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Exchange of Majority in Opinion Evolution on Complex Networks

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***Abstract*—This study considers a voting as a kind of opinion dynamic. People talk to and influence each other on a network basis until the end of the voting. During the process, the majority of the votes may transfer between two parties. The structure of the voter's network significantly impacts the frequency of the exchanges and the length of time between them. Five typical complex networks are built in this study. Opinions evolve on these networks. The exchanges of majority are observed on all networks. The relation between network structure and the exchange features are investigated. A new method to predict the voting result is suggested.**

Keywords—complex network, spectral analysis, ordinary differential equation, opinion dynamics, voting

I. Introduction

In nature and society, many complex systems can be represented as graphs or networks. These networks are composed of nodes and links between them. Nodes represent the elementary units of a system, while links stand for the interactions between the nodes. Complex networks have been a research focus in both science and engineering in the last decade.

Recently, attention has been given to the opinion consistency problem in social networks. This is associated with whether and how long it takes for an individual's opinion to reach a consistent status [1-3]. Knowledge of opinion dynamics is relevant to the prediction of collective behavior such as voting and election [4-6]. Interestingly, the original purpose of studying opinion dynamics was to predict the final voting result in realistic social networks [7-13]. An important feature of voting is that voters can't always reach a consensus before the end of the election process. That is why most voting processes present a non-neutral result, for instance, in the political election or the talent show we don't expect all participants vote for one candidate, instead, the one gaining the majority of the voters wins.

Previous studies have achieved conclusions on opinion models from different areas. However, the quantitative methods are rarely studied. This is partly due to the fact that analytical tools like graph theory have not been well developed in directed networks, especially in directed-weighted networks. For example, there are no standard definitions for the algebraic connectivity, which is an important measurement for the network synchronization and its speed, while its counterpart for undirected graphs has been extensively used to study the synchronization problem.

In this paper, we develop a framework of investigating the linear relations between the behaviors on networks and their topology structures. A model of voting between two parties is proposed. The voting process before the investigation is investigated. The influence of topological structures on the voting result is studied by the spectral analysis. Initially, opinions are set to be uniformly distributed. In this scenario, our observations indicate that the average degree or density of networks have a fundamental influence on the final voting result.

The paper is organized as follows: in the first section we give a brief introduction. In the second section we propose a differential-equation based model of opinion dynamics on complex networks. In the third section we give a detailed description of the network establishment, and consequently, some statistical features of these networks. In the fourth section, we simulate the opinion dynamics of voting on different networks. In the fifth section, we test the model on a real-world network and analyze the results. In the last section we propose the conclusion.

II. Opinion Dynamics Based on Differential Equation

The opinion networked dynamical models studied here are based on opinion model of Curtis and Smith [14]. Two

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people hold initial opinions x_1^0 and x_2^0 respectively and persuade each other by their so-called influence power b_1 and b_2. The opinions evolve under the dynamic equations:

$$\dot{x}_1 = b_2 (x_2 - x_1) \quad (1)$$

$$\dot{x}_2 = b_1 (x_1 - x_2) \quad (2)$$

and both converge on the limit $\frac{b_{12}x_2^0 + b_{21}x_1^0}{b_{12}+b_{21}}$ when time increases.

It is expanded to an N-person model in this study and the more complicated structure of interaction between people is allowed to have an impact therefore. The connection between every two person i and j is A_{ij}, which is 1 if i talks to j, and 0 if not. The corresponding adjacency matrix of the network is $A = A_{ij}$. Every node i holds an opinion xi which evolves by time. Then we get the general description for the system here:

$$\dot{x}_i = \sum_{j=1}^{N} b_j A_{ij} (x_j - x_i), i = 1,2,\ldots,N, t \geq 0 \quad (3)$$

Now we define a diagonal matrix B = Diag (b) and an all one vector 1, to difine the matrix M, the Laplacian matrix:

$$M = BA - Diag(BA\mathbf{1}) \quad (4)$$

which contains the information about the adjacency matrix and the influence ability between nodes. Consequently, the system can be written as the linear vector equation:

$$\frac{dX}{dt} = MX \quad (5)$$

where X is the vector of opinion x_i. In equation (3), when b_i for all nodes are positive, then all nontrivial eigenvalues of M are negative. There exist x_∞ so that

$$\lim_{t\to\infty} |x_i(t) - x_\infty| = 0 \quad (6)$$

for all i. In this study, we define every b_i a random value in (0, 1) to guarantee the convergence of the system.

