

Diversity on the Influence of Atmospheric Turbulence for the States Carrying Orbital Angular Momentum

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Abstract. We have proposed a spatial diversity scheme to mitigate the influence of atmospheric turbulence on the system employing Orbital Angular Momentum (OAM) state. The results show that, in comparison with the system without diversity, the ratio of SNR tends to two in weak turbulent region, and the ratio tends to one as the strength of turbulence increases. The diversity scheme has efficiently mitigated the influence of atmospheric turbulence at weak turbulence.

Keywords: Free-space optical communication, Orbital Angular Momentum, Atmospheric turbulence, Diversity.

1. Introduction

Orbital angular momentum (OAM) of light is an attractive degree of freedom for fundamental studies in quantum mechanics [1]. A light beam has spin angular momentum if all its polarization vectors rotate, whereas it possesses orbital angular momentum (OAM) if its phase structure rotates. In 1992, it was shown by Allen et al. that in the paraxial regime, an $e^{i\ell\theta}$ vortex phase structure of a circularly symmetric beam corresponds to $\ell\hbar$ units of OAM [2]. Since then, OAM of light has been found as a useful tool in a variety of applications [3-6]. It has been suggested that OAM encoding can be used alongside polarization to increase the channel capacity of communication systems [7]. The use of a multi-level encoding basis such as the OAM basis can increase the tolerance of quantum key distribution (QKD) systems to eavesdropping [8]. In addition, the multiplexing schemes for OAM states have demonstrated both in free-space optical communications (FSO) and fiber-optical communications system to increase the bandwidth efficiency [6, 9, 10].

However, OAM is susceptible to the atmospheric turbulence (AT) [7, 11]. Many methods have been studied to mitigate the influence of AT [12,13]. For example, Djordjevic et al. demonstrated to improve the performance of an OAM communication link under turbulent aberration by using LDPC-coded OAM [12]. Zhao et al. proposed two aberration correction methods to mitigate the effect of atmosphere turbulence in OAM state propagation [13]. On the other hand, diversity techniques can provide signal redundancy which mitigates the fading induced by atmospheric turbulence [14-15].

In this paper, we propose a mitigation scheme on the influence of AT by the diversity technique. The performance of the proposed diversity system is analyzed. In the scheme, a single simplifying assumption that the transmit apertures are small compared to the received diffracted beam spot and the turbulence coherence cell is employed which is valid for many practical free space optical (FSO) systems. No limitations are imposed on the link range, receive aperture diameter, or transmitter separation.

2. System Model

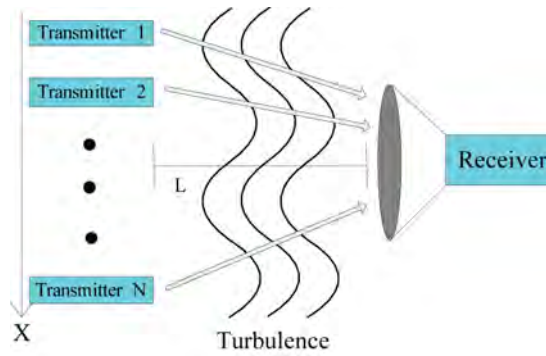


Fig. 1 A line-of-sight free-space multi-beam optical communication system carrying OAM, with N OAM transmitted sources and only one received aperture with radius R .

As illustrated in Fig.1, a line-of-sight multi-beam atmospheric optical communication system is considered where N laser sources located at the transmit plane with radius R' emit OAM beams towards a large R aperture located at the receive plane. Here, “large” aperture refers to an aperture with diameter on the order of or larger than the turbulence coherence length. The optical field collected at the large aperture is focused by the receiver lens to the photo-detection area located at the detector plane. The propagation distance is L . The i -th source is assumed as a pure vortex beam [11], located at $r' = t_i$ of the transmitted plane.

$$\xi_i'(r') = A \exp(jm_i\theta) \delta(r' - t_i), \quad (1)$$

where m_i represents the OAM state number, A denotes the amplitude of the pure vortex beam. Assume the total transmitted power is P , and all the sources have the same transmitted power. Assume L is long enough. The received field at the receiver aperture from the i -th source can be described as

$$\xi_i(r) = AW(r/R') \exp(jm_i\theta) \exp(j\phi(t_i, r)), \quad (2)$$

where $\phi(t_i, r)$ describe the phase fluctuations caused by AT. Hence, the total received field is then given by,

$$\xi(r) = \sum_i \xi_i(r). \quad (3)$$

Therefore, the total optical power collected by the receiver aperture with radius R is

$$\gamma = \int W(r/R) |\xi(r)|^2 dr, \quad (4)$$

where $W(x)$ is the aperture function.

