

# An Analysis on Optimizing Full and Part Routes of Urban Rail Transit under Peak Times

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**Abstract.** The cross-section passenger flow of urban rail transit has a large difference in space during the peak times and the operation mode of full and part routes should be adopted. By analyzing the passenger flow in the peak times, aiming at the minimum cost of the enterprise and the passenger waiting time, the multi-objective optimization model can be constructed. By introducing the time value coefficient, the passenger waiting time will be converted into the cost of generalized waiting time. The multi-objective optimization mode is transformed into a single-objective optimization model. The genetic algorithm is used to solve this problem and calculates the frequency of trains. Finally, the validity of the model is verified by an example.

**Keywords:** Urban rail transit, Full and part routes, Multi-objective planning, Genetic algorithm.

## 1. Introduction

Urban rail transit aims to improve passenger service performance by compiling the timetable and operation plan. It usually adopts a single operation mode and uses the maximum cross-section passenger flow as the basis of departure frequency, so that the full-load ratio in space is obviously imbalance. The full and part routes operation mode is a two-way nesting operation, in which one runs the whole line to form a large intersection, and the other runs part route to form a small intersection. In the operation process, full and part routes can be adjusted according to the different transportation capacity, to adapting the actual transportation demand, improving the efficiency of the vehicles and reducing the transport costs.

In terms of theoretical research, Yan Sun et al [1] set up a mathematical optimization model with the maximum train full-load ratio and the minimum number of operating hours to solve the lower bound of the number of vehicles in the multi-cycle section. Baohua Mao et al [2] conducted a theoretical study on the network-model of urban rail transit and analyzed the travel modes and path selection factors of passengers in various regions of the city and summarized the heuristic flow assignment method. Furth and Wilson [3] summarized four methods for train departure interval, and also established a nonlinear optimization model for determining the frequency of subway departures. Yuanyuan Wang et al [4] considered the cost of passenger travel and the minimum operating cost of

the enterprise, building part and full routes optimization model, and determined the train operation plan. Banks [5] proposed an optimization model for determining the bus departure interval. According to the condition that the demand function is unknown and the starting frequency depends on the demand. Oldfield and Bly [6] proposed a complex model to calculate the optimal vehicle size. The model assumes that passenger demand changes with travelling costs and that vehicle unit operating costs are linear with vehicle size.

The full and part routes meet the demand of passenger on the part route, but relatively it increases the waiting time of passenger on the other part of the route. It improves the efficiency of the vehicle and reduces the cost in the whole. In the implementation of the full and part routes planning scheme, it is also necessary to have certain flexibility, so that the transportation capacity should adapt the change of the passenger flow. In this paper, the frequency of trains and the number of serviceable vehicles are decision variables, and the full and part routes are constructed to balance the operational efficiency of enterprises with the passenger service performance.

## 2. Problem Description

### 2.1 Line Composition.

A line is shown in Fig. 1. There are  $N$  stations in the line. The train runs from the  $1$  station to the  $N$  station and it is in the direction of the uplink, with  $d=1$ , and vice versa, with  $d=2$ . The train in the full route runs the whole way, and the train returns from the  $1$  station to the  $N$  station; the part route runs part of the line, and the train returns from the  $P$  station to the  $M$  station.

The operation of the full and part routes are complicated, and the location of turn-back station will affect the capability of the whole line, if passenger flow at the starting point is very large, it will affect the passengers at other stations on part route, causing a backlog of passenger flow at the station. If the number of passengers at the terminal is too large, it will increase the burden of evacuating passengers and prolong the working hours. Therefore the selection of the turn-back station on the full and part routes should be coordinated with the capacity of the station and transportation demand. Generally, the turn-back station is set at the station where the passenger flow is half of the maximum passenger flow.

