76	2.14

4. According to the direct rough model proposed in Section 4.2, we can calculate the values of thresholds by using Equations (56–58) based on the data in Table 10, and obtain that $\alpha_s = 0.563$, $\beta_s = 0.264$, and $\gamma_s = 0.398$. The final results derived from four models can be depicted in Table 14 and Figure 4.

The above four results have different effects on the determination of the three-way decision-making rules. All results are summarized in Table 14. Table 14 shows that regardless of which decision-making method is selected, the final classifications of x_2 , x_3 and x_4 remain

unchanged, and they belong to the negative domain, the boundary domain, and the positive domain, respectively. That is to say, in all decision-making modes, x_4 is always a region with an influenza outbreak, whereas x_2 is not. For x_3 , no conclusion can be drawn and further data collection is required. For other regions, namely, x_1 , x_5 , x_6 , x_7 , and x_8 , when different decision-making models are selected, the three-way decision-making rules obtained are different. Thus, it is necessary to make a choice according to specific decision-making method and specific situation.

Table 14 | Results of decision-making.

Administrative Regions	Conditional Probabilities	Decision Results			
		Model 1	Model 2	Model 3	Model 4
x_1	0.54	$BND(C)$	$POS(C)$	$BND(C)$	$BND(C)$
x_2	0.21	$NEG(C)$	$NEG(C)$	$NEG(C)$	$NEG(C)$
x_3	0.32	$BND(C)$	$BND(C)$	$BND(C)$	$BND(C)$
x_4	0.65	$POS(C)$	$POS(C)$	$POS(C)$	$POS(C)$
x_5	0.58	$BND(C)$	$POS(C)$	$POS(C)$	$POS(C)$
x_6	0.26	$BND(C)$	$NEG(C)$	$NEG(C)$	$BND(C)$
x_7	0.25	$BND(C)$	$NEG(C)$	$NEG(C)$	$NEG(C)$
x_8	0.57	$BND(C)$	$POS(C)$	$POS(C)$	$POS(C)$

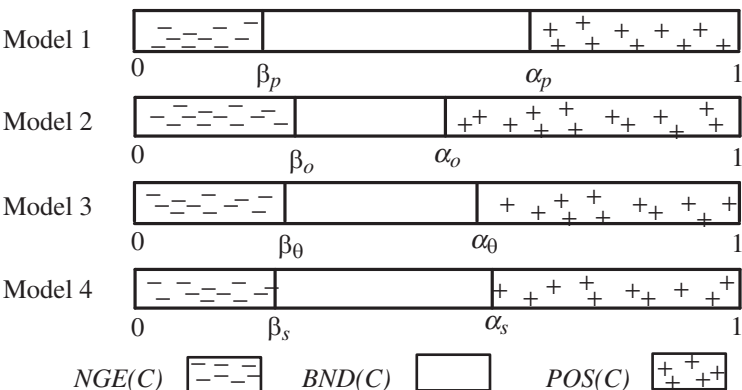


Figure 4 | Three-way decisions based on different models.

5.3. Comparison and Discussion

According to previous research, for three-way decision-making models with multiple decision-makers, methods such as taking the maximum value, the minimum value, and the weighted average can be used to aggregate the group information, and the group loss function can be obtained. Then the final three-way decision rules are obtained by using the three-way decision-making process. Here the results obtained by the weighted average method and the three-way group decision-making model proposed in literature [21] were selected and compared with those obtained using the methods in this study, which are shown in Tables 15 and 16.

Table 15 | Comparison of thresholds.

Methods	Thresholds		
	α	β	γ
Weighted average method	0.565	0.265	0.397
Method proposed by Liang [21]	0.571	0.218	0.381
Model 1	0.610	0.248	0.400
Model 2	0.526	0.270	0.388
Model 3	0.547	0.264	0.391
Model 4	0.563	0.259	0.394

Table 16 | Comparison of classifications.

Methods	Classification		
	POS(C)	BND(C)	NEG(C)
Weighted average method	$\{x_4, x_5, x_8\}$	$\{x_1, x_3\}$	$\{x_2, x_6, x_7\}$
Method proposed by Liang [21]	$\{x_4, x_5\}$	$\{x_1, x_3, x_6, x_7, x_8\}$	$\{x_2\}$
Model 1	$\{x_4\}$	$\{x_1, x_3, x_5, x_6, x_7, x_8\}$	$\{x_2\}$
Model 2	$\{x_1, x_4, x_5, x_8\}$	$\{x_3\}$	$\{x_2, x_6, x_7\}$
Model 3	$\{x_4, x_5, x_8\}$	$\{x_1, x_3\}$	$\{x_2, x_6, x_7\}$
Model 4	$\{x_4, x_5, x_8\}$	$\{x_1, x_3, x_6\}$	$\{x_2, x_7\}$

