

A New ISTA-net Solution to Inverse Problem of Electrocardiology

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Abstract. In this paper, we combined the traditional ISTA approach with deep learning, transferred the application domain from image inverse problem to ECG inverse problem because of its excellent performance to propose a new ISTA-net method to solve the inverse problem of ECG. The proposed method has quick convergence speed, steady performance and good extend capability. We created the ISTA-net algorithm model based on the ECG physical model, and transformed it into a neural network structure to achieve automatic parameter selection and more accurate result. We use a real human model on the ECGsim software to obtain the simulated potentials on the cardiac surface and body surface to train our model. The result of our experiment confirms that the proposed method has a better performance in reconstructing epicardial surface potentials distribution compared with the common regularization method like TTLS, Tikhonov, TSVD method.

Keywords: ISTA, ECG, inverse problem.

1. Introduction

The study of the inverse problem of ECG has a great importance in clinical studies of the heart, which aimed at non-invasively analyzing the cardiac transmembrane potential distribution by processing the potential values on the body surface. With the help of FEM(Finite Element Discretization) method, we can obtain the mathematical model by building the torso-heart volume conductor model. Thus the inverse problem of ECG can be expressed as in (1).

$$\Phi_E = A^{-1} \Phi_T \quad (1)$$

Here, $\Phi_E (M \times T)$ denotes the distribution of potentials on the cardiac surface, $\Phi_T (N \times T)$ refers the potential distributions artificially detected on the body surface, $A (M \times N)$ refers the transform matrix which connects Φ_E and Φ_T derived from the torso-heart volume conductor model. The matrix A which connects the given data and unsolved data is a singular matrix. The classical solutions are transforming the ill-posed problem into convex optimization problem like TSVD, TTLS[1], Tikhonov [2] and other methods[3]. The fundamental solution model can be expressed as in (2).

$$\hat{\Phi}_E = \arg \min_{\Phi_E} \frac{1}{2} \|\Phi_T - A\Phi_E\|_2^2 + \lambda \|\Phi_E\|_1 \quad (2)$$

As a classic iterative optimization algorithm, ISTA is widely used in many kinds of image inverse problems, In the ISTA algorithm model, the parameters like stepsize, play an important role in the accuracy of the results and the performance of the algorithm. To avoid the difficulty of parameter selection brought by the traditional L-curve method, we build a neural network structure based on the ISTA algorithm model to solve the annoying parameter selection problem and obtain more accurate results.

Deep learning method has been applied in ECG inverse problem such as the simple learning machine (SLM) model [4], which can only deal with some liner inverse problems. SVR[5] or CNN[6] model which considers the ECG inverse problem as a regression problem. In this paper, a neural network is built based on ISTA algorithm. The torso volume conductor model is used as a part of the neural network module to ensure the reliability of the results and the neural network method is used to

converge the parameters[7]. The experimental result shows that it has a significant effect on the cardiac potential reconstruction.

2. Method

2.1 ISTA

As the classical solutions, we build the mathematical model by using FEM method on the torso-heart volume conductor, then (2) can be rewritten as

$$\min \frac{1}{2} \|\Phi_T - A\Phi_E\|_2^2 + \lambda \|\Phi_E\|_1 \quad (3)$$

Do a derivation on $\|\Phi_T - A\Phi_E\|_2^2$, we can obtain the result $2A^T(A\Phi_E - \Phi_T)$, as the matrix A is a singular matrix, we try to get the resolution that $\|\Phi_T - A\Phi_E^k\| \leq \varepsilon$. According to the theory of ISTA algorithm[8], the solution process includes the following iterating steps:

$$v^k = \Phi_T - A\Phi_E^k \quad (4)$$

$$\Phi_E^{k+1} = \eta(\Phi_E + \beta A^H v^k; \lambda) \quad (5)$$

where $\beta \in (0, 1/\|A\|_2^2)$ is a stepsize, v^k the measurement error of the k th iteration, and $\eta(\cdot; \lambda)$ is the denoiser showed as:

$$[\eta(r; \lambda)]_j = \text{sgn}(r_j) \max\{|r_j - \lambda|, 0\} \quad (6)$$

At the beginning of the iteration, E is set to null matrix with the original size (M *T) and parameters like β and λ are preset by L-curve or other exhaustive method[9]. However, the selection of parameters is very complex, and ISTA algorithm with fixed step-size will lead to poor norm constraint. In order to improve precision and reduce computation load, we use the deep learning tool to build a neural network model to obtain the parameters needed by the training process.

