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The Application Research of Ant Colony Algorithm for Prediction of Bridge Deflection

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Abstract. It is very difficult to predict bridge deflection deformation accurately due to a variety of complex factors. Based on the TSP Ant Colony Algorithm model, combined with the actual deflection of a bridge, the Ant Colony Algorithm model of deflection prediction has eventually been established after determining the similar path selection mechanism, reasonable information function, heuristic function, pheromone updating strategy, ant-searching and ant-predicting mechanism. The model is tested by two different stages of this bridge. The results show that the model has high prediction accuracy as well as strong applicability, which would open up a new way for prediction of bridge deflection.

Keywords: Prediction of bridge deflection, ant colony algorithm, path selection, information function, heuristic function.

1. Introduction

According to relevant study, numerous bridges have shown different degrees of disease after putting into use for a period of time, among which the persistent deflection in the middle of the span has become a familiar problem. In China, the deflection of the auxiliary channel of Guangdong Humen Bridge had reached 22.2cm through 7 years operation, the deflection of Huangshi Bridge had reached 30.5cm through 7 years operation, while the deflection of Jiangjin Yangtze River Bridge in Chongqing had reached 31.7cm through 9 years operation [1]. The deflection of Parrotts Ferry Bridge had reached 63.5cm after operating 12 years, while the deflection of Koror-Babeldaob Bridge in Republic of Palau had reached 120cm after operating 12 years, and had collapsed soon after that [2].The deflection of the midspan of a bridge during 2003～2018 is shown in Fig.1. The maximum deflection had reached about 13cm, and had formed two distinct settlement troughs till 2018. Above all, the trend of deflection has not converged yet.

For bridges in operation, it is very difficult to establish a precise mathematical model of deflection prediction due to multiple complicated factors such as shrinkage and creep effect of the concrete, temperature, prestressed tension and loss, dynamic load, vibration as well as sunlight, wind power, wind speed, etc. The general approach is to consider some simple factors which are easy to analyze, or select some of the relative important factors through some means, and then establish Kalman filter model, neural network model or time-dependent assessment model to predict the deflection of the bridges[3-5]. Although these approaches have achieved some success, the prediction performances are usually not stable and have poor applicability because only part of the impact factors are considered, which means the models contain more human interference. In recent years, artificial intelligence algorithms have received wide attention with their huge advantage in solving complex, uncertainty and nonlinear problems. Among these algorithms, Ant Colony Algorithm [6] has been widely used in data mining, combinatorial optimization, IC distribution, image processing, network routing and many other fields. Ant Colony Algorithm is a kind of intelligent multi-agent system with self-organizing mechanisms, which makes it do not require detailed understanding on every aspect of the problem. Its essence is the dynamic process of increasing entropy in the absence of external action, which reflects the dynamic evolution from disorder to order. Undoubtedly, the self-organizing mechanisms will be very favorable to the prediction of bridge deflection, of which the factors are complex and multiple. In this paper, we will attempt to conquer the problem.