In the next section, we will set initial opinions uniformly distributed in (−1, 1) in each one of the networks. The nodes hold positive opinions and negative ones represent two confrontational parties respectively.

III. Generation of Complex Networks

In this Section, we will establish 5 presentative social networks, which have been frequently discovered as the structures of the real-life organizations. They are the Erdos-Renyi random graph (ER) [15], the small world network (WS) [16], and three classes of scale free (SF) networks, including the Barabasi Albert model (BA), the assortative scale free (ASSF) and the disassortative scale free (DSSF) [17-20]. Each network is generated by 1000 nodes. We build the SF networks with the same degree sequence.

The power-law, which defines the SF networks, has the discrete form $p(k) = k^{-\lambda}$ ($\lambda = 3$ in this study). The distribution of degrees can be obtained by normalization $p(k) \propto \frac{k^{-\lambda}}{\sum_{n=0}^{\infty}(n+k_{min})^{-\lambda}}$. In this study, to avert the possibility of the existence of an isolated cluster with two nodes, we define $k_{min} = 4$ and the degree sequence $D = \{d_1, d_2, \cdots, d_{1000}\}$ by the normalization we mentioned. The node i holds di numbers of prospective links initially. Every two prospective links connect to make an actual link randomly. The BA is established after all the half links are connected.

Then we generate the ASSF by rewiring the BA, as in Fig.1:

(i) At each step two links, so four nodes of the network are chosen at random;

(ii) The links between these nodes are rewired in such a way that one new link connects the two nodes with the smaller degrees and the other connects the two nodes with the larger degrees, as in Fig.1;

(iii) Repeat the two previous steps until a desired assortativity Pr is achieved:

$$Pr = \frac{\sum_{i=1}^{m} m_i^{-1} j_i^2 k_i^2 - [\sum_{i=1}^{m} m_i^{-1}(j_i + k_i)]^2}{\sum_{i=1}^{m} m_i^{-1}(j_i^2 + k_i^2) - [\sum_{i=1}^{m} m_i^{-1}(j_i + k_i)]^2} \quad (7)$$

where j_i and k_i are the degrees of the nodes at the end of the *i* th connection, $i = 1,2,\cdots m$. When Pr = 0, the probability that a link is connected to a node with a certain degree is independent from the degree of the attached node and the network is uncorrelated. On the contrary, Pr = 1 means the network is totally assortative.

To generate a DSSF network, two random links are chosen as for ASSF. However, the links are rewired to make sure that one link connects the two nodes with the smallest degree and the largest while the other connects the two remaining nodes. These steps are repeated till there is no change can be made during the rewiring. These algorithms ensure that the BA, ASSF and DSSF networks hold the same degree sequence.

For most of the real life networks, the assortativity Pr ranges from −0.3 to 0.3 [21]. Therefore, we set Pr = 0.3 for ASSF network and Pr=−0.3 for the DSSF network.

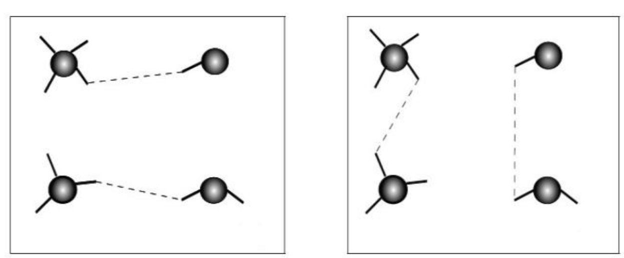

Fig. 1. Rewiring of links between nodes

The ASSF and DSSF algorithm: choosing two random connections as in the left figure, and rewiring them to guarantee one of them connects the two nodes with larger degrees while the other connects the two nodes with smaller degree as in the right figure. In contrary, the algorithm of DSSF is to take the step in contrary: choosing two random connections as in the right figure, and rewiring to connect the largest degree node and the smallest degree one. Then two remaining nodes are connected.

IV. Experiments and Analysis

In this study, all networks discussed are assumed to be connected networks. There is always a path between any two nodes i and j. The networks are all unweighted-undirected networks. Evolution and competition in the 5 networks are shown in the Fig.2-4. We use ODE45 in Matlab to solve the equation (3) in Section 2. In Fig.2-4, t denotes in the ODE45. If the time is long enough, all the opinions will converge at a value close to 0 [22,24,25]. Since the consensus can't be exact 0, we restrict the opinion values in 8 digits after decimal points, the opinions can converge at 0. In a real voting we divide those who hold the positive opinions and the negative ones into two opposing camps.