Table 15 illustrates the values of thresholds obtained by different methods. The values are various according to the selection of methods, but all the thresholds of each method satisfy $\beta < \gamma < \alpha$, so the alternatives can be classified by α and β . For α , the minimum value and the maximum value are obtained by model 2 and model 1, respectively, and the values obtained by other models are restricted in this interval. For β , the minimum value and the maximum value are obtained by method in [21] and model 2, respectively. According to the results of thresholds, we can obtain the classification of each alternative, which is exhibited in Table 16. The three-way decision rules obtained by different methods are diverse because of the difference of thresholds. For the weighted average method, the decision rule is equal to the result derived from model 3, meaning that for the eight alternatives the results are consistent when choosing the weighted average method or model 3. For the method proposed in [21], the decision rule is different from the others, but similar as the result derived from model 1 expect x_5 . It is easy to find that the negative regions obtained by method in [21] and model 1 are both $\{x_2\}$, while by other models the negative regions contain more than one alternative. The reason is that in experts' evaluating results, there is an abnormal value of λ_{NP} given by e_5 , while method in [21] and model 1 could not eliminate the effect of the noise in an effective way. The value of β is relevant to λ_{NP} , therefore the two models obtain lower values of β and only x_2 satisfies that the conditional probability is less than β .

The specific advantages of models in this paper are summarized as follows:

1. By using the rough number method to aggregate group information, it is possible to take advantage of the rough number method and to fully consider and explore the intrinsic relations among all information in the same evaluation category, which can effectively eliminate the effect of abnormal evaluating values.
2. The rough number method can reflect the objectivity of the rough set theory. No subjective prior knowledge is required, and only the intrinsic relations among information should be taken into consideration. Thus, the objectivity of the aggregation of group information is guaranteed, and the decision-making rules obtained are also more reliable.
3. Three-way group decision-making models based on rough numbers can utilize different decision-making modes. For example, the risk preferences of decision-makers in a decision-making situation can be taken into account, or the rough number calculation method can be employed when risk preferences are not considered. Decision-makers could select different three-way group decision-making models in various environments, which can result in decision-making rules that better address problems in real life and can offer greater flexibility.

6. CONCLUSIONS

Group decision-making involving multiple decision-makers is an important way to solve practical decision-making problems. This paper explored the integration of group decision-making with three-way decision-making models and their application. To handle the problem of group information aggregation in the process of group decision-making, the rough number method is adopted. Since the rough number method is objective and systematic, the estimation of the loss functions by all individuals in the group are aggregated to construct the rough number loss function and thus to describe the evaluation information of the group. In the construction stage of the three-way decision-making model, based on whether or not the decision-maker's risk preference should be taken into account, the optimistic method, the pessimistic method, the risk preference coefficient method, and the rough number calculation method were constructed respectively to examine the threshold calculation and the rule acquisition in three-way decision-making models. Finally, the three-way group decision-making model constructed in this paper was applied to an influenza emergency management problem to make the optimal decision with the lowest loss caused by errors. The three-way group decision-making model based on the rough number method is able to effectively aggregate the estimation of the loss function by the decision-making group to reflect the overall preference of the decision-making group and to choose different decision-making modes according to the actual problems to obtain decision-making rules. In future studies, three-way group decision-making models with group uncertain information, such as interval numbers and fuzzy numbers, will be further explored. Meanwhile, other emergency management problems will be selected for application.

ACKNOWLEDGMENTS

This paper is supported by the National Natural Science Foundation of China (Nos. 71771140 and 71471172), the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), and the Humanities

and Social Sciences Research Project of Ministry of Education of China (No. 19YJC630059).