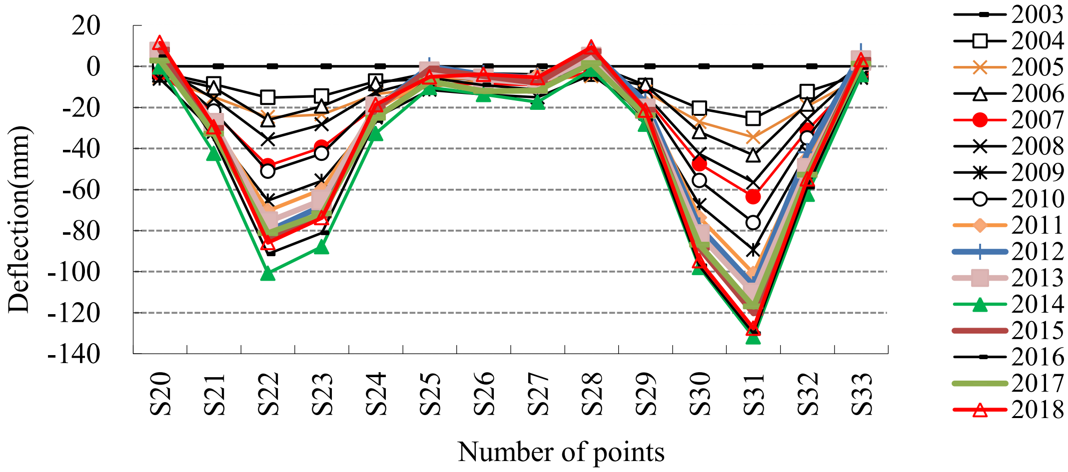

Fig. 1 The variations of the midspan deflection of a bridge

2. The Mathematical Model of Ant Colony Algorithm [7-8]

The Ant Colony Algorithm is a bionic algorithm generated by simulating the shortest path of the ant colony to the food source in the natural ant colony. Through long-term research, biologists have discovered that ants release a volatile substance called pheromone in the path, and ants tend to move along a larger concentration of pheromone when selecting a path. Individual ants communicate information through the pheromone to collaborate and complete complex tasks. Since the earliest Ant Colony Algorithm is mainly used for solving traveling salesman problem (TSP), we will illustrate Ant Colony Algorithm according to TSP.

2.1 Node Transition Rule.

We assume that there are N cities (nodes) and m ants in the plane. The goal of the TSP is to find a path in the plane that connects all cities once and the overall length is minimal.

$$m = \sum_{i=1}^{N} b_i(t) \tag{1}$$

where $b_i(t)$ is the number of ants in city i on time t.

$$\eta_{ij}(t) = 1/d_{ij} \tag{2}$$

where d_{ij} is the distance between the city i and the city j ,which is also the weight of edge (i, j). $\eta_{ij}(t)$ is the inverse of the weight of edge (i, j). Actually, $\eta_{ij}(t)$ is the visibility of the path (i, j), and we call it heuristic function. Ant k at node i determines to move to node j with the probability computed as follows:

$$p_{ij}^{k}(t) = \begin{cases} \dfrac{\left[\tau_{ij}(t)\right]^{\alpha}\left[\eta_{ij}(t)\right]^{\beta}}{\sum_{s \in allowed_k}\left[\tau_{is}(t)\right]^{\alpha}\left[\eta_{is}(t)\right]^{\beta}} & j \in allowed_k \\ 0 & j \notin allowed_k \end{cases} \tag{3}$$

where $\tau_{ij}(t)$ is the quantity of pheromone deposited on edge (i, j) ,which is defined as a uniform positive constant on every edge initially, α is a parameter controlling the influence of $\tau_{ij}(t)$ and reflects the collaboration between ants, β is a parameter controlling the influence of $\eta_{ij}(t)$ and reflects the independence of individual ants, $tabu_k$ is the tabu list of ant k, which contains the nodes that ant k had visited, $allowed_k$ is the list of cities ant k has not reached, and s stands for all the possible nodes that ant k can select.

2.2 Pheromone Updating Rule.

The ant repeatedly uses the decision shown in Eq. (3) to travel from one city to another, and modifies the pheromone concentration of the path after passing one city or traversing all cities. If there is too much residual information on the path, the heuristic information will be submerged. In order to encourage the ants to continue to search for other better paths, the residual information should be weakened, and the pheromone on the path will be volatilized over time. This mechanism can effectively reduce the algorithm stagnation. The pheromone of the edge (i, j) on time t+1 is:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t) \tag{4}$$

$$\Delta\tau_{ij}(t) = \sum_{k=1}^{m} \Delta\tau_{ij}^{k}(t) \tag{5}$$

where $\rho \in (0,1)$ is the evaporation rate of the pheromone, m is the total number of the ants, $\Delta\tau_{ij}^{k}(t)$ is the pheromone laid by the ant k on edge (i, j) in the current iteration. Dorigo proposed three different model according to different pheromone updating rules:

(1) Ant cycle system

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{Q}{L_k} & \text{ant k passed through the edge (i, j) at the iteration} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

(2) Ant quantity system

$$\Delta\tau_{ij}^{k} = \begin{cases} \dfrac{Q}{d_{ij}} & \text{ant k passed through the edge (i, j) between t and t+1} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

(3) Ant density system

$$\Delta\tau_{ij}^{k} = \begin{cases} Q & \text{ant k passed through the edge (i, j) between t and t+1} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where L_k is the route length and Q is a positive constant representing the total quantity of pheromone deposited during the tour. The model (1) uses the global information, which means ants update the pheromone after completing an iteration, while the model (2) and (3) use the local information, which means ants update the pheromone after each step. We should choose appropriate pheromone updating strategy for specific issue.