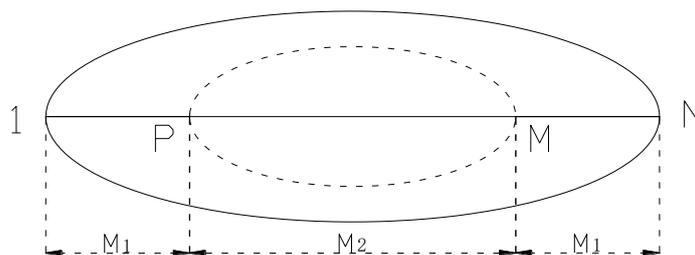


Fig. 1 Schematic diagram of the full and part routes

### 2.2 Basic Assumptions and Parameter Description.

Based on the characteristics of the line, this paper proposes the following assumptions:

(1) Trains on full and part routes have the same speed, and their turn-back operations are independent;

(2) Passengers arrive evenly, and the arriving passengers can take the first train without detention;

(3) Sharing ratio of passengers on the coincident route is determined by the departure frequency on each route;

(4) The formation of trains on full and part routes can change flexibly according to passenger flow;

(5) The vehicles of full and part routes are used independently.

Symbol definition:

$\lambda_{ik}$ : the number of passengers who gets on at the  $i$  station and gets off at the  $k$  station;  $\lambda_i^1$ : the number of passengers who gets on at the  $I$  station and gets off at the  $i-I$  station;  $\lambda_i^2$ : the number of passengers who gets on at the  $i$  station and gets off between  $i+I$  station and  $N$  station;  $f^1$ : single route downlink departure frequency;  $f^2$ : single route uplink departure frequency;  $f_1^1$ : full route downlink departure frequency;  $f_2^1$ : part route downlink departure frequency;  $f_1^2$ : full route uplink departure frequency;  $f_2^2$ : part route uplink departure frequency;  $N_0$ : maximum number of serviceable vehicles;  $n_1$ : the number of vehicles in a train on full route;  $n_2$ : the number of vehicles in a trains on part route;  $\eta_m$ : maximum full-load ratio of vehicle;  $\eta_h$ : maximum full-load ratio of vehicle at  $h$  intersection;  $L_1^d$ : the length of full route in direction  $d$ ;  $L_2^d$ : the length of part route in direction  $d$ ;  $t_{\max}$ : maximum departure interval time;  $t_{\min}$ : minimum departure interval time;  $M_{1,r}$ : passengers get on the train on full route and pass the  $r$  section;  $M_{2,r}$ : passengers gets on the train on part route and pass the  $r$  section;  $\sum t_l$ : the time of a whole operation.

### 3. Model Construction

#### 3.1 Objective Function.

In this paper, the optimization model is established with the minimum passenger waiting time and the minimum cost of the enterprise operating. The passenger waiting time is mainly determined by the passenger waiting at the station. In Figure 1, the passengers who are located in the  $M_1$  section can only take trains which are on the full route. The waiting time is only affected by departure frequency  $f_1$ . If passengers who are located in the  $M_2$  section arrive at the  $M_1$  section, they can only take trains on full route. The waiting time is only affected by departure frequency  $f_1$ . In the  $M_2$  section of the part route, one can take the trains on full route or part route. The waiting time is affected by both  $f_1$  and  $f_2$ . From this, the average waiting time for passengers on single route is:

$$t_w = \frac{1}{2} \left( \sum_{i=1}^n \frac{\lambda_i^1}{f^1} + \sum_{i=1}^n \frac{\lambda_i^2}{f^2} \right) \quad (1)$$

The passenger waiting time on full and part route is:

$$t_w = \frac{1}{2} \left[ \sum_{i=1}^{s_0-1} \frac{\lambda_i^1}{f_1^1} + \sum_{i=s_1}^{n-1} \frac{\lambda_i^1}{f_2^1} + \sum_{i=s_0}^{s_1-1} \left( \frac{\lambda_i^1}{f_1^1 + f_2^1} + \frac{\lambda_i^1}{f_1^1} \right) + \sum_{i=2}^{s_0} \frac{\lambda_i^2}{f_1^2} + \sum_{i=s_1+1}^n \frac{\lambda_i^2}{f_2^2} + \sum_{i=s_0+1}^{s_1} \left( \frac{\lambda_i^2}{f_1^2 + f_2^2} + \frac{\lambda_i^2}{f_1^2} \right) \right] \quad (2)$$

The operating costs of the enterprise mainly include the cost of train operation and the labor salary, which can be measured by the distance of vehicles travelling and the amount of serviceable vehicles. Therefore, the enterprise operation cost can be decomposed into two parts: vehicles travelling cost and serviceable vehicles cost.