As can be observed in Fig (a1), (b1), (c1), (d1) and (e1), with the same initial opinions $X^0 = x_1^0, x_2^0, \cdot\cdot\cdot, x_N^0$, the various of topologies lead to a same consensus $x_s = x_1^t = x_2^t = \cdots = x_N^t$ at time t, which is the average of the initial opinions. Given the dynamical equation (5), at the moment t when consensus is achieved,

$$X = e^{Mt}X^0 = Pe^{\Lambda t}P^{-1}X^0 \qquad (8)$$

where $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \cdot\cdot\cdot, \lambda_N)$, the diagonal matrix of eigenvalues of M, and P has the eigenvectors as rows. As M is symmetric, all eigenvectors are orthogonal and the one corresponding to zero eigenvalue should be $(1, 1, \cdot\cdot\cdot 1, \cdot\cdot\cdot 1, 1)^T$. So,

$$Pe^{\Lambda t}P^{-1}I = e^{\Lambda t}(1,1,\ldots,1)^T = (1,1,\ldots,1)^T \quad (9)$$

for all t.

Also we have $P^{-1} = P^T$, so a simple proof is:

$$N\bar{X} = X^T I = (P^{-1}X^0)e^{\Lambda t}P^{-1}I = (X^0)^T I = N\overline{X^0} \quad (10)$$

where $I = (1,1, \cdot\cdot;1)_{N*1}$, it is not hard to find that $x_s = x_1^t = x_2^t = \cdots = x_N^t = \overline{X^0}/N$.

The identity consensus on all the networks may mislead the prediction of the result. In fact, most social collative behaviors end before the consensus comes. Suppose the opinions of voters are affected by their friends in an election between two parties (negative and positive). The results will be totally different if the election ends at different time in the 5 networks.

The Fig (a1) shows the opinion evolution process in ER. The opinions with positive values and negative values are considered as two opposing parties in a voting. Given enough time, the opinions will all become zero as illustrated in equation (8)-(10). We mark them as "consensus" in the graph since they have achieved the consensus and will not change any more. Meanwhile, the numbers of people holding positive and negative opinions at time t are recorded in the simulation. The party with more people is the leading party while the other is the opposing party. The ER supports the fastest opinion convergence among all the networks, which leaves little time for the exchange to happen. In (a2) no obvious exchange can be observed.

Fig (b1) and (b2) shows the convergence and the competition of the opinions in WS. The convergence on WS takes the longest time among all, without the most frequent exchanges.

In Fig (c1), (d1) and (e1) are the opinion evolution in BA, Assortative SF and Disassortative SF. The Fig (c2), (d2) and (e2) show the competition of opinions. The difference between the convergence speeds is too small to be observed. However, as recorded, it is the fastest in ASSF among the three and the slowest in DSSF.