3. Ant Colony Algorithm Model for Prediction of Bridge Deflection

3.1 Node Transition Rule.

Firstly, Combined with the actual deflection of the bridge discussed above (shown in Fig. 1), we establish the similar path selection mechanism as the TSP Ant Colony Algorithm, which is the foundation of setting up a prediction model accurately. As mentioned above, the node transition rule is determined by the information function and the heuristic function, in which information function reflects the degree of influence that the accumulated experience of the overall ant colony has on path selection, while heuristic function reflects the degree of influence of individual experience from each ant on path selection. Hence, in the paper we regard the deflection measurement points on the bridge as the cities of the TSP, and consider the deflection value as the fundamental basis for ants to release pheromone. The ants tend to move towards the measurement points with large deflection. Thus, the cumulative deflection of a measurement point or the average cumulative deflection of some measurement points of a certain section can be regarded as the information function, which reflects the accumulated experience of the overall ant colony, while the difference between two adjacent monitoring of an individual point can be regarded as the heuristic function, which reflects the individual experience from each ant. Then the ants calculate the transition probabilities according to the information function and heuristic function, and release certain quantity of pheromone when they move to another node so as to increase the accumulated experience of the ant colony. That is, when an ant moves from one point to another, it will evaluate the path by releasing some pheromone on the edge. So a positive feedback, autocatalytic, self-organized path selection mechanism is formed.

3.2 Information Function and Heuristic Function.

1) The Initial Information Function $\tau_{ij}(0)$

In the Ant Colony Algorithm model of TSP, there is little information about the optimal path initially, so we define equal quantity of initial pheromone on each path. While in the model of prediction of bridge deflection, as we have previous monitoring information, we can improve the initial information function so as to obtain more consistent solutions with various runs and accelerate convergence.

It is observed from Fig.1 that in the first 6 years the bridge experienced the most dramatic stage of its deflection deformation, and we can call it the stage of initial formed of the settlement troughs, or simply, the first stage. In this stage the difference of deflection between two adjacent monitoring of the points can be regarded as the initial quantity of pheromone, but we have to modify it slightly:

$$\tau_{ij}(0) = (\Delta L_j - \Delta L_{\min})Q / \Delta L_{s\,u} \tag{9}$$

where ΔL_j is the difference of deflection between two adjacent monitoring of the point j, $\Delta L_{\min}$ is the minimum of the difference of deflection between two adjacent monitoring of the points within the section, ΔL_{sum} is the sum of the deflection between two adjacent monitoring of the points within the section, and Q is a constant representing the quantity of pheromone. Eq. (9) shows that during the first stage, the larger the difference of deflection between two adjacent monitoring of a point is, the higher quantity of initial pheromone will be released on the corresponding edge.

It can also be seen from Fig. 1 that 6 years later, the deflection of the settlement trough of the bridge has changed slightly. The down-deflection rate has slowed down, and occasionally, the opposite situation up-deflection may occur, but the cumulative deflection remains down-deflection. In the paper, we call it the stage of further development of the settlement troughs, or simply, the second stage. In this stage, the main factor that determines the initial quantity of pheromone is the cumulative deflection, which is different from the first stage. So we define

$$\tau_{ij}(0) = (L_j - L_{\min})Q / L_{s\,u} \tag{10}$$

where L_j is the cumulative deflection of point j, $L_{\min}$ is the minimum of the deflection of the points within the section, L_{sum} is the sum of the deflection of the points within the section, and Q is the same constant as in Eq. (9). Eq.(10) shows that during the second stage, the larger the cumulative deflection of a point is, the higher probability it will be selected.

2) The Heuristic Function $\eta_{ij}(t)$

The definition of the heuristic function $\eta_{ij}(t)$ is generally based on the distance between two cities in the model of TSP (see Eq. (2)). Here, $\eta_{ij}(t)$ should be defined according to the relative deflection of i and j. There are two different definitions for $\eta_{ij}(t)$ corresponding to the first and the second stages:

$$\eta_{ij}(t) = \Delta L_j - \Delta L_i \tag{11}$$

$$\eta_{ij}(t) = L_j - L_i \tag{12}$$

Actually, $\eta_{ij}(t)$ reflects the expectation that the ant located in point i moves to point j. Compared with i, the greater the deflection of j is, the higher probability t it will be selected.