2.2 ISTA-Net Model

We built a structure called ISTA-net trained with a set of D examples based on the ISTA method, where the test data are potentials on the body surface and verify data set are potentials on the heart surface simulated by the ECGsim software. In the proposed structure, each layer refers one time of iteration, and parameters need to be obtained by the L-curve method are trained in the ISTA-net. Since parameters are participated in updating while back-propagating, they are initially fixed in each iteration in the ISTA. Some trainable parameters are used to replace the complex parameter combination in the original algorithm. In order to simplify the structure of the network and better understand the relations between network structure (3), we can rewrite the iteration of ISTA as follows:

$$\Phi_E^{k+1} = \eta(U\Phi_E^k + B\Phi_T; \lambda) \quad (7)$$

$$\text{s.t. } B = \beta A^H \quad (8)$$

$$U = I - BA \quad (9)$$

Thus, the solution process based on the neural network has following steps:

Input: Transform matrix A, body surface potentials Φ_T .

Step 1: Initialize β and λ , determine the layer's size T.

Step 2: Build “coefficient” layer and “merge” layer to obtain matrix B by $B = \beta A^H$ and build “coefficient” layer and “merge” layer to obtain the matrix U by $U = I - BA$. Build the “thresholds” layer to update this layer’s output Φ_E by (7).

Step 3: Repeat step2 until the corresponding number of iterations

Output: Start training and adjust T, learning step, epoch size, batch-size and other parameters.

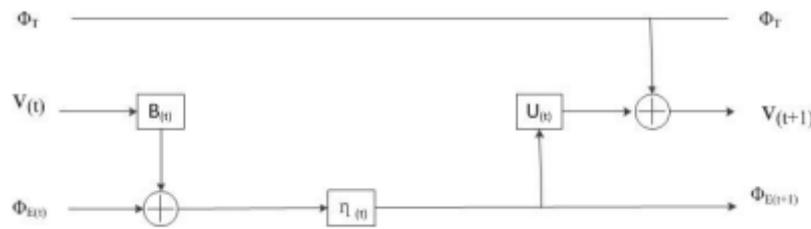


Fig. 1 The overall structure of neural network, which takes the four layers iterative structure as an example.

As the showed steps, ISTA-net’s overall model architecture is shown in Fig. 1. The inputs of this network are Φ_T and $\Phi_{E(0)}$, the output of is $\Phi_{E(T)}$ after T layers of iteration, the label of this network is the simulation data $\Phi_{(d)}$. However, the value of U and S are same at all layers, each iteration updates one time for parameters, the advantage of neural network is not brought into play. Therefore, we improved the structure of ISTA-net to update the parameters such as λ , β each layer independently.

The t th layer’s structure is showed as Fig. 1. Where $n_{(t)}$ denotes $\eta(\cdot; \lambda_t)$. Every parameter in t th layer is trained independently, which can speed up the convergence rate and improve the accuracy since the threshold function is different in each layer. Moreover, the structure of neural network also ensures that the automatic selection of parameters is more convenient and accurate.

We can define the loss function as in (10).

$$\text{loss} = \frac{1}{D} \sum_{d=1}^D \|\hat{\Phi}_{E(T)}(\Phi_T^{(d)}; \Theta) - \Phi_E^{(d)}\|_2^2 \tag{10}$$

where Θ denotes the set of learnable parameters, $\hat{\Phi}_{E(T)}$ denotes the output of the T-layer network with input Φ_T and parameters Θ . Train data and verify data in one iteration are trained as vectors because for ensuring the physiological significance of the data, each iteration corresponds one heartybeat time, and the training set and the verification set are approximately linear features. Thus we take Adam function as our loss function to train our data since adaptive learning rates can avoid the problem of result shocking and improve the accuracy of results and guarantee the speed of convergence.

Unfold the t th layer iteration network as shown in Fig.2. While $t=0$, Φ_E is set to null matrices with original size, parameters are set to standard value by the "uniform" mode of initialize. Φ_T and A remain unchanged in all layers. λ , β in B(t) and U(t) are the parameters need to be trained. According to the ISTA algorithm, η is the "soft thresholding" denoiser that operates component wise as: $\eta(r; \lambda) = \text{sgn}(r_j) \max\{|r_j - \lambda|, 0\}$ which corresponding to the soft threshold function in the ISTA algorithm is used to adjust the outliers in the matrix.