REFERENCES

- [1] Y.Y. Yao, Three-way decision: an interpretation of rules in rough set theory, *Rough Sets Knowledge Technol.* 5589 (2009), 642–9.
- [2] Y.Y. Yao, Three-way decisions with probabilistic rough sets, *Info. Sci.* 180 (2010), 341–53.
- [3] D. Liu, Y.Y. Yao, T.R. Li, Three-way investment decisions with decision-theoretic rough sets, *Int. J. Comput. Intell. Syst.* 4 (2011), 66–74.
- [4] F. Jia, P.D. Liu, A novel three-way decision model under multiple-criteria environment, *Info. Sci.* 471 (2019), 29–51.
- [5] D. Liu, T.R. Li, D. Ruan, Probabilistic model criteria with decision-theoretic rough sets, *Info. Sci.* 181 (2011), 3709–22.
- [6] G. Lang, D. Miao, M. Cai, Three-way decision approaches to conflict analysis using decision-theoretic rough set theory, *Info. Sci.* 406 (2017), 185–207.
- [7] H. Li, L. Zhang, B. Huang, X. Zhou, Sequential three-way decision and granulation for cost-sensitive face recognition, *Knowledge Based Syst.* 91 (2016), 241–51.
- [8] H. Li, L. Zhang, X.Z. Zhou, B. Huang, Cost-sensitive sequential three-way decision modeling using a deep neural network, *Int. J. Approx. Reason.* 85 (2017), 68–78.
- [9] M. Nauman, N. Azam, J.T. Yao, A three-way decision making approach to malware analysis using probabilistic rough sets, *Info. Sci.* 374 (2016), 193–209.
- [10] Y.F. Chen, X.D. Yue, H. Fujita, S. Fu, Three-way decision support for diagnosis on focal liver lesions, *Knowledge Based Syst.* 127 (2017), 85–99.
- [11] D. Liu, D.C. Liang, C.C. Wang, A novel three-way decision model based on incomplete information system, *Knowledge Based Syst.* 91 (2015), 32–45.
- [12] H.R. Zhang, F. Min, B. Shi, Regression-based three-way recommendation, *Info. Sci.* 378 (2016), 444–61.
- [13] H.R. Zhang, F. Min, Three-way recommender systems based on random forests, *Knowledge Based Syst.* 91 (2016), 275–86.
- [14] D. Liu, T.R. Li, H.X. Li, Interval-valued decision-theoretic rough sets, *Comput. Sci.* 39 (2012), 178–181, 214.
- [15] D.C. Liang, D. Liu, Systematic studies on three-way decisions with interval-valued decision-theoretic rough sets, *Info. Sci.* 276 (2014), 186–203.
- [16] D.C. Liang, D. Liu, W. Pedrycz, P. Hu, Triangular fuzzy decision-theoretic rough sets, *Int. J. Approx. Reason.* 54 (2013), 1087–106.
- [17] D.C. Liang, Z.S. Xu, D. Liu, Three-way decisions with intuitionistic fuzzy decision-theoretic rough sets based on point operators, *Info. Sci.* 375 (2016), 183–201.
- [18] D.C. Liang, Z.S. Xu, D. Liu, Three-way decisions based on decision-theoretic rough sets with dual hesitant fuzzy information, *Info. Sci.* 396 (2017), 127–43.
- [19] D.C. Liang, W. Pedrycz, D. Liu, P. Hu, Three-way decisions based on decision-theoretic rough sets under linguistic assessment with the aid of group decision making, *Appl. Soft Comput.* 29 (2015), 256–69.
- [20] D. Liu, D.C. Liang, Three-way decisions in ordered decision system, *Knowledge Based Syst.* 137 (2017), 182–95.
- [21] D.C. Liang, D. Liu, A. Kobina, Three-way group decisions with decision-theoretic rough sets, *Info. Sci.* 345 (2016), 46–64.
- [22] B.Z. Sun, W.M. Ma, X. Xiao, Three-way group decision making based on multigranulation fuzzy decision-theoretic rough set over two universes, *Int. J. Approx. Reason.* 81 (2016), 87–102.
- [23] S.L. Liu, X.W. Liu, J.D. Qin, Three-way group decisions based on prospect theory, *J. Oper. Res. Soc.* 69 (2018), 25–35.
- [24] L.Y. Zhai, K.P. Khoo, Z.W. Zhong, A rough set based QFD approach to the management of imprecise design information in product development, *Adv. Eng. Info.* 23 (2009), 222–8.
- [25] L.Y. Zhai, L.P. Khoo, Z.W. Zhong, Towards a QFD-based expert system: a novel extension to fuzzy QFD methodology using rough set theory, *Expert Syst. Appl.* 37 (2010), 8888–96.
- [26] G.N. Zhu, J. Hu, J. Qi, C.C. Gu, Y.H. Peng, An integrated AHP and VIKOR for design concept evaluation based on rough number, *Adv. Eng. Info.* 29 (2015), 408–18.
- [27] F. Jia, X.Y. Wang, BWM-TOPSIS multi-criteria group decision-making method based on rough number, *Control Dec.* 31 (2016), 1915–20.
- [28] W.Y. Song, X.G. Ming, Z.Y. Wu, An integrated rough number-based approach to design concept evaluation under subjective environments, *J. Eng. Des.* 24 (2013), 320–41.
- [29] Y.Y. Yao, S.K. Wong, A decision theoretic framework for approximating concepts, *Int. J. Man Mach. Stud.* 37 (1992), 793–809.