3.3 Pheromone Updating Rule

When solving the TSP, Ant cycle system (Eq. (6)) is usually chosen because this model is on the basis of global information, that is, the pheromone increment $\Delta\tau_{ij}^k$ is only related to the whole length of the tour L_k, and has no direct relationship with the distance between i and j d_{ij}. Therefore, the shorter L_k is, the faster the ant completes the tour within the same time and the more pheromone will be released on the path, which will stimulate more ants move toward the direction. But in the deflection prediction model, it is the deflection that determines the ants' selection. The pheromone increment $\Delta\tau_{ij}^k$ is only related to the each step of the ants (local information), and has nothing to do with the route of the tour. So Ant quantity system (Eq. (7)) is suitable for updating pheromone in deflection prediction model. That is to say, as soon as ant k moves from i to j, it will release some quantity of pheromone on the edge immediately. We define

$$\Delta\tau_{ij}^k = \frac{L_j - L_i}{L_{sum}} Q \tag{13}$$

Eq. (13) shows that the quantity of pheromone is related to the difference of the cumulative deflection between j and i.

Taking the release and evaporation into account, the pheromone on edge (i, j) can be updated by the rule of Eq. (4) and Eq. (5).

3.4 The Parallel Searching Mechanism of Ant Colony.

Firstly, all the points should be divided into several sections according to the deflection of the bridge, and large deflection part can be subdivided. We place a number of ants within each section. The ants' initial position is randomly determined, but we should try to make them distributed evenly in each section. Each ant is equipped with a tabu list for storing the points that the ants have reached. Then make all the ants search path parallel and update pheromone respectively as the rules mentioned above so that the algorithm can be evolved in a self-organizing manner. If the tabu list is full, but has not met the stopping criteria yet, then the tabu list should be emptied and enter the next iteration. The stopping criteria can be adjusted appropriately according to the number of points and the prediction accuracy. When the number is small, the stopping criteria can be set for all the ants having gathered to one point; otherwise, the stopping criteria can be set for a pre-specified number of ants having gathered to one point. Besides, we can also terminate the search by setting the maximum number of iterations.

3.5 Deflection Prediction Mechanism.

When the searching program terminates, the arrival frequency of the ants is recorded on each of the points. As the ants tend to move toward the large deflection points, we can estimate the deflection deformation of each point through establishing the correspondence between the frequency and deflection deformation. Comparing predicted and measured data, if the prediction deviation is within the allowable range, the predicted data can be used as known data, with which we can continue to predict subsequent deflection of the bridge. If the prediction deviation is out of the allowable range, we have to consider repositioning the initial position of ants, modifying the information function, heuristic function, pheromone updating rule, or changing some of the parameters of the model.

3.6 Parameters Selection of the Model[9-11].

In the deflection prediction model, the following 5 parameters will directly affect the global searching ability and convergence rate of the algorithm: m, Q, α, β, and ρ. Since the vast parameters space of the algorithm and the correlation between each parameter, how to determine the optimal combination of the parameters for the best performance has always been a very complex optimization problem. There is no perfect theory currently. Based on previous experience, referring to the "three-step" approach, we determine the parameters through repeated trials at a certain step.

1) Determine ant colony size m. When multiple ants are used, the global searching ability and stability of the algorithm can be improved, but in the meanwhile, the quantity of pheromone on each path will vary relatively uniform, and the positive feedback of the algorithm will be weaken. On the contrary, when few ants are used, the convergence rate of the algorithm can be improved, but the searching randomness of the algorithm will be greatly weaken, and the global searching ability and stability of the algorithm will be reduced, which will lead to premature stagnation. According to the

literature, the algorithm is satisfying when the ratio of ant colony size to city size is about 0.6 to 1, which can provide guidance for determining the ant colony size m .

2) Coarse adjustment, which means to adjust the parameter Q , α and β , whose range is relatively large. Q is a system initialization parameter, which reflects the initial energy of the system. If set too small, it will lead to bad performance because the quantity of pheromone on some path will become extreme small, and affect the searching ability of the algorithm. Thus, the value of Q is generally determined depending on the scale of the problem. In the latter case, we set $Q \in (0,10000]$, set the step as 500, and conduct repeated trials so as to obtain the optimal value of Q .

α controls the global intelligence (determined by $\tau_{ij}(t)$) and β controls the local heuristic (determined by $\eta_{ij}(t)$). A larger α will make the algorithm pay too much attention to the initial random fluctuations, resulting in poor calculation results, but the convergence rate is slower if it is too small. A larger β allows the ants to quickly select a better path and speed up the convergence of the algorithm, but it weakens the randomness of the search and makes the algorithm fall into local optimum. In the latter case, we set α and β range from 0 to 5, and set the step as 0.5.