On the other hand, the received beam $\xi(r)$ can be decomposed into various OAM mode n states,

$$\xi(r) = \sum_n A_n \exp(jn\theta), \quad (5)$$

where A_n is the coefficient at OAM n state,

$$A_n = \frac{1}{\pi R^2} \int W(r/R) \xi(r) \exp(-jn\theta) dr. \quad (6)$$

Without loss of generality, two transmitted sources with distance d is considered for the diversity model for simplification, where the sources are located at $r_1' = d/2$ and $r_2' = -d/2$. And assuming $m_1 = m_2 = m$. Since the aberrations introduced by AT are normal random variables, the total received optical power can be expressed by the ensemble average,

$$\langle \gamma \rangle = \frac{PR^2 + PR^2 e^{-D(d,0)/2}}{R^2}, \quad (7)$$

and the averaged received optical power for the n OAM component is,

$$\langle \gamma_n \rangle = \frac{P}{\pi R^2} \int_0^R r dr \int_0^{2\pi} e^{j(m-n)\theta} (e^{-D(0,2r\sin(\theta/2))/2} + e^{-D(d,2r\sin(\theta/2))/2}) d\theta, \quad (8)$$

where $D(\rho, \rho')$ is the wave-structure function [16], given as

$$D(\rho, \rho') = 0.295\pi^2 k^2 C_n^2 \int_0^L \left| \rho \frac{z}{L} + \rho' \left(1 - \frac{z}{L}\right) \right|^2 dz, \quad (9)$$

where L is propagation distance, k is wave number, and C_n^2 is the refractive index structure constant.

The average Signal-to-noise rate (SNR) of the received n OAM state field is defined as

$$\langle SNR \rangle = \frac{\langle \gamma_n \rangle}{\sigma^2}, \quad (10)$$

where σ^2 is the variance of the noise.

In comparison, the total received optical power and the received n OAM state optical power with only one transmitted source are given as following, where the transmit source is assumed to locate at $t=0$,

$$\langle \gamma \rangle = \frac{PR^2}{R^2}, \quad (11)$$

$$\langle \gamma_n \rangle_1 = \frac{P}{\pi R^2} \int_0^R r dr \int_0^{2\pi} e^{j(m-n)\theta} e^{-D(0,2r\sin(\theta/2))/2} d\theta, \quad (12)$$

and

$$\langle SNR \rangle_1 = \frac{\langle \gamma_n \rangle_1}{\sigma^2}. \quad (13)$$

The ratio of $\langle SNR \rangle$ and $\langle SNR \rangle_1$ with different d is shown in Fig.2. The parameters are of the simulation are: propagating distance $L=10\text{km}$, wavelength $\lambda=800\text{nm}$, radius of received aperture $R=30\text{cm}$.

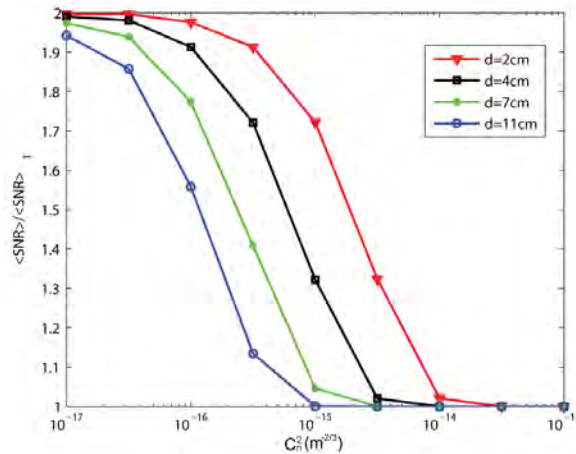


Fig. 2 The ratio of $\langle SNR \rangle$ and $\langle SNR \rangle_1$ with different d .