The distance of vehicles travelling on single route is:

$$S = L_1 n (f^1 + f^2) \quad (3)$$

The distance of vehicles travelling on full and part route is:

$$S = f_1^1 L_1 n_1 + f_2^1 L_2 n_2 + f_1^2 L_1 n_1 + f_2^2 L_2 n_2 \quad (4)$$

The amount of serviceable vehicles is:

$$N = \frac{\sum t_l}{t} m \tag{5}$$

The objective function of the enterprise operating cost and passenger service performance can be expressed as:

$$\text{min } z = \varphi_1 t_w + \varphi_2 S + \varphi_3 N \tag{6}$$

### 3.2 Constraints.

The constraints required to be met by operation plan are as follows:

$$\frac{60}{f} \leq t_{m a} \tag{7}$$

$$\frac{60}{f} \geq t_{m i} \tag{8}$$

$$\eta_h < \eta_m \tag{9}$$

$$1 \leq \xi < \xi \leq N \tag{10}$$

$$N_1 + N_2 \leq N_i \tag{11}$$

$$f_{h,r} = \max \left\{ \frac{\beta_1 \sum_{od \in M_{1,r}} q_{od} + \beta_2 \sum_{od \in M_{2,r}} q_{od}}{C n_h \eta_m} \right\}, h = 1, 2; \beta_1 = \begin{cases} 1, & h = 1 \\ 0, & \text{else} \end{cases}; \beta_2 = \begin{cases} f_1 / (f_1 + f_2), & h = 1 \\ f_2 / (f_1 + f_2), & \text{else} \end{cases} \tag{12}$$

(2-7) is a constraint for frequency; (2-8) is a constraint for passing capacity of the line; (2-9) is a constraint for full-load ratio; (2-10) is a constraint for part route section; (2-11) is a constraint for the amount of vehicles; (2-12) is a constraint for passenger flow.

### 3.3 Model Solving.

In this paper, the optimization model of full and part route is a multi-objective nonlinear optimization model, in which the decision variable is the frequency of the full and part route including four discrete variables. The objective function is the sum of the cost of the passenger waiting time  $t_w$ , the vehicle traveling distance  $S$  and the amount of vehicles used  $N$ . However, waiting time and enterprise cost are different in dimension. The calculation cannot be directly performed. Therefore, we can introduce the time value coefficient  $\delta$  and convert it into a generalized waiting time fee. The total waiting time fee of the passenger is:

$$\delta = \frac{S}{a \times b} \tag{2-13}$$

Where:  $S$  is the average monthly income of city passengers, yuan/day;  $a$  is month working days, calculated according to 22 days, days;  $b$  is daily working time, calculated by 8h, h.

The genetic algorithm is a global random search and optimization method developed by Holland [6] in 1975 by imitating the biological evolution mechanism of nature. Due to its wide versatility and efficient computation, it is currently widely used to solve optimization problems in the transportation field.

The result accuracy of this problem is  $\epsilon=0.1$ . According to the composition of the result  $x$ , it can represent any kind of operation plan. According to the minimum and maximum interval of train departure, it can be determined that the range of  $x$  is  $[10, 30]$ . It is divided into  $(30-10)/0.1 = 200$  copies, because the binary code is used, the actual required bits of code is at least 8 bits. Binary strings 00000000 and 11111111 are used to represent the two endpoint values of the interval. Respectively, we set enterprise benefit cost as the adaptive grade. The fitness calculation formula is:

$$f(x) = \varphi_1 t + \varphi_2 S + \varphi_3 \tag{2-14}$$

Use the roulette method to select filial generation. And the probability that each individual is selected is:

$$p_j = \frac{f(x_j)}{\sum_{j=1}^n f(x_j)} \tag{2-15}$$

Where,  $f(x_j)$  is the sufficiency of the  $j$  individual;  $\sum_{j=1}^n f(x_j)$  is total sufficiency.