3) Fine adjustment. It means to determine the parameter ρ , whose range is relatively small. The value of ρ should also seek a balance between the global searching ability and convergence rate. In the latter case, we set ρ range from 0 to 1, and set the step as 0.05, and conduct repeated trials so as to get the optimal value.

Through the above "three-step" approach, we can acquire a more balanced combination of the 5 parameters through repeated trials.

3.7 Main Flow of the Model.

Fig. 2 shows the main flow of establishing the Ant Colony Algorithm prediction model of bridge deflection.

4. Case Study

A cross-river bridge, with a total length of over 2000 meters, is a cross-river traffic artery. The deflection of the bridge has been monitoring regularly since 2003. 54 monitoring points were emplaced along both the upstream and downstream sides symmetrically. In the first period, it was monitored every 3 months, and every 6 months 4 years after. Combined with Ant Colony Algorithm above and existing monitoring data we write the prediction program based on Visual Basic. Through repeated tests, we ultimately determined that the deflection prediction algorithm had the most balanced performance when the values of the 5 parameters were: $m = 50$, $Q = 6000$, $\alpha = 2.5$, $\beta = 3.5$, $\rho = 0.40$.In the first stage, the deflection of 2006 was predicted by the two-period monitoring data of 2005, while in the second stage, the deflection of 2018 was predicted by the two-period monitoring data of 2017. Here we just list and analyze the prediction performance (shown in Table 1) of the 14 points in the midspan due to space limitation of the paper.

It is observed from Table 1 that there is not much deviation between the predicted values and measured values. In the predicted result of 2006, the absolute deviation maximum is 1.9 mm, the minimum is -1.2mm, the average value is only 0.0 mm, and the average of absolute value of absolute deviation is 0.7mm, while the relative deviations are between 0.0% and 33.3% with an average of 9.7%. In the predicted result of 2018, the absolute deviations are between-2.3mm and +2.7mm with an average of 0.2mm, and the average of absolute value of absolute deviation is 1.2mm, while the relative deviations are between 0.9% and 23.4% with an average of 9.0%.The prediction results of two different stages show that model has good applicability and high prediction precision.