The results show that the ratio value tends to two in weak turbulent region, and the ratio tends to one as the strength of turbulence increases. It is indicated that the diversity can efficiently mitigate the influence of weak AT, and the diversity scheme has a better performance when d is smaller.

3. Conclusion

In summary, we have proposed a diversity scheme to mitigate the influence of AT on the system using OAM state. The results show that, in comparison with the system without diversity, the ratio

of $\langle SNR \rangle$ tends to two in weak turbulent region, and the ratio tends to one as the strength of turbulence increases. The diversity scheme have efficiently mitigated the influence of AT in weak turbulent region, and the diversity scheme has a better performance when d is smaller.

References

- [1]. M. Mirhosseini, M. Malik, Z. Shi, and R. W. Boyd, Efficient separation of the orbital angular momentum eigenstates of light[J], *Nat. Comm.*, 2013, 4, 2781.
- [2]. L. Allen, M. W. Beijersbergen, R. Spreeuw, and J. P. Woerdman, Orbital angular momentum of light and the transformation of Laguerre-Gaussian laser modes[J]. *Phys. Rev. A* 1992, 45, 8185~8189.
- [3]. A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Entanglement of the orbital angular momentum states of photons[J]. *Nature* 2001, 412, 313~316.
- [4]. G. Molina-Terriza, J. P. Torres, and L. Torner, Twisted photons[J]. *Nat. Phys.* 2007, 3, 305~310.
- [5]. J. T. Barreiro, T.-C. Wei, and P. G. Kwiat, Beating the channel capacity limit for linear photonic superdense coding[J]. *Nat. Photon.* 2008, 4, 282~286.
- [6]. J. Wang, et al. Terabit free-space data transmission employing orbital angular momentum multiplexing. *Nat. Photon.* 2012, 6, 488~496.
- [7]. M. Malik, et al. Influence of atmospheric turbulence on optical communications using orbital angular momentum for encoding. *Opt. Express* 2012, 20, 13195~13200.
- [8]. M. Bourennane, A. Karlsson, and G. Bjork, Quantum key distribution using multilevel encoding. *Phys. Rev. A* 2001, 64, 012306.
- [9]. N. Bozinovic, Y. Yue, Y. Ren, M. Tur, P. Kristensen, H. Huang, A. E. Willner, S. Ramachandran. Terabit-Scale Orbital Angular Momentum Mode Division Multiplexing in Fibers, *Science* 2013, 340, 1545.
- [10]. P. Boffi, P. Martelli, A. Gatto and M. Martinelli, Mode-division multiplexing in fibre-optic communications based on orbital angular momentum, *J. Opt.* 2013, 15, 075403.
- [11]. G. A. Tyler and R. W. Boyd, Influence of atmospheric turbulence on the propagation of quantum states of light carrying orbital angular momentum, *Opt. Lett.* 2009, 34, 142.
- [12]. I. B. Djordjevic and M. Arabaci. LDPC-coded orbital angular momentum (OAM) modulation for free-space optical communication, *Opt. Express* 2010, 24, 24722.
- [13]. S. M. Zhao, J. Leach, L. Y. Gong, J. Ding, and B. Y. Zheng, Aberration corrections for free-space optical communications in atmosphere turbulence using orbital angular momentum states, *Opt. Express* 2012, 20, 452.
- [14]. S. M. Navidpour, M. Uysal, and M. Kavehrad, BER performance of free-space optical transmission with spatial diversity, *IEEE Trans. Wireless Commun.*, 2007, 6, 2813~2819.
- [15]. E. Lee and V. Chan, Part 1: optical communication over the clear turbulent atmospheric channel using diversity, *IEEE J. Sel. Areas Commun.*, 2004, 22, 1896~1906.
- [16]. D. L. Fried, Optical heterodyne detection of an atmospherically distorted signal wave front, *Proc. IEEE* 1967, 55, 57.