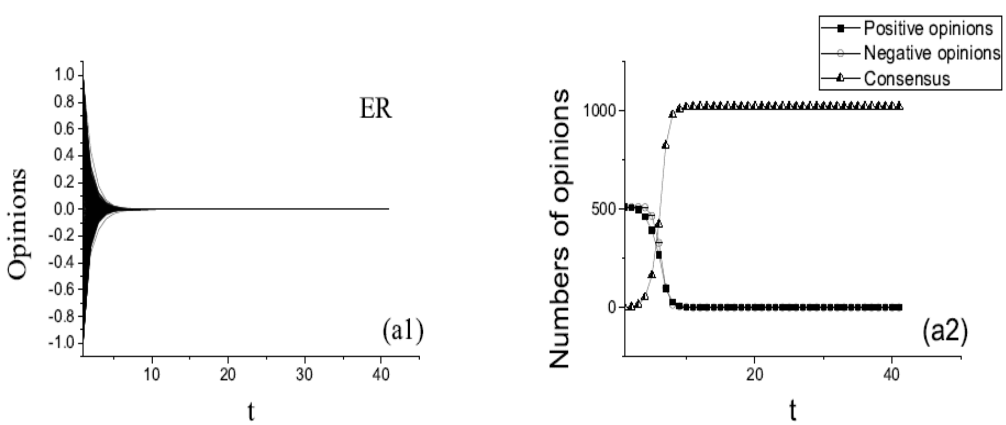

Fig. 2. The opinion evolution process in ER

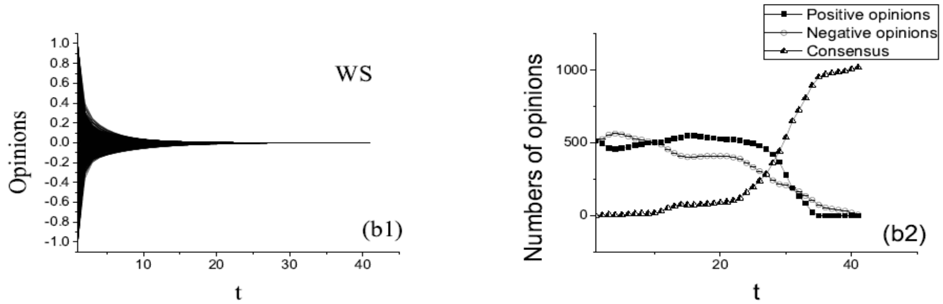

Fig. 3. The convergence and the competition of the opinions in WS

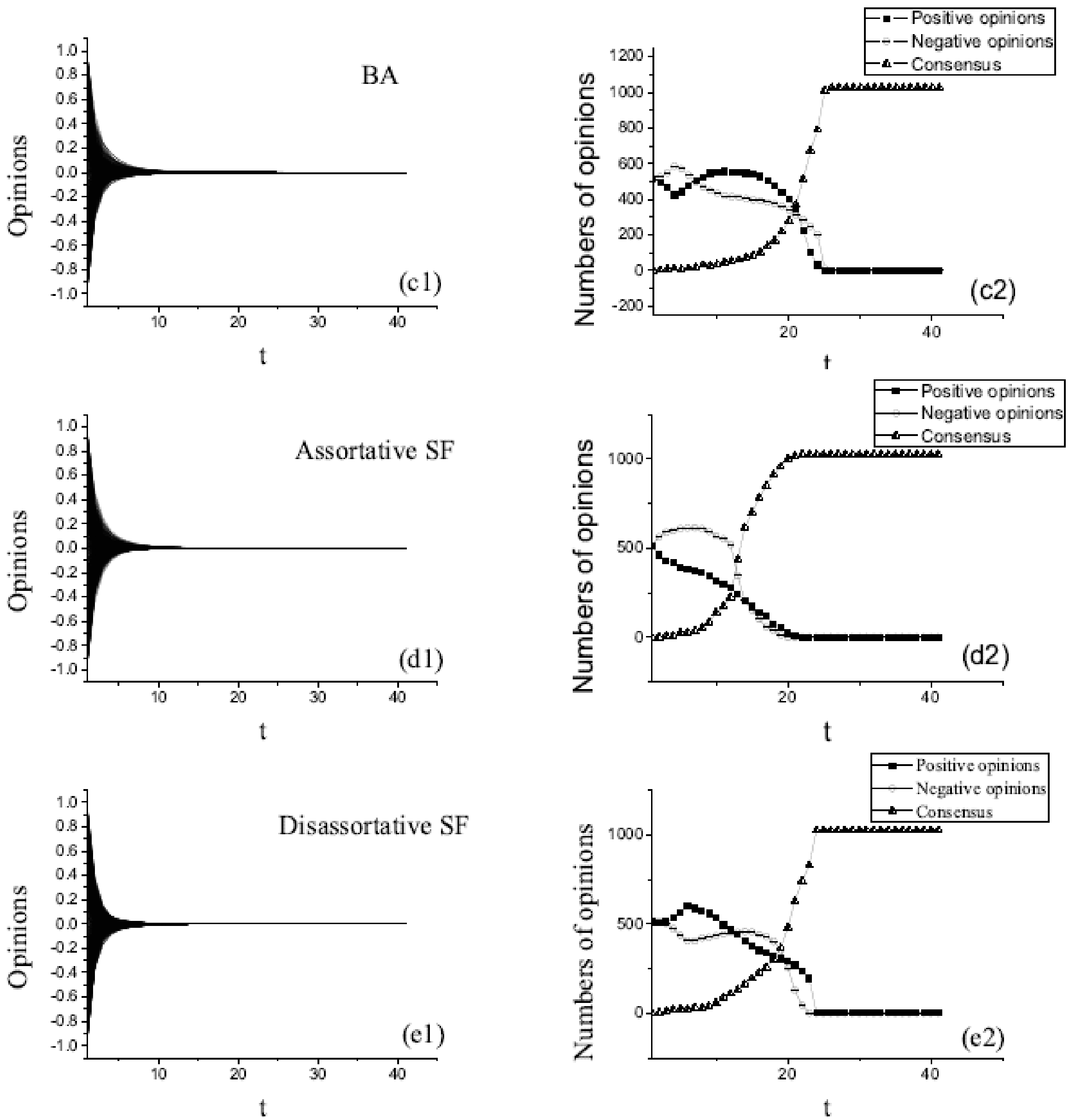

Fig. 4. The opinion evolution in BA

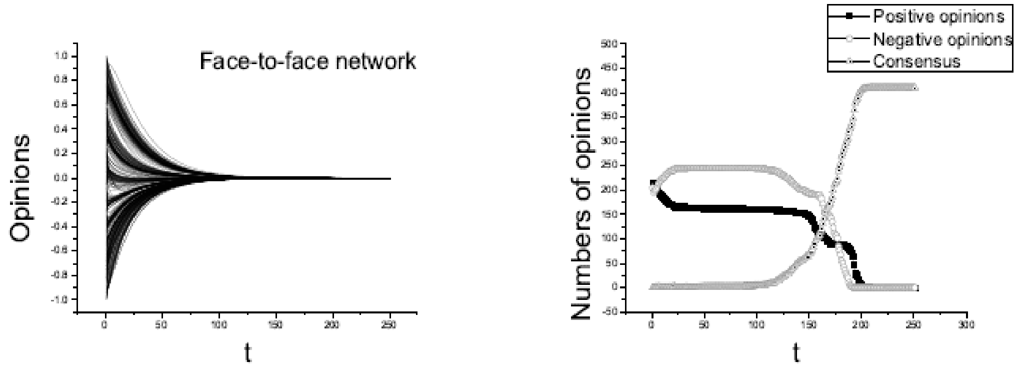

Fig. 5. Opinion evaluation in Face-to-Face Network

V. Application and Discussion

In [26], a behavioral network of face-to-face contacts in a long running museum exhibition was tracked. The network is consisting of 251 nodes and 5530 links. We give every node a random initial opinion between (−1, 1) and simulate the process of their talks. The degree correlation of the network is 0.755, which means it has strong assortativity. Some obvious exchanges of majority have been observed in this opinion process. If a decision or a voting is going to be made based on the face-to-face communications, when to collect the opinions and put an end to the event will significantly impact the final result. The results are illustrated in Fig. 5.

The opinion evolutions approach the same value of consensus with varying speed on the five networks. During the process, a significant phenomenon is the exchange of the majority between positive and negative opinions, which is caused by the local consensus before the global one [28, 29]. From Equation (8) we can get:

$$x_i^t = \sum_{j=1}^{N} P_{ij} e^{\Lambda_j t} x_j^0 \qquad (11)$$

So the difference of opinions between any two nodes i and m is:

$$|x_i^t - x_m^t| = \sum_{j=1}^{N} |P_{ij} - P_{mj}| e^{\Lambda_j t} x_j^0 \qquad (12)$$

$$\leq \sum_{j=1}^{N} |P_{ij} - P_{mj}| e^{\Lambda_j t} \qquad (13)$$

The equation (12) and (13) illustrate how the network topology impacts the process of opinion convergence. When time is long enough, all exponentials are zero and all the opinions get identity. During the process, the small eigenvalues ensure those nodes with similar projections on the eigenvectors to get synchronized eventually. In another word, the small communities in the network will achieve a local consensus before they arrive at the global consensus together. When the opinions of a community move together from positive to negative, or in contrary, the exchange of majority may happen.