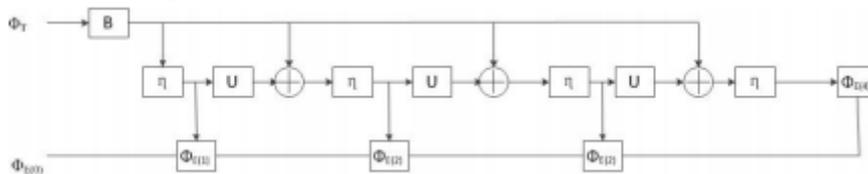


Fig. 2 The t th layer of iteration network

After all the training processes, the results and parameters of the neural network can be preserved as the exclusive solving tool for the ecg inverse problem of this conductor model of human thoracic volume[10], which avoid the complex and imprecise L-curve method to calculate the parameter[11], and can obtain better generalization ability and more accurate results.

3. Experiment

3.1 Experiment Setup

We take the real human torso volume conductor model and use ECGsim software to obtain the simulated cardiac surface potential and body surface potential data. Each heartbeat produces 500ms data, and the cardiac potential data size is 257*500, and the body surface potential data size is 64*500. A 19500ms data is generated by adjusting the source of cardiac surface potential at all. We use cross-validation method by selecting 80% of the data randomly as the training data, remains are used as validation data. The number of layers of the network is set to 6 at the beginning since the training set and verification set are approximately linear, and the size of parameters each layer is set the same as the first dimension of the datasets they are associated with, which can obtain more accurate result than classical methods' 1*1 and avoid over- fitting at the same time. The results are compared with the other three main regularization methods and visualized through Map3D and MATLAB R2013a.

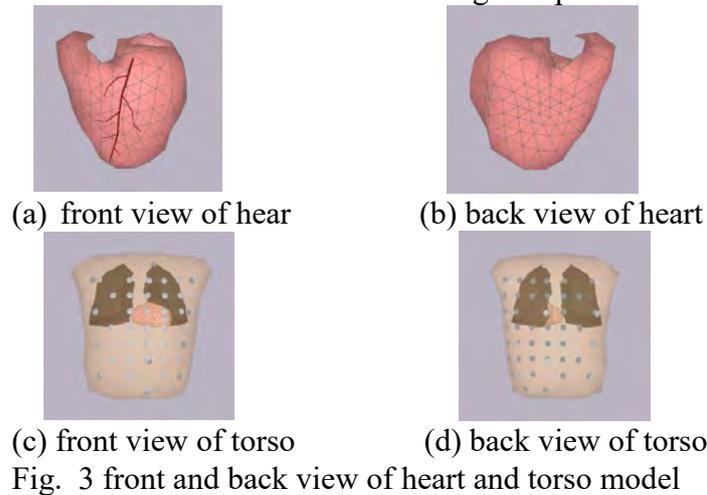


Fig. 3 front and back view of heart and torso model

3.2 Error Assessment

We quantitatively evaluate the accuracy of solving the ECG inverse problem by using two standard performance metrics: relative error (RE) and correlation coefficient (CC)[12]. The two performance metrics are defined as follows:

$$RE = \frac{\|\Phi_E - \hat{\Phi}_E\|_2}{\|\hat{\Phi}_E\|_2},$$

$$CC = \frac{\sum_{k=1}^{N_k} (\Phi_E(k) - \bar{\Phi}_E(k))(\hat{\Phi}_E(k) - \bar{\hat{\Phi}}_E(k))}{\sqrt{\sum_{k=1}^{N_k} (\Phi_E(k) - \bar{\Phi}_E(k))^2 \sum_{k=1}^{N_k} (\hat{\Phi}_E(k) - \bar{\hat{\Phi}}_E(k))^2}}$$

where N_k denotes the total number of nodes on the geometry surface of the heart, Φ_E denotes the potential value of the surface. The superscript '^' refers to its reference value and the superscript '-' refers to its mean value, and represents L2-norm. The RE measures the deviation percentage, and CC measures the spatial differences between the estimator and estimated.

3.3 Model Training and Results

The keras platform is employed to build the neural network and tensorflow is used as the backend tool of keras. The size of training data and validation data are 15600 and 3900, and the size of input and output are 257*1 and 64*1, We take Adam as the convergence method of our model. The geometric model of the heart and body surface is shown in Fig. 3.

Layer size of our network is preset to 6 and adjusted to 7 8 which can obtain good results and save time because the error of validation set increases as the layer become deeper, which is caused by the transform matrix A will become less and less weighted in the entire network as shown in Fig. 6.

After calculation and comparison, the average RE and CC of our method and other three classical methods are shown in Fig. 4. We can see that our method obtained the best result and more accurate than using other three classical methods.