In the process of cross-variation, the chromosome crossover probability  $p_c$  is determined. Two parental chromosomes generate two filial chromosomes by exchanging chromosome fragments, and meanwhile generate random numbers  $a \in (0, 1]$ . If  $a \leq p_c$ , in the case of single-point crossover, two individuals are randomly selected for crossover. As the Figure2 shows:

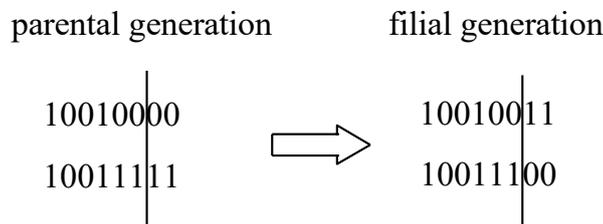


Fig. 2 Recombination of gene segment

The selected chromosome should be mutated, and if the sufficiency score of filial chromosome is greater than the sufficiency score of parental chromosome, the parental chromosome is replaced, as Figure 3 shows. Cycling through the algorithm until the algorithm reaches the fitness expectation or maximum evolution algebra, the algorithm ends. And the final chromosome is decoded.

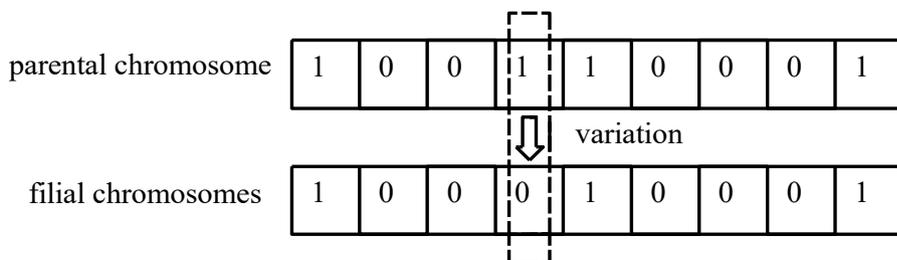


Fig. 3 Single point mutation operation

#### 4. Model Verification

##### 4.1 Examples and Parameter Settings.

Taking Tianjin Rail Transit Line 4 as an example, there are 30 stations on the whole line. The transfer station is set at the 6th and 24th station. The length of each platform meets the needs of 8

groups of B-type vehicles, and the maximum passenger capacity of each vehicle is 240. The length of each interval is shown in Table 1:

Table 1 The length of each interval

Interval number	1	2	3	4	5	6	7	8	9	10	11
Length(km)	1.6	0.9	1.8	2.1	2.3	1.7	2.8	1.6	2.4	2.9	1.6
Interval number	12	13	14	15	16	17	18	19	20	21	22
Length(km)	2.2	1.8	2.6	2.4	1.9	2.3	1.0	2.1	1.6	1.9	2.4
Interval number	23	24	25	26	27	28	29				
Length(km)	1.6	2.1	2.3	2.4	1.8	1.4	2.2				

By analyzing the passenger flows of each section in morning peak (8:00-9:00) and evening peak (18:00-19:00) and optimizing the frequency on full and part route, we can improve the benefit of enterprises. In order to study the impact of the frequency to benefits, it is stipulated that the trains on full route are fixed at 8A, and the trains on part route are fixed at 6A. Compare the single route and the full and part route, as shown in Table 2:

Table 2 Passenger flow in morning peak time

Section number		1	2	3	4	5	6	7	8	9	10
Passenger flow (ten thousand/h)	Uplink	0.15	0.29	0.34	0.57	2.28	2.03	1.89	1.67	2.27	3.21
	Downlink	0.19	0.26	0.36	0.46	2.17	2.14	1.76	2.56	1.89	2.45
Section number		11	12	13	14	15	16	17	18	19	20
Passenger flow (ten thousand/h)	Uplink	1.48	2.42	3.14	2.55	2.64	2.34	3.05	2.51	2.74	1.85
	Downlink	1.67	1.87	2.47	2.34	1.75	2.84	3.21	2.34	2.43	1.48
Section number		21	22	23	24	25	26	27	28	29	30
Passenger flow (ten thousand /h)	Uplink	0.48	0.98	1.25	1.48	0.57	1.68	0.85	0.74	0.35	0.17
	Downlink	0.74	1.02	2.01	1.65	0.85	1.47	0.57	0.62	0.24	0.20

Table 3 Passenger flow in evening peak time

Section number		1	2	3	4	5	6	7	8	9	10
Passenger flow (ten thousand/h)	Uplink	0.21	0.32	0.37	0.64	2.32	2.24	1.74	1.79	2.37	3.17
	Downlink	0.18	0.28	0.36	0.76	2.41	2.47	1.94	2.43	2.03	3.22
Section number		11	12	13	14	15	16	17	18	19	20
Passenger flow (ten thousand/h)	Uplink	1.57	2.38	3.24	2.57	2.57	2.27	3.34	2.74	2.85	2.03
	Downlink	1.74	1.96	2.79	2.65	1.85	2.96	3.27	2.62	2.35	1.74
Section number		21	22	23	24	25	26	27	28	29	30
Passenger flow (ten thousand/h)	Uplink	0.86	1.48	1.86	1.54	2.33	1.98	0.74	0.84	0.29	0.28
	Downlink	0.69	1.58	2.23	1.87	1.08	1.47	0.89	0.77	0.34	0.36

#### 4.2 Calculation Results.

It can be seen from Table 2 and Table 3 that in the morning and evening peak times, the difference between the maximum and minimum section passenger flows is 30,600 and 33,400

respectively, while in the normal times, the difference between the maximum and minimum section passenger flows is 7,900, indicating the peak time are more suitable for full and part routes operation mode, while the normal times is more suitable for the single route operation mode, and the transfer station is set at the 4th and 22nd stations. According to the model, the results are as follows:

Table 4 Operation plan in peak and normal times

Turn-back station	Operating time	Mode	Number of vehicles		Frequency(pair/h)	
			Full route	Part route	Full route	Part route
4,22	Morning peak time	Full and part routes	8A	6A	15	10
	Evening peak time	Full and part routes	8A	6A	17	10
	Normal time	Single route	8A	-	12	-

Table 5 optimized results of operation plan

Operating time	Mode	Passenger waiting time (h)	Vehicles travel length (km)	Number of vehicles	Objective function	Rate of change of passenger waiting time	Rate of change of vehicle travel length	Rate of change of vehicles	Rate of change of the results
Morning peak time	Single route	2785.2	15479	382	2928	-	-	-	-
	Full and part routes	2945.3	13296	321	2745	+5.7%	-14.1%	-15.9%	-6.3%
Evening peak time	Single route	2341.9	18419	436	3241	-	-	-	-
	Full and part routes	2651.6	16542	374	2964	+9.6%	-10.2%	-14.2%	-8.5%

In the morning and evening peak time, the full and part routes operation mode was adopted. The trains on full route are grouped at 8A, and the trains on part route are grouped at 6A. Compared with the single route, passenger waiting time is increased by 5.7% and 9.6% respectively. However, the objective function has been optimized significantly, it can be decreased by 6.3% and 8.5% respectively.

### 5. Conclusion

This paper studies the frequency of full and part routes under peak times to balance the passenger service level and the operational efficiency of enterprise, and obtains the following conclusions:

(1) Adopting full and part routes operation mode will increase passenger waiting time than single route operation mode.

(2) When determining the location of the transfer station, the length of part route section is too long or too short which may cause greater loss of passengers' interests. It cannot save the cost of enterprise and make both passengers and enterprises benefits maximized.

(3) When the location of the transfer station and the number of vehicles in a train are certain, adopting full and part routes operation mode can reduce the cost of enterprise than single route operation mode.

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