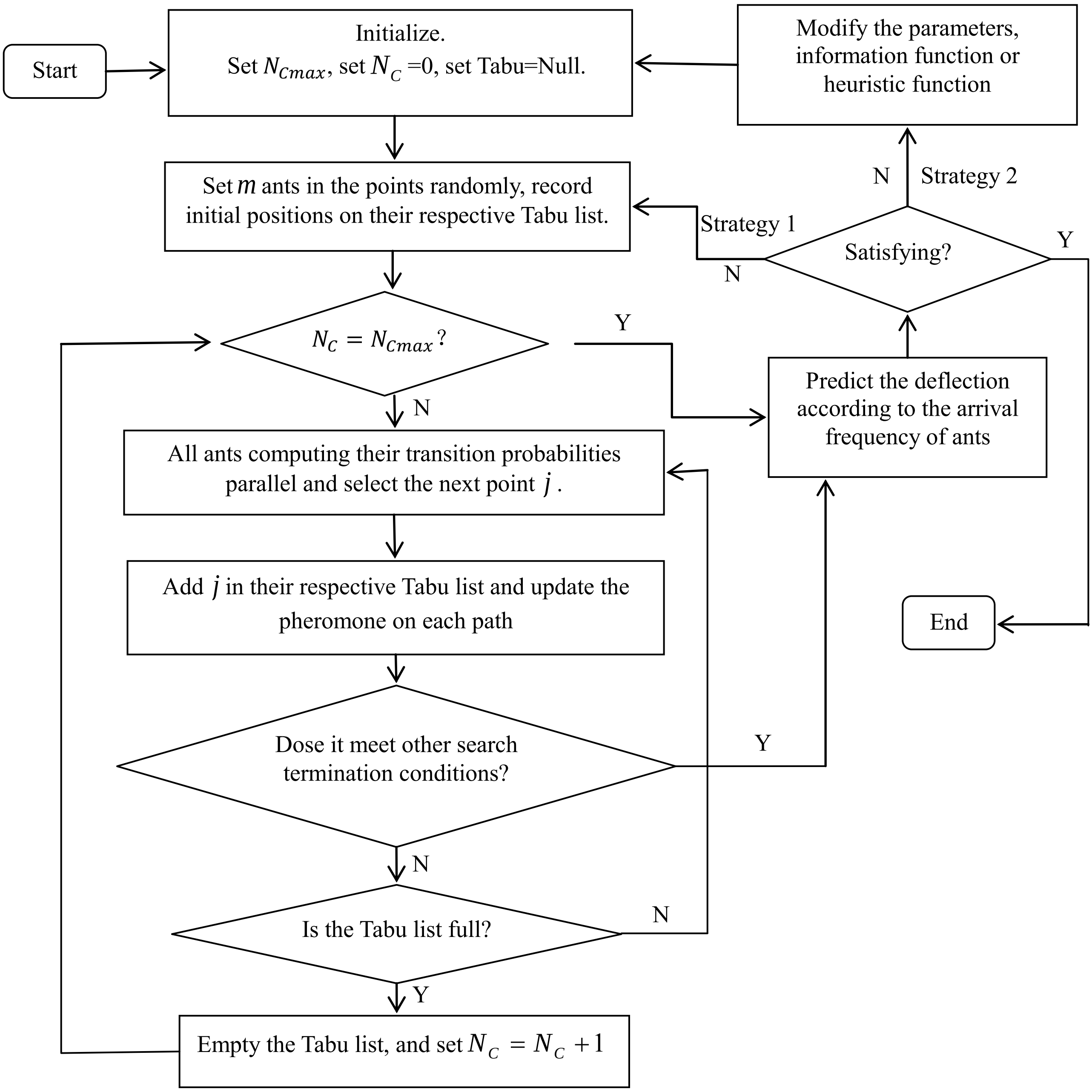

Fig. 2 Flow chart of Ant Colony Algorithm prediction model of bridge deflection

In order to reflect the difference between the predicted and measured deflection (or called actual deflection) of the bridge clearly, the comparison curves representing the absolute deviations that are enlarged 10 times are plotted (shown in Fig.3). As is seen from Fig.3 the predicted deflection trend is very consistent with the actual deflection, which indicates that the deflection deformation of the bridge is in accord with its internal deformation rule, and there is no abnormal condition, from which we can infer that the structure of the bridge is in good condition. Meanwhile, we can also see from Fig.3 that the actual deflection is slightly larger than that of the predicted at the settlement trough S20~S24 in 2018, which indicates the deflection deformation of this part is still weakly developed, while at the settlement trough S28~S33, the situation is slightly different. The actual deflection is slightly smaller than that of the predicted at S28~S31, which indicates the deflection deformation of this part tends to be stable gradually, but it appears the opposite situation at S32~S33. So it can be inferred that the deflection of the two settlement troughs in the midspan of the bridge is still not converge completely, and it may need further monitoring and analysis to ensure its safety.

Table 1 Deflection prediction performance of upstream side points in the midspan unit: mm

No.	2006 Predicted values	2006 Measured values	2006 Absolute deviations	2006 Relative deviations (%)	2018 Predicted values	2018 Measured values	2018 Absolute deviations	2018 Relative deviations (%)
S20	-4.9	-4.2	-0.7	16.7	6.1	5.6	0.5	9.1
S21	-10.0	-10.4	0.4	3.8	-28.0	-29.4	1.4	4.8
S22	-24.2	-26.1	1.9	7.3	-85.1	-85.9	0.8	0.9
S23	-20.2	-19.4	-0.8	4.1	-72.8	-73.7	0.9	1.3
S24	-9.1	-8.5	-0.6	7.1	-18.3	-18.7	0.4	2.0
S25	-2.9	-3.1	0.2	6.5	-6.2	-5.1	-1.1	21.7
S26	-4.2	-3.3	-0.9	27.3	-4.4	-3.9	-0.5	13.1
S27	-3.4	-3.8	0.4	10.5	-4.6	-5.4	0.8	15.6
S28	1.2	1.2	0.0	0.0	7.9	9.4	-1.5	16.3
S29	-8.0	-9.1	1.1	12.1	-23.7	-21.4	-2.3	10.8
S30	-33.0	-31.8	-1.2	3.8	-92.1	-94.8	2.7	2.8
S31	-42.6	-43.2	0.6	1.4	-129.4	-127.6	-1.8	1.4
S32	-18.7	-18.4	-0.3	1.6	-53.5	-54.9	1.4	2.5
S33	0.8	0.6	0.2	33.3	4.0	3.2	0.8	23.4