In this study, we test how the network topology impacts the exchange frequency F of the leading opinions and the ratio of the longest leading time R in the synchronization process. Two statistical characters are chosen to represent the network topology: the clustering coefficient C and average path length L. Both of them will be adjusted by changing the average degree of the nodes K. In Fig.6 we illustrate how F and R change by the increasing of K, C and L. When K goes from 3 to 5, the exchange frequency F are all dropping in 5 networks, while the longest leading time is enhanced. The clustering coefficient C which has a positive correlation with K effects the F and R in the same way while the average path length L in the contrary way.

Since we don't observe clear exchanges in ER, we will focus on the other four networks. Because of the small world feature, the community structure is unclear in WS. There are not too many chances for the small groups of opinions to cross the zero together. For the same reason, it's harder to replace the majority in WS than in any other networks. So the WS holds relatively lower exchange frequency F and longer leading time R.

For the classes of SF networks, the community structures are clearer. Although ASSF and DSSF are generated from BA, the varying of assortativity causes different average path length L, clustering coefficient C in the three networks. The DSSF, with longer L and higher C, provides the most frequent exchanges with very short leading times. There are many small communities with similar sizes in the DSSF. Once an exchange happens, it's easy for the next one to replace it. The ASSF with small numbers of huge communities is in contrary.