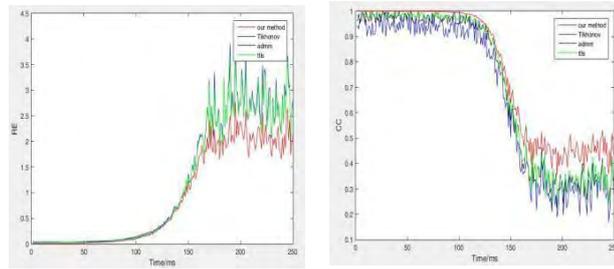


Fig. 4 RE and CC of continuous 250ms data base on our method and other three methods

To make the results more intuitive, we make a comparison figure which presents the visualization of reference and calculated electric potentials mapping on the heart surface and is shown in Fig. 5. Different colors parts represent different potentials, the similarity with the reference image shows the reconstructive potential effect of each algorithm. It is demonstrated that our method obtains the best result compared with the other three regularization methods in reconstructing the cardiac surface in all selected instants.

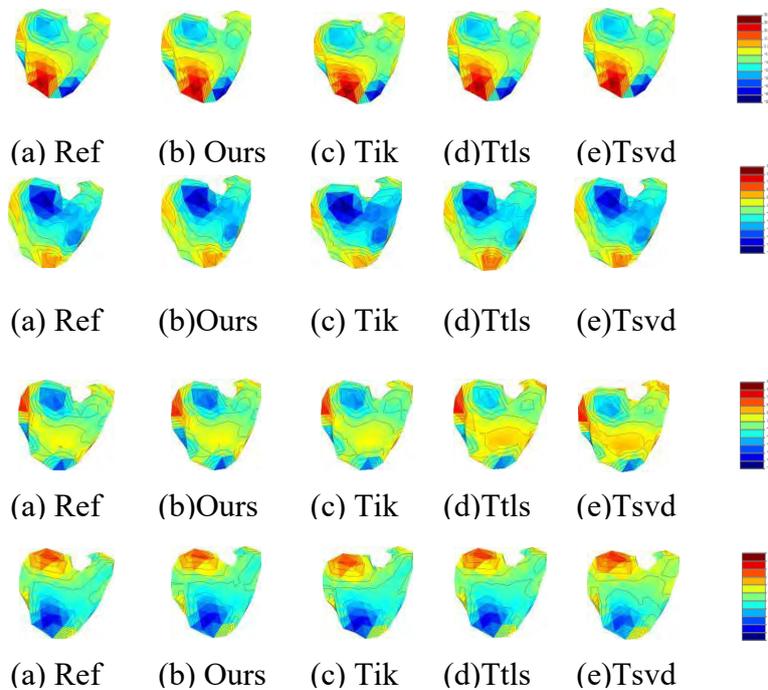


Fig. 5 Each row visualizes the epicardial potentials on the heart surface reconstructed from the data at 35, 316, 803 and 932ms respectively from the top down.

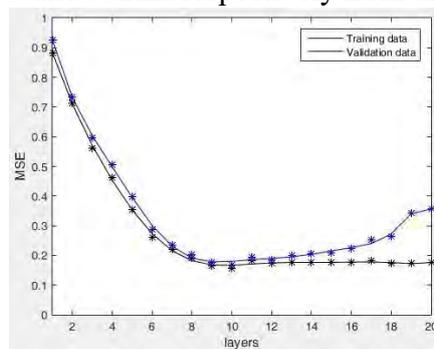


Fig. 6 the MSE versus layer of our method

Table 1 Comparison of Average Re And Average CC For Our Method And The Other Three Regularization Methods.

	Tikhonov	TTLS	TSVD	our method
RE	0.202648	0.184597	0.182132	0.164582
CC	0.882315	0.906793	0.916554	0.940623

At the same time, we compare the RE and CC of our method with the other three regularization methods which is shown in Fig. 6 by selecting continuous 250ms data in the validation dataset, from these experimental results, we can get the conclusion that our method can reconstruct epicardial potentials more accurately compared with the other three regularization methods.

4. Conclusions

By our proposed method, the parameter selection and calculation of the ISTA algorithm or other regularization methods are simplified and be precise, the cardiac potentials reconstruction results can be trained. Whether from result data or image reconstruction result, by analyzing our method compared with other three classical regularization methods, we find that the ISTA method combined with the deep learning method can obtain excellent results. This research is of great significance for the further study of cardiac pathology and for the use of more mainstream tools to solve the inverse problem of ECG.

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