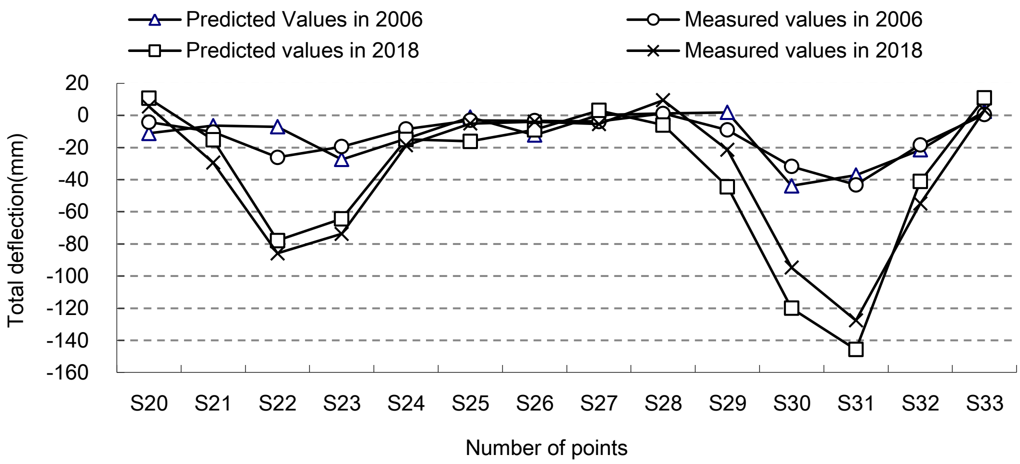

Fig. 3 Comparison curves between predicted values and measured values

It should be noted that in addition to the midspan of the bridge, for the deflection prediction performance of other parts, the absolute deviations of all points is within ±3.0 mm, which further indicates that the model has strong applicability and reliability. In addition, it should be pointed out that when predicting the deflection of 2018, we found that the precision of using the information function defined by Eq. (10) and heuristic function defined by Eq. (11) is slightly higher than using Eq. (10) and Eq. (12), which is probably because Eq. (12) only considers the spatial difference of points without considering the time characteristics, while Eq. (11) takes both the space and time characteristics into account, and it can reflect the deflection development of the bridge more fully, so the prediction performance based on Eq. (11) is better.

5. Conclusions and Discussion

1) Based on the TSP Ant Colony Algorithm model, combined with the actual deflection of a bridge, the Ant Colony Algorithm model of deflection prediction has eventually been established after determining the path selection mechanism, reasonable information function, heuristic function, pheromone updating rule, ant-searching and ant-predicting mechanism. The Ant Colony Algorithm model is tested by deflection prediction from two different stages of a bridge. The results show that the ACO model has high precision as well as strong applicability and reliability.

2) Ant Colony Algorithm self-organization mechanism makes it possible that the model does not require detailed understanding of all complex factors that affect the deflection deformation of bridges, just express the issue of deflection prediction as a kind of standardized format. Each individual ant has the ability of "exploration" and "exploitation", and determines its movement direction according to pheromone. At the same time, the ants update the pheromone on their respective paths according to a certain pheromone updating rule. Then the behavior of the ant colony (i.e. the direction of movement) can be planned out from an overall perspective. Again and again, they can get the optimal solution.

3) The Ant Colony Algorithm model we build in this paper is simple, effective, less data required, and easy to program. What's more, the model can be used for the deflection prediction of other bridges as long as we modify the model slightly depending on the circumstances.

4) In the paper, we try to explore the prediction of bridge deflection based on Ant Colony Algorithm and achieve some success, but there are still many problems such as the definition of information function, heuristic function, the formulation of pheromone updating rule, and the establishment of parallel searching and deflection prediction mechanism. Their forms may be relatively simple, and they may not reflect the correlation between the points, so there is still large space for improvement.

5) In the Ant Colony Algorithm model, the positive feedback effect, convergence rate, and global searching ability greatly depend on the parameters. We get a relatively balanced combination of the parameters through repeated trials, which obviously not only reduce the efficiency of the algorithm greatly, but also hinder the further popularization of the algorithm. Therefore, how to obtain the optimal combination of the parameters also deserves further exploration and research.

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