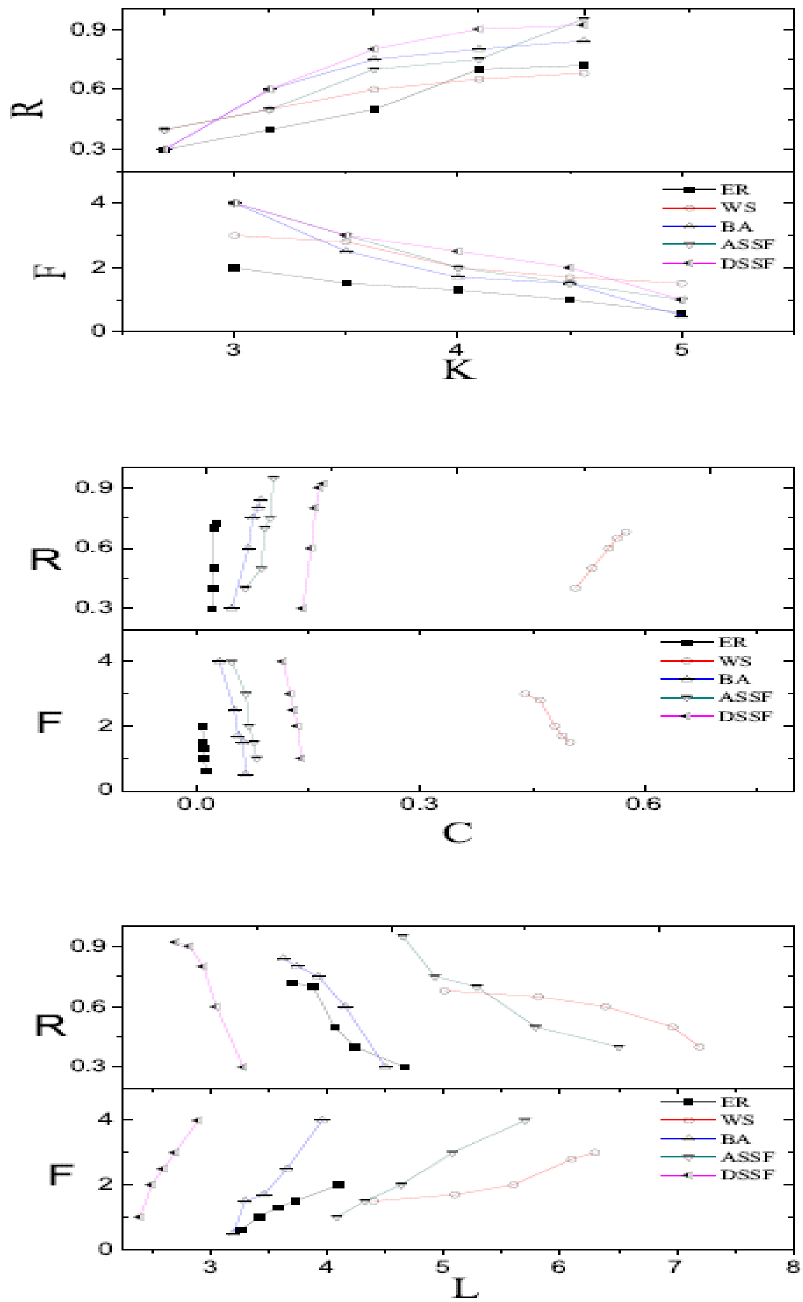

Fig. 6. How F and R change by the increasing of K, C and L

When increasing the clustering coefficient, the exchange frequency drops and the longest majority grows. The increasing of the average path length decreases the exchange frequency while increase the longest majority.

In real-life voting, the leading party may want to maintain the superiority while the opposing party may hope the next exchange to come soon. As can be observed in Fig.5, for the leading party, any behavior to prevent C from dropping or L from increasing may help, for instance, the establishment of small clusterings, the communications between large degree people and the isolated ones.

VI. Conclusion

In this study, we investigate the exchanges of the advantage between two parties in a voting. We consider all the people participating in as a network. Five typical networks are selected to describe the most possible structures of real-life networks. The opinion evolution during a voting is simulated on the five networks. We have found that the structure of the networks will significantly impact the frequency of the exchanges and the time length between every two exchanges. A new method to predict and manipulate a voting is suggested.

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